MAIN SEQUENCE AND H-SHELL BURNING: CONVECTION AND SEISMOLOGY

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Abstract: Uncertainties in the treatment of convection in lower main sequence stars and red giants remain a major problem in understanding the evolution of these stars and in making comparisons with observation. In intermediate mass stars, convective core overshoot near the main sequence and its implications on evolutionary lifetimes and luminosity functions present the main uncertainty. We discuss new efforts to improve on the mixing length theory, primarily with the help of 3D numerical simulations of convection, and how seismology (at the moment helioseismology, but eventually asteroseismology as well) provides new ways of exploring the structure of the outer layers, especially the temperature gradient in the superadiabatic region where the uncertainty is the greatest.

1 Introduction

In studying low mass stars near the main sequence and in their evolution toward the giant branch, the focus is increasingly toward problems of their internal dynamics. One wants to explore their rotational history, the associated internal mixing, and the importance of diffusion and its interaction with other mixing processes. These important problems are discussed by others at this Colloquium. In this talk we shall focus on convection and on the science of seismology which is beginning to provide new ways of testing our understanding of convection in the outer layers of the Sun and sun-like stars. Our poor understanding of stellar convection is a long standing problem: it affects the calculation of stellar radii of cool stars from first principles, with implications for the determination of the ages of the oldest stars in globular clusters and the fixing of the position of the giant branch (Hayashi line) in old stellar systems. It also affects the depletion of the light elements, magnetic activity cycles and some mass loss mechanisms. As befits the theme of this Colloquium, we shall primarily describe some current research and take a look forward into the future.

What is usually referred to as “convective overshoot” at the edge of the core is discussed in Section 2. Section 3 considers the problem of the depth of convective envelopes, the importance

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of understanding the highly superadiabatic transition region near the surface, and prospects from seismology to probe these layers in the Sun and similar stars. Finally Section 4 briefly describes the promise of seismology on sun-like stars.

2 Convective cores

The lifetimes of intermediate mass stars, say with masses in the range of 1.2 to $3M_\odot$, are still uncertain by a factor of up to fifty per cent because of the uncertainty of the size of the convective core. Understanding stellar evolution in open star clusters is important for galactic structure and stellar population studies. Even more importantly, these clusters are testbeds of stellar physics theory; understanding better the details of the physics of stellar evolution increases our confidence in the reliability of the theory in its most varied applications to stellar and galactic astronomy, and to cosmology.

It is generally believed that core “convective overshoot” takes place at the convective-radiative boundary defined by the Schwarzschild criterion (for a recent discussion, see e.g. Demarque et al. 1994). Comparison with open cluster color-magnitude diagrams yield information on the extent of the hydrogen phase of evolution near the main sequence, which follows directly from the size of the mixed region in the stellar core. Comparison of model isochrones with observation have consistently suggested that mixing extends beyond the Schwarzschild boundary (see e.g. Demarque et al. 1994), although there has been recent indications that the OPAL opacities, which are larger than previous opacities, do not require as much overshoot as earlier radiative opacities to match the observations of the turnoff gap (Stothers & Chin 1991).

But first let us clarify one point of physics. If the convective elements that travel into the radiative region dissipate rapidly, the results is simply overmixing beyond the Schwarzschild boundary, with the local temperature gradient in the mixed zone remaining radiative. This situation is best called ‘overshoot’. On the other hand, if the temperature gradient in the mixed region is adiabatic, we have ‘convective penetration’ (Zahn 1991). Roxburgh (1978, 1989) has set a limit on the maximum extent of this convective penetration, of the order of a quarter of the local pressure scale- height at the convective-radiative boundary. This limit is in good agreement with estimates of convective overmixing derived from fitting theoretical isochrones to the color-magnitude diagram of well-observed star clusters, like NGC 2420 (Demarque et al. 1994) or NGC 3680 (Koziurina-Platais 1995). One question is to what extent the local temperature gradient in this region, which is likely to be somewhere in between the two extremes, and in a way which depends on the local physics in complex ways, affect the structure.

Another issue is the extent of overshoot or penetration. Is it realistically described by assigning it to be equal to a fraction of the pressure scale height at the convective-radiative boundary? Or should it be a fraction of the radius of the convective core, as suggested by Zahn (1991)? Stellar modeling shows that from the operational point of view, these two approaches are practically equivalent (Bromm 1995). Is there a maximum extent for penetration? These issues are difficult to answer observationally because open clusters are sparse, but increasingly better photometry and membership studies make some comparisons possible, although not definitive.

A potentially more powerful tool is asteroseismology. At this point, only the brightest field stars can be considered for such tests, but it may become possible in the future to observe sequences of stars in nearby open clusters. A detailed study of the evolutionary state and pulsation properties of Procyon reveals the extreme sensitivity of $p$-mode frequencies to the extent of convective overshoot. Slightly more evolved stars in the subgiant phase should further exhibit the phenomenon of “mode bumping” (see e.g. Scuflaire 1974; Guenther 1991; Audard
et al. 1995). This is because of the presence of a $\mu$-gradient at the edge of the exhausted core traps $g$-modes in the interior and gives rise to modes with a mixed $p$- and $g$-mode character. This affects the $p$-mode frequencies in a way which is sensitive to the size of the helium core.

It appears that mode bumping has indeed been observed in the sun-like subgiant $\eta$ Bootis (Christensen-Dalsgaard et al. 1995; Guenther & Demarque 1996). Although the results are preliminary, there seems little doubt that the phenomenon has indeed been observed, lending support for the reality of the observations. We shall return to the $\eta$ Bootis observations in Section 4, and consider the remarkable ability of seismology to constrain fundamental stellar parameters.

Seismology should eventually also provide information about internal rotation from $p$-mode rotational splitting. In stars with a convective core near the main sequence, differential rotation is likely to occur outside the core as the star evolves. Rotational shear may induce turbulent mixing in this region, which from the global point of view of stellar evolution in the color-magnitude diagram and of star cluster luminosity functions, is indistinguishable from mixing due to overshooting.

3 Convection zones, the superadiabatic peak and seismology

We shall begin by reviewing some of the approaches which have been adopted to model the convection zone in the Sun and sun-like stars, as seen from the point of stellar structure calculations rather than from the point of view of the detailed dynamics of the convection zone. Because the problem of the convection zones is so complex and only approximate solutions are possible at this time, it is profitable not to try and solve all aspects of the problem at once; for example, a model which may not be acceptable for the computation of convective cell patterns or dynamo numbers, might nevertheless provide an acceptable approximation to the run of the temperature gradient.

3.1 The calculation of stellar radii and the mixing length approximation

It has become a standard procedure in the last few years to use the mixing length approximation (MLA) to describe convection in the outer layers. The MLA combines a phenomenological description of convection based on the notion that, for all practical purposes, the convective energy is carried by convective elements of a single size and symmetric upward and downward flows. Radiation is treated in the diffusion approximation, which is only valid in the optically thick layers and becomes quite incorrect in the atmosphere (Vitense 1953; Böhm-Vitense 1958; Gough & Weiss 1976).

The MLA correctly predicts a very small superadiabaticity in the deep convective layers. This means that in most of the convective zone, for the purpose of calculating the envelope structure and radius, the temperature gradient is equal to the local adiabatic temperature gradient. In the transition region between the outer primarily radiative layers and the region of deep (nearly adiabatic) convection, there is a thin highly superadiabatic layer. It is in this layer that the MLA is used to regulate the efficiency of convection. In constructing the models, the height of the superadiabatic peak depends on the details of the local physics (primarily the local equation of state and opacities) and the choice of the mixing length parameter; all of which control the efficiency of convection in the SAL. In a laboratory fluid, the effective
mixing length is usually found to equal the size of the convective region. In stellar models, where fluids are compressible, and convection can extend over many scale heights, the mixing length is usually set equal to a fraction of the local pressure scale height. The choice of $\alpha$ (mixing length parameter) is a measure of the efficiency of convection in the SAL, and is used to determine the local temperature gradient.

There is a fundamental inconsistency in this approach in that the MLA, in addition to its simplification of the thermodynamics and flows in the convection zone, is only applicable to the deep convective layers where the coupling between radiation and convection can be safely ignored (see the discussions of Chan and Sofia 1989, and Kim et al. 1995a,b,c). This not the case in the highly superadiabatic layer where its validity in calculating the temperature gradient breaks down (see Kim et al. in these Proceedings). And yet, the superadiabatic layer is the very region where the MLA is mostly relied on in constructing stellar models.

The choice of $\alpha$ also determines the calculated model radius. We emphasize here that because the SAL is itself thin compared to the total radius, its thickness is not a significant factor in determining the radius (at least for stars near the main sequence). But the precise structure of the SAL, specifically the run of the thermodynamic variables on the adiabatic inner side of the superadiabatic peak, determines the specific entropy in the adiabatic envelope. It has long been known that the specific entropy of a stellar convection zone determines its depth (Schwarzschild 1958; Gough and Weiss 1976; and for a more general physical argument, see also Larson 1974). In fact, many authors have been content to ignore the existence of the superadiabatic layer in calculating solar models, and have simply adjusted the specific entropy in the solar convection to fit the solar radius. It is therefore the sensitivity of the specific entropy of the convective envelope to $\alpha$ which is the origin of the radius sensitivity to $\alpha$. It is now common practice to vary $\alpha$ to produce solar models with a radius that matches precisely the solar radius (Demarque & Percy 1964). The resulting value of $\alpha$ depends on the details of the model atmosphere (which determines the surface boundary condition) and on the local opacities (see e.g. Guenther et al. 1992). In this scheme, a single free parameter, the mixing length parameter $\alpha$, is used to compensate not only for the distortions introduced by the MLA in modeling convection, but also for the uncertainties in modeling the outer layers. This is done simply by adjusting the structure of the SAL in the solar model. This procedure highlights the flexibility of the MLA in constructing solar models and why it is so convenient in stellar evolution calculations. It should be remembered, however, that the MLA approach with a constant universal $\alpha$ is based on a convenient fudge to the solar radius. There is no physical justification for using the same $\alpha$ to other stars, or even as a function of depth within a given star. The fact that theoretical isochrones can be constructed under the assumption of constant $\alpha$ is an indication that the effective $\alpha$ in the critical superadiabatic layer does not vary much from star to star or during evolution. However, because the fitting process to cluster color-magnitude diagrams is sensitive to $\alpha$ at some level (see the contribution by Chaboyer in these Proceedings), this point should be remembered in star cluster age determinations.

3.2 The Canuto-Mazzitelli (CM) approach

Canuto and Mazzitelli (1991,1992) have generalized the MLA by taking into account the whole spectrum of convective wavelengths. In this sense, the CM formalism is an improvement on the MLA. The parameters of CM's description of convection are based on the results of laboratory experiments of incompressible convection extrapolated to stellar conditions. As in the MLA, CM describe radiation in the diffusion approximation. Using the laboratory analogy, CM argue in favor of a mixing length equal to the local distance to the surface convection boundary,
although some of their calculations related the mixing length to the pressure scale height. The CM theory has been cast in a convenient form for stellar evolution calculations (similar to the MLA formulation) by Stothers and Chin (1994). The CM models discussed in Section 3.5 were constructed using the Stothers-Chin equations implemented in the Yale stellar evolution code. For a fixed mixing length parameter, the CM models differs from the MLA in that the efficiency of convection decreases toward the surface. And although the radiation is decoupled from convection in this scheme, the result is to increase the relative importance of radiation closer to the surface than in the MLA, leading to a more sharply peaked superadiabatic temperature gradient, and yielding better agreement with the observed $p$-mode spectrum (Paterno et al. 1993).

### 3.3 The Convective Flux Approximation

The Convective Flux Approximation (CFA) (Lydon et al. 1992,1993) is based on the numerical simulations of deep convection by Chan & Sofia (1989), from which it draws expressions for the convective flux. The CFA models represent the first attempt to incorporate the results of numerical simulations of compressible convection into solar models. Another unique feature in the CFA approach is that it has no recourse to a free parameter (such as the mixing length) in constructing the convection zone and adjusting the radius. As in the MLA, radiation is decoupled from convection, and is treated by the diffusion approximation. Because the Chan-Sofia simulation was applicable to deep convection only, the properties of convection had to be extrapolated outside the range of validity of the simulation, to the shallow layers of the solar model. In this sense, the CFA is very similar to the MLA, and it yields a temperature gradient very similar to the MLA for the Sun. The SAL in the CFA model is less peaked and located slightly deeper below the photosphere than the MLA model. This is at odds with the CM model and the models inspired from the shallow convection simulations described in Section 3.4 which produce more peaked superadiabatic layers. Although the CFA model is important as a first step in the parameterization of convection from numerical simulations of convection, the model (like the MLA) fails to describe the superadiabatic peak because it assumes that the properties of deep efficient convection can be extrapolated to the critical shallow layers.

### 3.4 Experiments inspired from numerical simulations

Kim et al. (1995a,b and these Proceedings) have performed numerical simulations of convection for a compressible, radiation-coupled, non-magnetic, gravitationally stratified medium, using a realistic equation of state. The spatially and temporally averaged thermodynamic quantities of the simulation have been compared to the equivalent linear relationship in the MLA. Because the calculation made use of the diffusion approximation for radiation, only the inner part of the superadiabatic peak was modeled. The results show that the MLA overestimates the convective energy transport near the top of the convection zone, thus requiring a sharper superadiabatic peak than for MLA models. They also show that the temperature gradient in the simulation could be mimicked by the MLA with a decreasing $\alpha$ toward the surface. We emphasize here that one should not expect the MLA with variable $\alpha$ to be equivalent to the numerical simulation in all respects (the MLA flows and thermodynamics are fundamentally different). But it is possible to mimic some of the properties of the numerical simulation with the variable $\alpha$ MLA. In this case, we mimic the temperature gradient, which is what matters in a one-dimensional stellar model.
Figure 1: The peak of the superadiabatic excess for several treatments of convection (see text). The abscissa is the radius fraction.

3.5 Helioseismology as a probe of the outer envelope structure

Since the advent of helioseismology, it has become possible to test the sensitivity of the structure of the superadiabatic peak in the Sun directly. We have carried out comparison tests of the sensitivity of the $p$-mode oscillation frequencies to the choice of the convection formulation in the model. All models discussed here compared to a standard solar model (ssm) constructed with the OPAL (Rogers and Iglesias 1994) opacities and the Bahcall-Pinsonneault (1992) nuclear energy generation rates.

All models are closely calibrated to the sun. Their temperature gradient superadiabatic peaks are plotted in Figure 1. The model labeled SSM refers to a standard solar model constructed using the OPAL equation of state and opacities, and including the effects of helium diffusion. The label "YC mlt slope" refers to models in which the mixing length parameter $\alpha$ is taken to be proportional to the local pressure scale height, as indicated in the left hand lower panels of Figure 2. The "C&M mlt" models make use of the Stothers-Chin (1995) formulation of Canuto and Mazzitelli. We see that the solar radius constraint is satisfied for $\alpha = 1.06$ in the case of a mixing length proportional to the local pressure scale height, and for $\alpha = 1.26$ when the mixing length is proportional to the local distance to the top convective boundary, as favored by CM.

Figure 2 shows comparisons of the $p$-mode frequency differences (model minus observed) plotted against the observed frequencies. Each model is labeled in the following way: ssm; (diff+opal) in which in addition helium diffusion and the OPAL equation of state (Rogers, Swenson and Iglesias 1995) have been included; the C&M mlt panels refer to the use of the Canuto-Mazzitelli mixing length formalisms; the (mlt+diff+opal) panel refer to the variable alpha models based on the Kim et al. (1995a) convection numerical simulations; the (diff+opal+kappa) panel, in which an ad hoc increase in the atmospheric opacities has been added, shows the
sensitivity to the atmospheric structure. We see how the introduction of the helium diffusion and the OPAL plus the OPAL equation of state improve the agreement between the calculated frequencies and the observations. There remains a well-known slope in the frequency difference diagram (see e.g. the discussions of Guenther 1994, Chaboyer et al. 1995, or Guenther et al. 1995) which is due to the inadequacy of the solar model outer layers, by which we mean both the mostly radiative atmosphere and the region of the highly superadiabatic layer. Figure 3 labeled [C&M mlt (l = α × z)] shows the CM model with a mixing length proportional to the distance to the top of the convection zone. Returning to Figure 2, we see that the stronger superadiabatic peak of the variable mlt models agrees better with observation than the models based on the MLA such as ssm and (diff+opal). Note also the better fit of the CM model constructed with a pressure scale height based mixing length than with a zone thickness based mixing length.

All the above models should be viewed as exploratory. In particular, there are good physical reasons for expecting some convective penetration into the atmospheric layers. But we note that none of the standard MLA, CFA and CM models discussed here included this effect. Our test of the sensitivity of the SAL structure to the atmospheric opacities should be viewed in this context; not as a statement on the uncertainty in the atmospheric opacities themselves, but as an indication of the uncertainty in the treatment of our adopted convective-radiative boundary, which is clearly incorrect in our models. Another indication of the importance of convective penetration is reflected in the need for a zero-point offset in the variable mixing length relation to match the observed acoustic frequencies in the Sun (see also the Kim et al. paper in these Proceedings).

In view of the present uncertainties in the input physics, in our understanding of convection, and of the fact that the models neglect some obvious and possibly significant factors, such as magnetic fields, the agreement with observation of the best models described above is encouraging. It now remains the formidable task of refining convection simulations and our
understanding of the full convection zone, particularly near the surface where the coupling of convection with radiation controls the structure. Research in this area is now under way by one of us (YCK in collaboration with K.-L. Chan) to perform 3D numerical simulations for the solar outer layers. Eventually, our objective is to extend this work to the outer layers of other sun-like stars with different physical parameters.

4 Convection and seismology in sun-like stars

Another way of gaining insight on the best approaches to improve modeling of the outer convective layers is to make use of other stars similar to the Sun, the so-called solar-stellar connection. The recent seismic observations of the sun-like subgiant $\eta$ Boo provide just such an opportunity. It can also be shown that Kjeldsen et al.'s (1995) $p$-mode frequencies can be used to put much stronger constraints on the mass, helium abundance and distance, than conventional techniques can provide (Guenther & Demarque 1995). The HR-diagram for $\eta$ Boo is shown in Figure 4. We see that $\eta$ Boo is a subgiant, just beyond the core exhaustion phase.

The $\eta$ Boo data also show evidence for mode bumping, as would be expected in a subgiant star with a helium core exhausted of hydrogen. The theoretical models (Christensen-Dalsgaard et al. 1995; Guenther & Demarque 1995) also predict mode bumping for some $p$-modes, thus lending support for the reality of the observations. Figure 5 (after Figure 6 of Guenther & Demarque) shows the same frequency difference diagram as shown in Figures 2 and 3, this time for $\eta$ Boo models. Figure 5 makes two points. The first point is that the slope of the lines of constant $l$ have a slope with the opposite sign of the solar slope. This result maybe a clue as to where our treatment of the outer layer goes wrong in the sun and $\eta$ Boo. The second point is the evidence for "mode bumping", which is apparent in three of the panels in Figure 5.

It is clear that future refinements in the observations and modeling of $\eta$ Boo and a few
Figure 4: Post-main sequence evolutionary tracks in the theoretical HR-diagram. The position of $\eta$ Bootis is marked, with error bars.

Figure 5: Frequency difference diagrams for four possible models of $\eta$ Bootis, each corresponding to a different adopted value for the parallax.

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other luminous sun-like stars will enable us to predict the $p$-mode frequencies more reliably, learn about core overshooting and about $\eta$ Boo's superadiabatic peak (which is much closer to the surface than in the sun) and as a result will greatly strengthen our confidence in stellar structure theory.

In addition to observing a few bright field stars, it should eventually be possible to observe stars along the main sequence in the brightest clusters. As always, observations in star clusters, where the stars are coeval, at the same distance, and of the same chemical composition, should prove invaluable (see Sills 1996, for a theoretical seismological study).

In conclusion, the convergence of two new ways to explore convection in the interiors of sun-like stars, the numerical simulation of radiation coupled convection using realistic local physics, and seismology show great promise for the future. Many more and more sophisticated simulations are needed, and grids of model envelopes covering the relevant parts of the HR-diagram for stars of different compositions and masses will need to be constructed. It is unclear at this point how best to incorporate the results of these simulations into stellar models. Perhaps we shall be fortunate and be able to parameterize the relevant aspects of the 3D simulations for use in 1D stellar models in a convenient way similar to the MLA. If not, the convective envelope models themselves will have to be used as surface boundary conditions in stellar evolution calculations.

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DISCUSSION

E. SCHATZMAN: Please, can you tell again how you describe the surface layers, the radiation transfer in region where the fitting with the convection zone takes place.

P. DEMARQUE: The outer layers are described by a simple grey atmosphere (in the Eddington approximation). We use Rosseland mean opacities all the way to the surface.

N. GREVESSE: You showed that tuning the opacity in the outer solar layer could make the O-C curve for the frequencies become horizontal around zero. Does it not simply mimic the fact that your model atmosphere is not the most appropriate one, e.g. use of a theoretical model rather than of an empirical model?

P. DEMARQUE: Yes, in all likelihood.

I. ROXBURGH: Long ago, Hoyle and Schwarzschild used a model of convection in which the layer was radiative and then switched to adiabatic, the change being when the total energy could be carried with convective velocities or (freely chosen) fraction of the sound speed. Have we really made any progress since then?

P. DEMARQUE: We have not solved the problem yet, but it is now better posed.

A. NOELS: In the Sun, diffusion seems to be necessary to improve the agreement with the p modes. Could the lack of agreement and even the change of slope you found in η Boo be due to diffusion, which would of course act in a different way in a star more massive than the Sun and with a different age?

P. DEMARQUE: It is quite possible. Diffusion should be very effective in η Boo, because the convection zone is shallow. We have not yet included diffusion in our η Boo models.

M. STEFFEN: Do you have any idea how temperature fluctuations in the surface layers (granulation) might affect the p mode frequencies?

P. DEMARQUE: This question has been looked at by Balmforth. He estimates an uncertainty of 5 μHz in the p mode frequencies.