Convective penetration in the Sun in presence of microscopic diffusion

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Abstract: We have computed calibrated solar models with penetrative convection and microscopic diffusion (Morel et al., 1995) and we give some helioseismological consequences of the effect of the convective penetration in presence of microscopic diffusion.

1 Introduction

A penetration of convective motions is generally expected beyond the level where the Schwarzschild criterion predicts the base of the solar convective envelope and it has been described by Zahn (1991). The possibility to infer the extent of this convective penetration below the convection zone from helioseismic data has been much discussed. In a first approach (Berthomieu et al., 1993) we have given an upper limit of this extent from direct comparison between observed and theoretical frequencies, but such a method do not allow to separate opacity effects from convective penetration.

Further papers are based on the idea that any rapid variation ("discontinuity") of the stellar structure induces an oscillatory behavior of the frequencies of p-modes that penetrate beyond the discontinuity level (Gough, 1990). Such a discontinuity occurs at the base of the solar convection zone of a standard solar model, due to the transition between an adiabatic stratification and a radiative one: the temperature and hence the sound speed has a discontinuous second derivative (Fig.1a). In presence of convective penetration, the first derivative of the sound speed is discontinuous due to a jump of the temperature gradient from adiabatic to radiative value (Fig.1a). An additional source of discontinuity, generally neglected, occurs in presence of microscopic diffusion, due to the discontinuity of the derivative of the chemical composition located at the base of the convection zone (Fig.1b).

Estimations of the extent of the convective penetration below the convection zone have been derived by extracting the oscillatory component of the frequencies or that of the 2n-th differences of frequencies (Basu et al., 1994; Basu and Antia, 1994, Monteiro et al., 1994; Roxburgh and Vorontsov, 1994; Christensen-Dalsgaard et al., 1995). Here, following the method developed by Basu et al. (1994), we estimate the effect of convective penetration on the amplitude of the oscillatory part of the 6-th differences of frequencies for sequences of solar models with different extent of convective penetration, in presence of microscopic diffusion.

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2 Characteristics of solar models

We have computed calibrated solar models with penetrative convection and microscopic diffusion with the same physics as Morel et al. (1995). The extent \( L_p \) of the convective zone due to convective penetration is characterized by the parameter \( \zeta \) (Zahn 1991):

\[
L_p = \frac{\zeta}{\chi_p} H_p
\]

\( H_p \) is the pressure scale height, \( \chi_p = (\partial \ln \chi / \partial \ln p)_{ad} \), \( \chi \) being the radiative conductivity (\( \chi = 16\pi T^3/(4\kappa p) \)). For a solar model \( \chi_p \) is \( \sim 1.8 \) at the base of the convection zone.

Models \( M_i \) and \( D_i \) (Table 1) are characterized by an overshoot parameter \( \zeta = i/10 \). The models \( D_i \) are computed with helium microscopic diffusion, according to Proffitt and Michaud (1993). The heavy element content is assumed to be constant. \( R_{ZC} \) corresponds to the position of the lower part of the convection zone according to the Schwarzschild criterion and \( R_C \) gives the radius of the extended adiabatic region. For the \( M_i \) models, the value of \( R_C \) becomes of the order of the “observed” value 0.713 (Christensen-Dalsgaard et al., 1991) for \( \zeta \) of order 0.5; except \( D_0 \), the models \( D_i \) have a too deep convection zone even without convective penetration.

3 Helioseismic analysis

According to Basu et al. (1994) the sixth differences of frequency \( \delta^6 \nu \) can be considered as a combination of a slowly varying component \( \delta^6 \nu_{\text{smooth}} \) and of an oscillatory part \( \delta^6 \nu_{\text{osc}} \).

With \( \nu_m = \nu - \gamma \ell (\ell + 1) / 2 \nu \), then \( \delta^6 \nu_{\text{osc}} = \delta^6 \nu - \delta^6 \nu_{\text{smooth}} \approx \)

\[
\left( a_0 + a_1 \left( \frac{\bar{\nu}}{\nu_m} \right)^2 + a_2 \ell (\ell + 1) \left( \frac{\bar{\nu}}{\nu_m} \right)^2 + a_3 \ell (\ell + 1) \left( \frac{\bar{\nu}}{\nu_m} \right)^4 \right) \sin(2\tau \nu_m + \psi)
\]

\( \bar{\nu} \) is a scaling frequency (hereafter \( \bar{\nu} \equiv 2.5 \text{mHz} \)). The form of this relation comes from the asymptotic theory (Monteiro et al., 1994; Roxburgh and Vorontsov, 1994; Christensen-Dalsgaard et al., 1995). The terms in \( \nu_m^{-2} \) and \( \nu_m^{-4} \) are respectively related to the discontinuities of the first and second derivatives of sound speed. According to the asymptotic analysis:

\[
\tau = \int_{r_d}^{R} \frac{dr}{c}, \quad \gamma = \int_{r_d}^{R} \frac{cdr}{r^2}.
\]
Figure 2: (a) Sixth differences of frequency $\delta^6\nu$ as a function of the frequency $\nu$, for the standard model $M_0$. The continuous curve corresponds to the smooth component $\delta^6\nu_{\text{smooth}}$; (b) Resulting oscillatory part $\delta^6\nu_{\text{osc}}$ as a function of the frequency $\nu$, for the same model. The points are almost symmetrically distributed; (c): $\delta^6\nu_{\text{osc}}$ as a function of $\nu_m$. The continuous curve corresponds to the fit.

Figure 3: Variation of the coefficients characterizing the oscillating part of frequencies derived from sixth differences $\delta^6\nu_{\text{osc}}$ analysis, as a function of $\zeta$, for models $M_i$ (full circle) and $D_i$ (star): (a) $\tau$ (sec); (b) amplitude coefficient $a_1$ ($\mu$Hz); (c) $A = a_0 + a_1$ ($\mu$Hz); (d) $A_{\ell} = a_2 + a_3$ ($\mu$Hz); (e) $A = a_0 + a_1$ as a function of $\tau$. The symbol $\odot$ corresponds to values derived from Libbrecht's data.
We have computed $\delta^6 \nu$ in the range $5 \leq \ell \leq 20$ and $2 mHz \leq \nu \leq 3.5 mHz$. These modes are almost unaffected by the surface uncertainties and are observed with a good accuracy. In a first step we determine $\delta^6 \nu_{smooth}$ by progressive smoothing using spline functions (Fig.2a). This component contains in particular contributions from the surface layers and a damped oscillatory component due to the rapid variation of the sound speed in the helium ionization zone.

In a second step, the values of the parameters $a_{i, i=0,3, \tau, \gamma}$ and $\psi$ are obtained by a least square fit of the resulting $\delta^6 \nu_{osc}$ (see Fig.2b) to the expression (1). To visualize the quality of the fit (Fig.2c), we have considered the part of the oscillatory component which depends on the degree $\ell$ only through the reduced frequency $\nu_m$ (Fig.2c):

$$\overline{\delta^6 \nu_{osc}} \equiv \delta^6 \nu_{osc} - \ell(\ell + 1) \left( a_2\left(\frac{\bar{\nu}}{\nu_m}\right)^2 + a_3\left(\frac{\bar{\nu}}{\nu_m}\right)^4 \right) \sin(2\tau\nu_m + \psi) \simeq \left( a_0 + a_1\left(\frac{\bar{\nu}}{\nu_m}\right)^2 \right) \sin(2\tau\nu_m + \psi)$$

(4)

4 Results and discussion

We have applied the above seismic analysis both to solar models $M_1$ and $D_1$. For each model, we have obtained estimated values for the parameters $a_{i, i=0,3, \tau, \gamma}$ and $\psi$ (Table 1). As expected, the values of $\tau$ (Fig.3a) and $\gamma$ increase with $\zeta$ for each set of models, due to deeper convection zone. The $\tau$ values are smaller for models $M_1$ than for models $D_1$. The values obtained for $\tau$ and $\gamma$ are in good agreement with the predicted asymptotic values. Nevertheless the $\tau$ value is systematically higher, due to surface effects (Monteiro et al., 1994).

All the $a_i$ coefficients have the same behavior when varying $\zeta$ for the two series of models (see Table 1), except $a_1$. The coefficient $a_1$ decreases with $\zeta$ for the models $M_1$ while it increases for the models $D_1$ (Fig.3b). As this coefficient is related to the discontinuity of the first derivative of the sound speed, this behavior could be due to the presence of the gradient of chemical composition in diffusion models, which has been neglected in their asymptotic approach by Christensen Dalsgaard et al. (1995).

The amplitude of $\delta^6 \nu_{osc}$ depends on the frequency and on the degree through the coefficients $a_i$. It writes $A + \ell(\ell + 1)A_{\ell}$ with $A = a_0 + a_1(\bar{\nu}/\nu_m)^2$ and $A_{\ell} = a_2(\bar{\nu}/\nu_m)^2 + a_3(\bar{\nu}/\nu_m)^4$. It decreases with the degree $\ell$ in the considered frequency range due to negative values of $A_{\ell}$. The values of $A$ and $A_{\ell}$ around $\nu = \bar{\nu}$ are given as a function of $\zeta$ in Figure 3c and d. $A$ is analogous to the average amplitude defined by Basu et al. (1994). It increases with $\zeta$ from 4 to 7 $\mu$Hz. $A_{\ell}$ is smaller around -0.01 $\mu$Hz and slightly decreases with $\zeta$. The two series of models are well separated. As in previously published results, we find that, for a given $\zeta$, the amplitude is significantly higher for models including microscopic diffusion. Figure 3e represents the amplitude $A = a_0 + a_1$ as a function of $\tau$ for the same models. The value obtained by the same method with the Libbrecht’s data 1 limits very strongly the amount of convective penetration in the sun.

These preliminary results are in agreement with previous papers, taking into account the fact that our convective penetration parameter is smaller by a factor about 1.8 than the usual one. Nevertheless our models correspond to systematically smaller $\tau$ values than those of Basu et al. (1994). In conclusion it appears that the effect of $\mu$ gradient due to microscopic diffusion is small, but it seems that it affects particularly one of the coefficients of the amplitude of the oscillatory part of the frequencies. However other transport processes like turbulent diffusion have been ignored.

1p-mode data acquired by Ken Libbrecht and Martin Woodard, Big Bear Solar Observatory, Caltech

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Table 1: Solar models properties and fitting coefficients

<table>
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<tr>
<th>$M_i$</th>
<th>$\zeta$</th>
<th>$R_{\odot}$</th>
<th>$R_\odot$</th>
<th>$\tau$ (sec)</th>
<th>$\gamma$ (mHz)</th>
<th>$a_0$ (mHz)</th>
<th>$a_1$ (mHz)</th>
<th>$a_2$ (mHz)</th>
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<td>0</td>
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<td>0.733</td>
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<td>0.712</td>
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Acknowledgements

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References

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