SEISMOLOGY OF THE SOLAR SURFACE REGIONS

Colin S. Rosenthal
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond,
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark;
E-mail: rosentha@obs.aau.dk

Jørgen Christensen-Dalsgaard
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, and
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark; E-mail: jcd@obs.aau.dk

Günter Housèk
Institut für Astronomie, Universität Wien, Austria, and
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond,
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark; E-mail: hg@obs.aau.dk

Mário J.P.F.G. Monteiro
Astronomy Unit, School of Mathematical Sciences, Queen Mary and Westfield Colleges, London, and
Grupo de Matemática Aplicada da Faculdade de Ciências, and Centro de Astrofísica,
Universidade do Porto, Rua do Campo Alegre 823, 4150 PORTO, Portugal; E-mail: njmontei@ncc.up.pt

Åke Nordlund
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, and
Niels Bohr Institutet for Astronomi, Fysik og Geofysik, Astronomisk Observatorium, Københavns Universitet,
Øster Voldgade 3, DK-1350 København K, Denmark; E-mail: aak@astro.ku.dk

Regner Trampedach
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark; E-mail: art@obs.aau.dk

ABSTRACT

We investigate the influence of dynamical and nonadiabatic effects occurring in the superadiabatic region near the top of the solar convection zone on the frequencies of solar p modes. Taking as our baseline a standard hydrostatic solar model, we calculate frequency changes resulting from a number of different formalisms, involving modifications of the superadiabatic temperature gradient, turbulent pressure and/or nonadiabatic effects. We compare these various methods of calculating the effect of convection on solar-oscillation eigenfrequencies with each other and with the measured frequency residuals.

1. INTRODUCTION

The near-surface regions of the Sun are dominated by the extremely rich and complex physics of the interaction of convection, radiation, magnetism and rotation. This region is also the principal site of the generation, damping, and possibly scattering, of p and f modes.

Our hope is that the properties of these modes may be used to probe the structure of this region. We begin here by attempting to understand the influence on mode eigenfrequencies from

- the effect of convection on the mean stratification,
- the absence of hydrostatic equilibrium, on the assumption of which the usual oscillation equations are based, and
- nonadiabatic effects, including the coupling of the oscillations to both radiation and convection.

Our approach is to consider several different strategies for modelling these effects, incorporating a variety of different physical assumptions varying from the simplest mixing-length theory to a sophisticated hydrodynamical simulation. Specifically, we consider

1) A hydrostatic model computed using a non-standard parameterised model of convection.
2) Adiabatic oscillations of a nonlocal mixing-length model of the convection zone under the assumption that the Lagrangian variation in the turbulent pressure vanishes.
3) A nonlocal time-dependent mixing-length calculation of the oscillations including radiative damping and the effect of the perturbations to the convective flux and turbulent pressure.
4) Adiabatic oscillations of a time-averaged hydrodynamical model of the convection zone, using the same assumption as in (2).
5) A variational calculation of the effect of convective inhomogeneities on the mode frequencies using the results of a hydrodynamic simulation.


© European Space Agency • Provided by the NASA Astrophysics Data System
2. BASELINE MODEL

The baseline model is a standard solar model constructed using OPAL and Kurucz opacities, the MHD equation of state and Böhm-Vitense mixing-length theory, and taking into account the effects of helium diffusion and settling. The model includes an atmosphere in hydrostatic equilibrium, based on a simple analytical $T - r$ relation. Further details on the model are given by Basu et al. (these proceedings).

Figure 1 shows a comparison between the adiabatic eigenfrequencies of this model and measured frequencies. The observations were BISON results from Elsworth et al. (1995) for $l \leq 3$ and BBSO frequencies (Libbrecht et al. 1990) for higher degree. The frequency residuals have been scaled by the mode mass, normalized to the value for a radial mode at the same frequency. The scaled residuals are well represented by a function of frequency, for degrees $l \leq 500$, indicating that they are predominantly due to inaccuracies in the model confined to a narrow region near the solar surface (e.g. Christensen-Dalsgaard 1986).

We investigate the effects of changes in the superficial layers by means of pairs of models: each pair consists of a model incorporating the given change and a reference model using otherwise the same physics and calibrated to have the same depth of convection zone, by adjusting the mixing-length parameter. By keeping the convection-zone depth fixed, we ensure that the structure of these two models is essentially the same, apart from the superficial region.

3. MODIFICATIONS TO THE MEAN STRUCTURE

We have computed eigenfrequencies of two modified models resulting from the use of different treatments of convection. The first model uses a theory developed by Canuto & Mazzitelli (1991), based on modern theories of turbulence, which models the spectrum of energy-bearing eddies in a considerably more sophisticated manner than standard mixing-length theory. In the form used here, this theory does not include the effect of turbulent pressure on the hydrostatic stratification.

The second model is based on the results of a hydrodynamical simulation of solar convection (e.g. Stein & Nordlund 1989), averaged over horizontal surfaces and time, and matched continuously to a model of the deeper envelope calculated in the usual fashion. The hydrodynamical simulation leads to a non-zero turbulent pressure. Also, the resulting mean model depends on the precise procedure used in averaging the quantities resulting from the simulation. Consequently some care is required in defining the averages; in addition, one must make some assumption about the perturbation to the turbulent pressure in the oscillation equations.

To be precise, we here assume that in the equilibrium model we can define the gas pressure $p_{\text{gas}}$ and the turbulent pressure $p_{\text{turb}}$, such that the total pressure $p = p_{\text{gas}} + p_{\text{turb}}$ satisfies hydrostatic equilibrium,

$$\frac{dp}{dr} = -g \rho,$$

where $g$ is the gravitational acceleration and $\rho$ is the (suitably averaged) mean density in the equilibrium model. We furthermore assume that, for the purpose of calculating the real part of the frequency shift, the Lagrangian perturbation $\delta p_{\text{turb}}$ in turbulent pressure may be ignored. This assumption is justified by the observation from nonlocal mixing-length theory and from the simulation, that $\delta p_{\text{turb}}$ varies approximately in quadrature with the other force terms in the momentum equation, and hence contributes only to the imaginary part of the frequency shift. With this assumption, the Lagrangian perturbation in the total pressure $p$ is determined by

$$\frac{\delta p}{p} = \frac{\delta p_{\text{gas}}}{p} = \Gamma_1 \frac{\delta p}{\rho},$$

where $p_{\text{gas}}$ is assumed to respond adiabatically, and $\Gamma_1 \equiv (\rho_{\text{gas}}/\rho) \Gamma_1$, $\Gamma_1$ being the usual adiabatic exponent. With these assumptions, the only modification
of the adiabatic oscillation equations is the replacement of $\Gamma_1$ by $\bar{\Gamma}_1$.

By averaging the Euler equations for the simulated convection, one finds that equation (1) is satisfied, provided $p_{\text{gas}}$ and $\rho$ are defined by straight horizontal and temporal averages of the fluctuating gas pressure and density, and $p_{\text{turb}} = \langle p u^2 \rangle$. We have verified that this is indeed approximately the case for the simulations. Since $\Gamma_1$ defines the thermodynamic response of $p_{\text{gas}}$ to changes in $\rho$, we calculate it from the equation of state, evaluated at the mean density and internal energy.

The hydrodynamical simulation only covers the outer 2% of the solar radius. The averaged model is extended with an envelope assuming adiabatic convection, matching the averaged simulation continuously in sound speed and extending to 0.2$R$. The corresponding reference model uses mixing-length theory, calibrated to give the same depth of convection zone as in the matched model. The Canuto & Mazzitelli model is constructed using the same calibration procedure for the same depth of the convection zone as in the two other models.

Monteiro, Christensen-Dalsgaard & Thompson (1995) and Christensen-Dalsgaard & Thompson (in preparation) have shown that the effects of near-surface changes in the model are conveniently expressed in terms of the variable $v \equiv \Gamma_1/c$, $c$ being the adiabatic sound speed. In particular, the frequency changes are largely determined by the change in $v$, as evaluated at fixed mass coordinate in the models (the so-called Lagrangian changes). Figure 2 shows such Lagrangian changes for the two models considered here, relative to the respective reference models, in $\Gamma_1$ and $v$. The changes are clearly quite large (10% for the Canuto & Mazzitelli model and 20% for the simulation). The width of the region where the changes occur is considerably larger in the simulation.

To understand the behaviour of the differences, we note that $v^2 = \Gamma_1/u$, where $u = p/\rho$ is essentially proportional to temperature. In the Canuto & Mazzitelli model, the change in $\Gamma_1$ arises because the superadiabatic temperature gradient is considerably steeper than for the usual mixing-length model, resulting in a more rapidly increasing temperature, a more rapid onset of hydrogen ionization, and hence a sharper decrease in $\Gamma_1$ with depth in the upper parts of the convection zone. This, together with the corresponding sharper increase with depth in $u$, causes the sharp decrease in $v$ relative to the reference model. A superficially similar effect is produced in the hydrodynamical simulation (and in the nonlocal mixing-length model discussed below) by the effect of turbulent pressure on $\bar{\Gamma}_1$; in the latter models the mean temperature gradient is similar to that obtained from mixing-length theory.

Figure 3 shows the frequency shifts for the adiabatic eigenmodes of the two models. Both models show similar behaviour to the data at low frequency. The shifts on the Canuto & Mazzitelli model show a distinct tendency to flatten out at higher frequency. However, this trend is much less evident in the model based on the simulation, because the model changes in this case are large well up into the solar atmosphere. The frequency residuals based on the simulation are larger than those observed while those based on the formalism of Canuto & Mazzitelli are smaller.

4. NON-LOCAL MIXING-LENGTH MODELS

We have also calculated solar envelope models using the nonlocal mixing-length theory of Balmforth (1992). In local mixing-length theory, the convective flux at a given point depends only on conditions at that point. In contrast, the nonlocal description takes into account that the properties of a convective eddy depend upon an average of the mean stratification over the range of heights spanned by the
eddy; also, the flux at a given point is obtained as the combined result of the eddies, originating at different heights, which pass this point. The calculations also include a consistent treatment of the effect of turbulent pressure on the stratification. Further details of these calculations are described by Houdek et al. (these proceedings).

We have calculated eigenmodes of this envelope in two different ways: one assumes adiabatic oscillations and treats the pressure perturbation as indicated in equation (2); in the second, nonadiabatic effects and the perturbation in turbulent pressure are taken into account. The resulting frequencies are compared with adiabatic frequencies for a standard local mixing-length model computed with the same physics and having the same convection-zone depth as the nonlocal model.

The results are shown in Figure 4. In the adiabatic case the frequencies are depressed above about 2 mHz, with a maximum deficit of about 15 mHz. This effect is of the same nature as that found for the averaged hydrodynamical simulation in Figure 3; as in that case, it is dominated by the depression in $\Gamma_1$ induced by turbulent pressure. The nonadiabatic treatment, described by Balmforth (1992), includes a nonlocal version of the time-dependent mixing-length theory originally developed by Gough (1977); this provides physically consistent expressions for the perturbations in the convective flux and the turbulent pressure. Radiation was treated in the Eddington approximation. It is evident from Figure 4 that this produces much smaller downshifts, suggesting that the inclusion of nonadiabatic effects sufficiently increases the mode frequencies that the effect due to the change in envelope structure caused by turbulent pressure is nearly cancelled.

If we believe that the difference between the two curves in Figure 4 can be interpreted as the nonadiabatic correction to the mode eigenfrequencies, then its effect is to bring the residuals based on the simulation (Figure 3) closer to the data, while the residuals from the Canuto & Mazzitelli model would then be in poorer agreement with the data. However, such a conclusion must be treated as tentative since it involves combining the results of two quite different theories of convection.

5. FREQUENCIES FROM A VARIATIONAL CALCULATION

We have also developed a variational technique to calculate frequency changes arising from convective fluctuations (Rosenthal et al., these proceedings). The approach is based on linearising the full fluid-dynamical equations about the convecting state. The equations for the acoustic motions then contain cross terms between the acoustic part of the solution and the convective fluctuations. On the assumption that the time variation of the convective
terms may be neglected, these terms can be treated as time-independent perturbations using the variational method. The resulting frequency shift is

$$\frac{\delta \omega}{\omega} = \frac{2\pi R_o^2}{\omega^2 (\xi (R_o))^2} \int_0^{R_o} \left[ \frac{d \xi}{d r} \frac{d}{d r} (p_{\text{gas}} - p) \right] dr + \left( \frac{d \xi}{d r} \right)^2 (p_{\text{gas}} \Gamma_1 - p < \Gamma_1 >)
+ \xi \rho' \left( - \frac{1}{\rho} \frac{d p_{\text{gas}}}{d r} - g \right) dr,$$

where $M_{\text{mode}}$ is the mode mass and $\xi$ and $\rho'$ are the displacement and Eulerian density perturbations in the eigenfunction of the unperturbed model. This expression determines the change in eigenfrequencies relative to those of a hydrostatic model with the same density stratification.

The results obtained from evaluating equation (3), using the same numerical simulation as used above, are shown in Figure 5. The frequency shifts obtained vary substantially between timesteps in the simulation, which is due to the presence of fluctuations in the average model on time scales of minutes, notably due to the presence of p-mode oscillations in the simulations (Stein & Nordlund 1991). Nevertheless, it is clear that the general trend is to decrease the mode frequencies, and that the variation of the frequency shift with frequency follows a broadly similar trend to that seen in the other calculations.

The mean shift obtained is rather smaller than that seen in the data and shows a distinct tendency to flatten out for mode frequencies above about 3 mHz.

6. CONCLUSIONS

The observed solar frequencies are typically lower than those predicted by standard solar models for modes with frequency greater than 2 mHz. The frequency residuals, scaled by the normalised mode mass, increase in magnitude to about 20 $\mu$Hz at a frequency of 5 mHz and are predominantly a function of frequency. The lack of visible dependence on degree indicates that the residuals arise largely from errors in the modelling of the superficial layers of the Sun. We note, however, that inverse analysis shows small differences between the Sun and the model also in the deeper layers (Basu et al., these proceedings).

The standard models are based on treating convection by means of local mixing-length prescriptions. Frequency shifts of roughly the correct magnitude and dependence on frequency result from three different modifications to the treatment of the superadiabatic parts of the convection zone. In the case of the more sophisticated formulation developed by Canuto & Mazzitelli (1991) this is caused by a strong increase in the temperature gradient, relative to the mixing-length model, in the upper parts of the convection zone; this causes a corresponding decrease in the adiabatic exponent $\Gamma_1$ due to the rapid onset of hydrogen ionization. In the model using non-local mixing-length theory, and the model resulting from averaging aodynamical simulation of convection, the frequency shifts are dominated by a similar change in the effective $\Gamma_1$, but here induced by turbulent pressure. We note that in the latter two models, the temperature gradient is similar to that in the local mixing-length model, and unlike the steep gradient obtained from the Canuto & Mazzitelli formulation. The frequency effects are largest for the averaged hydrodynamical model, where the shifts are close to those obtained for the observed frequencies.

We have evaluated the effects of nonadiabaticity and the perturbations in the turbulent pressure by applying a time-dependent nonlocal mixing-length theory. These effects apparently nearly compensate for the changes in the equilibrium model computed with the nonlocal mixing-length theory, resulting in frequencies close to the adiabatic frequencies of the corresponding local mixing-length model.

Finally, we have applied a variational procedure based on the fluctuations found in the numerical simulation. This produces frequency shifts similar to those resulting from the adiabatic calculations for the modified models, although rather smaller than those in the data, particularly at high frequency.
Zhugzhda & Stix (1994) have also attempted to model the effect of convection on mode eigenfrequencies using a model consisting of alternating layers of upflowing and downflowing material. Their model therefore includes the advection of wavefronts by the fluid motions, but does not include the effect of turbulent pressure on the mean structure. With a reasonable choice of parameters, they are able to obtain a good fit to the data.

The present situation is therefore somewhat confusing in that, whilst different treatments of the surface layers produce frequency shifts of the correct general form and magnitude to explain the data, the precise behaviour of the shifts varies substantially depending on the assumptions used in the modelling. The results of the time-dependent mixing-length theory are particularly significant as they tend to imply that nonadiabatic effects, which are usually ignored, may be as important as changes to the stratification. We plan to explore the role of nonadiabaticity more thoroughly using numerical simulation.

ACKNOWLEDGMENTS

This work was supported in part by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Center. GH acknowledges funding from the Fonds zur Förderung der Wissenschaftlichen Forschung (project 8776-PHY) Österreichische Akademie der Wissenschaften. During part of this work, MJPFGM was co-supported by a Programa PRAXIS XXI grant from JNICT-Portugal.

REFERENCES