LOCAL AREA ANALYSIS OF HIGH-DEGREE SOLAR OSCILLATIONS:
NEW RING FITTING PROCEDURES

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ABSTRACT

Local–area analysis of five–minute solar oscillations using ring diagrams to determine subphotospheric velocity flows is on the brink of becoming an important tool in understanding convection zone dynamics. One of the main problems up to this point has been the large computational burden posed by the task of fitting the rings. We present here a faster method for carrying out the ring fits using data obtained with the High-\(\ell\) Helioseismometer at Kitt Peak. We first eliminate serious sources of noise, then use a perturbation approach to fit the azimuthally averaged spectrum. The parameters so determined are held constant while fitting the entire ring diagram.

Keywords: Helioseismology, Ring diagrams.

1. INTRODUCTION

The mapping of subphotospheric solar velocity flows can provide us with information necessary for a better understanding of convection zone dynamics. One way of carrying out this mapping is to analyze frequency splittings of the local wave field from many small (square) areas on the sun (Hill 1988, 1990, Patrón et al. 1995). Large–scale horizontal flows, caused by a combination of rotation and convection, can advect the waves, causing asymmetric shifts in the observed oscillation frequencies. As long as the size of the areas over which these flows are averaged are modest compared to the horizontal scale over which the flow fields vary, we can use inversion techniques to find a measure of the mean horizontal velocity vector over a range of depths for these areas.

To search for these frequency shifts we take three-dimensional Fourier transforms of the data (two in space, one in time) for each small area and build a power spectrum with respect to the frequency \(\omega\) and the horizontal wavenumbers \(k_x\) and \(k_y\). The power in such a diagram is distributed along curved trumpet-like surfaces rather than ridges and a cross-sectional cut of the data at constant frequency shows a series of rings. Each ring is produced by modes having the same radial order \(n\). The underlying flow field influences both the shapes and the displacement of the rings so measurements of these quantities can be used in inversion procedures to estimate the flow field. One can construct a mosaic map of large–scale convective motions, including meridional and spatially varying zonal flows, by using data from many localized sites.

Ring diagram analysis, as this technique has been called, has already been used to study convective flows beneath the solar surface (Hill 1988, 1990, Patrón et al. 1995), yielding several interesting results: a thin layer of high-speed flow near the surface and evidence of flows changing direction with depth in a spiral pattern. In order to determine whether these results are actually solar in origin or the results of systematic errors, and to see how these flows change with time, we need to look at more data observed with different instruments. Further, given data sets should also be analyzed in alternate ways. We need to determine how sensitive the results are to varied types of data analysis.

There is already a large amount of data from Mt. Wilson, the South Pole, and Kitt Peak that can be used to study the upper layers of the solar convection zone and the ring analysis technique itself. However, the main obstacle to these tests to date is that the ring–fitting procedure itself has been very expensive computationally. The data throughput problem will be compounded when data from the SOI-MDI instrument on SOHO as well as from GONG, and TON become available for analysis in the next year or so. It is imperative that we find efficient ways of carrying out the local–area analysis so that we will be prepared for actually studying the sun in a timely fashion given the vast quantity of data that will soon have to be dealt with.

One way to speed up the fitting procedures is to calculate power spectra whose rings are almost circular and are as free from background noise as possible. We have previously shown (Haber et al. 1995) that changing the coordinate system from a latitude-longitude grid to a ‘great-circle’ grid onto which the local wave fields are projected cleans up the power spectrum considerably for regions far from disk cen-
The rings become more circular and more complete when the data have been mapped onto a localized great-circle grid. This allows us to look for flows at higher latitudes. However, there is still substantial background power at low wavenumbers $k$, along with a degradation in the amount of power at high values of $k$ due to the presence of atmospheric seeing. There is also an angular component to the background field due to astigmatism in the instrument optics and noise due to the changing spatial resolution caused by foreshortening towards the solar limb. These sources of noise can make fitting the rings difficult and it behooves us to eliminate or at least reduce as many of these noise sources as possible before attempting to speed up the fit to the rings.

2. DATA ACQUISITION AND ANALYSIS

2.1 Spatial Filtering

The $1024 \times 1024$ intensity images employed for this analysis were obtained using the NSO High-$\ell$ Helioseismometer (HLH) (Harvey et al. 1991) on 22 June 1993. The data were taken with a 60 s cadence and for each image the modulation transfer function (MTF) was calculated for the full disk of the sun using the GONG GRASP analysis package (Toner & Jeffries 1993). The data were restored using the MTFs and the Wiener filter restoration procedure described in Toner, Jeffries & Duvall (1995) (also from the GRASP package). Next a subraster ($\sim 30^\circ$ on a side) was tracked throughout the day at the solar differential rotation rate appropriate to the central latitude of the subraster as determined by the coefficients given in Snodgrass (1984). The subraster at each time step was fitted with a two-dimensional 10th-order Chebyshev polynomial and was then divided by this smooth surface to spatially filter the data. This filtering removes limb darkening and transparency gradients across the subraster.

Next, the subrasters are spatially apodized and then temporally filtered by taking the ratio at each pixel to the same pixel at the next time step and then subtracting 1 from this ratio. Finally, Fast Fourier Transforms (FFTs) are performed in both spatial directions and in time so that a three-dimensional power spectrum can be computed. The dramatic effects of the MTF correction on power spectra can be seen when comparing cross-section cuts of power spectra calculated with and without the MTF correction. In Figure 1 we see that without the MTF correction the background power at low wavenumbers dominates the ring structure. The apparent hole at the centers of the diagrams is due to the effect of the high-pass spatial filter.

2.2 Ring Fitting

Once the power spectrum is computed we move to the more difficult task of finding the frequencies associated with each $k_x$, $k_y$, and $n$ value so that we will have the information necessary for inverting the

Figure 1: Effects of the Modulation Transfer Function (MTF) correction process on the rings. The slices of the power spectra shown here are taken at 3.4 mHz and a high-pass spatial filter has already been applied in both cases. Case a) shows a ring diagram from data without the MTF correction, whereas b) shows a ring diagram from data with the MTF correction. The ring structures at low wavenumbers are accentuated after the MTF correction.
data to determine flow velocities with depth. The technique reported here is an iterative one. First, an azimuthal average at every wavenumber is carried out so that we have what looks like a normal \( k - \omega \) diagram. A peak-finding routine, from the GRASP analysis software (Anderson, Jefferies, & Duvall 1990), is then used to fit Lorentzians to the averaged ridges at each \( k \) value in order to determine the frequency (\( \nu \)), the full-width at half-maximum (\( \Gamma \)), the peak power (\( P \)), and the terms for a linear fit to the background under each peak (\( b_0 + b_1 \nu \)). Figure 2 shows an example of these fits for \( \ell = 511 \).

Figure 2: Sample Lorentzian fits of an azimuthally averaged ring diagram for 22 June 1993 at \( \ell = 511 \) and \( n = 0 \) thru \( n = 4 \), generated by the peak fitting procedure of the GONG GRASP package. The solid line is the actual data while the dashed line is the fit.

These variables are parameterized along each ridge using a least-squares fit to a polynomial in \( \log(k) \). These functional fits are then used to determine the values of \( \nu, \Gamma, p, b_0, \) and \( b_1 \) at every point in each ring. Figure 3 shows these functional fits for the \( n = 2 \) ridge.

These values are kept fixed during the fit of the entire ring diagram which is carried out one ring at a time by taking a small volume in both \( k \) and \( \nu \) surrounding the ring and performing a maximum-likelihood fit to the data in this volume. When the parameters have been found for this region, the donut-shaped box is moved up in frequency and the procedure is repeated. This averaging in frequency helps reduce the noise in the inferred velocities. A maximum-likelihood fit is used that represents solar oscillations as randomly forced damped harmonic oscillations whose statistical distribution of power is \( \chi^2 \) with 2 degrees of freedom (Duvall & Harvey 1986). Woodard (1984) showed that a \( \chi^2 \) distribution with 2 degrees of freedom describes solar oscillations well.

In the maximum likelihood technique used here we construct the joint probability density for the outcome of the ring fit in terms of a number of model parameters. In this case, if \( P_i \) is the model \( P \) (given below) for a given frequency \( \nu \), and wavenumbers \( k_x \) and \( k_y \), the probability density at this point is given by

\[
\frac{1}{P_i} \exp(-O_i/P_i),
\]

where \( O_i \) is the observed power at the same \( \nu, k_x, \) and \( k_y \). (Duvall & Harvey 1986). The joint probability density is found by taking the product of these individual probabilities over all the independent frequencies and wavenumbers:

\[
L = \prod_i \left[ \frac{1}{P_i} \exp(-O_i/P_i) \right]
\]

which when we take the logarithm becomes

\[
-\ln L = \sum_i \left( \ln P_i + O_i/P_i \right) \equiv S.
\]

Maximizing the likelihood function \( L \), which is equivalent to minimizing \( S \) for the given observation, then gives the maximum likelihood estimates for the model parameters we are interested in.

The model \( P \) being fit for each ring is:

\[
P = \frac{ET}{2\pi} \left[ \frac{(\nu - H + (U_x k_x + U_y k_y)/(2\pi))^2 + (\Gamma/2)^2}{\nu^2 + (\nu - H)^2 + (\nu - c_0 - c_1 k)^2} \right] \cos(2\theta + \phi).
\]

Here \( H \) and \( \Gamma \) are the functional fits to the frequencies and fwhm described previously. \( E \) is the integrated power over the Lorentzian fit at the given \( n \) and \( k \) and is related to \( p \) by the formula \( E = p\Gamma/2\pi \). \( U_x \) and \( U_y \) are the components of the flow (averaged over depth) in the east-west and north-south directions respectively. The terms on the second line are all contributions to the background power, where \( b_0 \) and \( b_1 \) are from the functional fits to the background with respect to frequency and are held constant, while \( c_0, c_1 \) are parameters to be fit to the background as it varies in \( k \). \( \phi \) is a parameter to be fit which tells how the background varies as a function of angle for a given ring. Thus for any given ring there are five parameters to be fit: \( U_x, U_y, c_0, c_1, \) and \( \phi \) while five others are held constant: \( H, \Gamma, E, b_0, \) and \( b_1 \). To carry out the multidimensional minimization required we use the downhill simplex method given in Press (1989).

An iterative process will then take place, as any nonzero values of \( U_x \) and \( U_y \) indicate that the centers of the rings are offset and thus the azimuthal average taken at the beginning of the procedure should be taken about this offset point. Substantial flows
Figure 3: Parameterized fits for the quantities held fixed in fitting the $n = 2$ ring for MTF corrected data taken on 22 June 1993. The diamonds are the values found by the peakfinding program while the lines are the parameterized fits.
also mean that the trumpet patterns of power will be tilted with respect to the temporal frequency axis and that the cross-sectional cuts (at first at constant frequency) over which the ring fits are carried out should also be tilted and the fits computed over slices where the frequency for a given $k$ value changes as a function of the angle around the ring. A further development will be to add in higher order velocity terms and magnetic effects which can also affect the frequencies of the waves.

3. RESULTS & DISCUSSION

The time it takes to compute the ring fits with this technique is substantially faster than the method described by Patrón (1994), mainly because there are less parameters being fit in the three-dimensional power spectrum. The preliminary steps of taking the azimuthal average, fitting Lorentzians to the averaged spectrum, and then parameterizing the variables found in these fits take about 20 minutes on a Sun Sparstation 10/40 for 8 ridges for one day of data. The full ring fit takes about 80 minutes of computing for a single ridge over 35 frequencies from a $256 \times 256$ sub raster for one day of data. It is not a straightforward procedure to compare these numbers with the ring fitting technique carried out by Patrón et al. (1995) because of the different sizes of the arrays (they used $135 \times 135$ size sub rasters) and the different frequency resolution due to their use of four days of data strung together. However, if we assume that the timing scales linearly with these factors, then the method presented here is at least 7 times faster than the previous method.

Figure 4 shows the flows $U_x$ and $U_y$ determined from a ring fit of the $n = 2$ ring for 22 June 1993. These flows are for an area $\sim 15^\circ \times 15^\circ$ near disk center covering $128 \times 128$ pixels. We find a north-south flow, $U_y$, on the order of 90 m/s and an east-west flow, $U_x$, averaging between 10 - 20 m/s. These are averages over depth and do not reflect the actual flow speed at the solar surface. Bogart et al. (1995a), using different programs for generating the power spectra, and a completely different method for finding the averaged flows, found answers for a similar area of the sun that were comparable for both techniques. The large north-south velocity may be influenced by some systematic error in how we apply the differential rotation part of our tracking algorithm, or it may be an indication of the Ekman-like spiralling of the velocity vector with depth as found by Hill (1990) and Patrón et al. (1995). Until a real inversion of the data is carried out (which entails fitting all the rings from $n = 0$ to $n = 7$ and calculating a better estimate of the errors involved) we cannot really say what is happening on the sun.

The iterative process outlined in §2.2 has not yet been implemented and only rough estimates of the errors are plotted here. These estimates were determined by running the fitting procedure a number of times with different initial guesses for the downhill simplex method (dsm) used for carrying out the maximum-likelihood fit. The differences in the results give the estimate of the errors plotted in Figure 4. The dsm is fairly slow, as it searches parameter space somewhat like an amoeba, and calculating the errors in the fit is nontrivial. Yet, it is very good at finding the global minimum rather than various local minima. We are planning to try a Newton-Raphson minimization approach to further increase the speed of the fitting but for this to work one needs to be relatively close to the global minimum. However, the error calculations for Newton-Raphson are much more straightforward. A combination approach, wherein the dsm is used to find the parameters for the first frequency and then taking these parameters as the starting point for the Newton-Raphson gradient approach may be appropriate.

Figure 4: Velocities determined from fitting $n = 2$ ring as a function of frequency. $U_x$ and $U_y$ indicate flows in the east-west and north-south directions respectively, where $U_x$ is positive in the direction opposite solar rotation and $U_y$ is positive towards the south.

It is clear that the MTF correction improves the ring diagram by bringing out the ring structure at lower spatial frequencies, but it is also computationally expensive. Some of the same effects of this type of filtering are achievable by simply multiplying the power diagram by a power of $k$ as in Bogart et al. (1995b). We need to assess the effect of both these filtering processes on the frequencies determined from the ring-fitting procedures and on the results of an actual inversion in order to determine whether they are truly necessary.

Simulations are now underway so that we can test the ring-fitting procedure outlined here on known velocity flows. This will be crucial for understanding and
calibrating these local area analysis techniques.

Acknowledgements. NOAO is operated by AURA under cooperative agreement with the National Science Foundation. This work is supported in part by the SERC(UK), by NSF, and by NASA through grants NAG5-2258 and NAS5-30386.

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