ACOUSTIC WAVES IN RANDOM ENSEMBLES OF MAGNETIC FLUXES

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Abstract

To analyze the observational data and provide the appropriate diagnostic procedure for photospheric manifestation of solar oscillations it is necessary to take into account strong inhomogeneity of solar atmosphere with respect to distribution of magnetic fields. We study the collective phenomena in the propagation of acoustic waves and unsteady wave packets through quite regions, sunspots and plages, including time-dependent response of these regions to solar oscillations, the energy transfer mechanisms, frequency shift effects and reradiation of the acoustic waves in higher layers of atmosphere. We show that the dynamics of differently magnetized regions, their dispersion properties, and their response to the propagation of acoustic waves are completely different. We describe the effects caused by the specific distribution and randomness of magnetic flux tubes, which can be observed and which can provide the tools for diagnostic goals.

1. INTRODUCTION

Rich observational data on the acoustic wave propagation in the different magnetic regions of solar atmosphere obtained in the past few years form a new discipline of acoustic spectroscopy (Ulrich 1970; Leibacher and Stein 1971; Leibacher et al. 1985; Duvall et al. 1993), and evidently show that the effects of the wave propagation are different in differently magnetized regions (Braun et al. 1987; Braun et al. 1992; Brown et al. 1994). The observed difference in the response of differently magnetized regions to solar oscillations and understanding of the origin of this difference may give a reliable basis for the diagnostic methods.

We model solar atmosphere as strongly inhomogeneous medium containing the random ensembles of magnetic flux tubes. We distinguish two major classes of ensembles: 1) it widely spaced flux tubes randomly distributed in space and over their physical parameters - the model corresponding to quite regions (Fig.1a); 2) tightly settled bundles of magnetic fluxes - the model of fibril sunspot as a dense conglomerate of flux tubes (Fig.1b); we model plage regions as a loose mosaic of magnetic domains.

In quite regions the ensembles of widely spaced magnetic flux tubes are embedded in almost nonmagnetized plasma and may possess a wide variety of physical parameters - radius, magnetic field strength, plasma density inside and outside them; we take into account their noncollinearity as well. The characteristic transverse dimension $R$ of flux tubes is much smaller than the characteristic distance $l$ between them:

$$R \ll l$$

Magnetic filling factor of these regions is much less than unity, $\alpha = R^2/l^2 \ll 1$. We distinguish two subclasses in widely spaced ensembles and predict different effects depending on the actual distribution of flux tubes; for example, the maximum energy input and the observed frequency shift are different for regions with $l \ll \lambda$ (more populated regions regions) and $l \geq \lambda$ (less populated regions), where $\lambda$ is the acoustic wave length.

Second type of region contains tightly settled bundles of magnetic flux. Here also we distinguish two subclasses, namely, sunspots and plage regions, which should manifest significantly different properties: (a) we model fibril sunspots with highly variant physical parameters as dense conglomerate of flux tubes with magnetic filling factor close to unity, and (b) plage regions as the random ensembles of less tightly settled magnetic domains with magnetic filling factor $\alpha \approx 0.1 - 0.3$. The dispersion properties and the observational spectroscopy of these regions, as well as the mechanisms of energy transfer from lower to upper layers of the atmosphere are completely different.

2. QUITE REGIONS

The most important role in acoustic wave propagation in an ensemble of widely spaced flux tubes is played by resonant interaction - both absorption and scattering of sound waves by flux tubes (Ryutov and Ryutova 1976; Ryutova and Priest 1993 a,b). For resonance absorption (an effect similar to Landau damping in collisionless plasma) which requires a broad distribution function with respect to physical parameters of random flux tubes, the energy of acoustic wave is accumulated in the system of magnetic flux tubes and remains for a long time in the form of flux tube oscillations. Oscillating flux tubes then radiate accumulated energy as secondary acoustic waves in the upper layers of atmosphere.

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Fig. 1.— Classes of ensembles of magnetic structures: (a) widely spaced magnetic flux tubes; (b) dense conglomerate of random magnetic flux tubes

Since radiative damping rate depends on flux tube parameters, different flux tubes radiate in different times and at different heights. In the propagation of unsteady wave packet this leads to the spreading of the energy input region over scales much larger than the initial wave packet and results in clear morphological effects, such as, for example, as observed acoustic halos (Braun et al. 1992; Fernandes et al., 1992; Brown et al., 1992; Toner and Labonte, 1993).

2.1. Resonance Absorption and Scattering of P-modes

The key assumption in the theory is the introduction of a broad distribution function \( f(\Xi) \) for magnetic flux tubes, in order to reflect the great variety of observed magnetic structures and to set up an adequate theory for wave-propagation in such an inhomogeneous medium. \( \Xi \) is the set of flux tube parameters completely determining the individuality of separate flux tubes, namely their radius \( R \), plasma density ratio inside and outside flux tube, \( \eta = \rho_i/\rho_e \), magnetic field \( B \), and flux tube inclination \( n \), where \( n \) is the unit vector along the flux tube axis. The distribution function is normalized to the magnetic filling factor, \( \alpha \), of the medium (containing many flux tubes):

\[
\alpha = \int_0^\infty f(\Xi) d\Xi
\]

As it is mentioned earlier, the most important effect in the propagation of acoustic waves through the ensembles of widely spaced magnetic flux tubes is the resonant interaction - both absorption and scattering of the sound wave by flux tubes. The energy of sound waves is absorbed by magnetic flux tubes due to an effect similar to Landau damping: it consists of the resonant excitation of oscillations propagating along the flux tubes. The resonance condition is determined by sound speed and parameters of flux tube:

\[
\omega = (k \cdot n) v_{ph}
\]

where \( \omega \) and \( k \) are the frequency and wave vector of the acoustic wave, \( v_{ph} \) is the phase speed of the flux tube oscillation, and \( n \) is a unit vector along the tube axis. Here \( v_{ph} \) corresponds either to a kink mode, with \( v_{ph}^{m=\pm 1} = \sqrt{\eta/(1+\eta)} v_A \), or to sausage mode, with \( v_{PH}^{m=0} = c_i v_A \sqrt{c_i^2 + v_A^2} \) (\( c_i \) is the sound speed inside flux tube, and \( v_A \) is the Alfvén velocity). Thus, each acoustic wave with its own frequency and wave vector in a whole acoustic spectrum chooses from a variety of magnetic structures those flux tubes whose parameters satisfy the resonance condition (3) (Fig 2a). Damping rate of this process is proportional to magnetic filling factor of a region and atmospheric sound speed, \( c_{sa} \): \( \nu_L \approx \alpha c_{sa} \). The oscillations of flux tubes once excited can be radiatively damped through the emission of secondary waves (Ryutov and Ryutova 1976; Ryutova 1981; Ryutova and Priest 1993 a,b). The corresponding damping rate is proportional to characteristic radius of magnetic structures: \( \nu_{rad} \sim \omega(kR)^2 \). Thus, the energy of the incident acoustic waves remains for a long time

\[
\tau_{rad} \approx \frac{1}{\omega(kR)^2}
\]

in the form of flux-tube oscillation energy. In a time \( \tau_{rad} \) flux tube radiate secondary acoustic waves.

Obviously the different flux tubes radiate secondary acoustic waves over different times: for example, thicker tubes radiate sooner than thinner tubes; and, of course, the radiated waves have random phases (Fig. 2 b):

\[
\Delta \omega \approx \omega_L
\]

This process is a major one for those regions where magnetic filling factor \( \alpha \gg (kR)^2 \); that is, if characteristic distance between flux tubes \( \ell \) is less or much less than acoustic wave length, - more populated regions. In this case nonlinear effects in the collective phenomena become very important. Namely, the frequency shift becomes essentially nonlinear: \( \Delta \omega = \omega(k\xi)^2 \) (\( \xi \) is flux tube displacement). In the opposite case of less populated regions with \( \ell \ll \lambda \) the process of energy transfer to medium occurs through direct scattering of acoustic waves by flux tubes which leads to linear frequency shift and to the appearance of incoherent noise without a preliminary build up of the wave energy in flux tube oscillations. This case is somewhat similar to the case of narrow distribution function and,
the prediction of theory is close to those obtained for the model of identical flux tubes (Parker 1982; Bogdan and Zweibel 1987). However, the results obtained for the model of identical flux tubes are of a general interest, and can not be applied for diagnostics of randomly magnetized solar atmosphere. It seems, that even in our model of random magnetic flux tubes the scattering process which occurs mainly in regions with $l \ll \lambda$, plays a minor role in solar atmosphere dynamics: recent observations of sub-arcsecond photospheric bright points (Berger et al. 1995) show, that small-scale magnetic flux tubes occupy granular boundaries with mean distance $l \ll \lambda$. Therefore, the dominant magnetic effect in the p-mode propagation through quite regions is the resonance absorption and subsequent reradiation of the acoustic waves with random phases.

2.2. Unsteady Wave-Packets

Non-stationary representation of the acoustic modes is the important agent in helioseismology. The study of time-dependent response of randomly magnetized solar atmosphere to the propagation of unsteady wave packets (Ryutova & Priest 1993b) reveals some simple and remarkable phenomena which seem very promising in the inference of plasma parameters and morphology of different regions from observational results. For example, the interaction of an unsteady wave-packet with a random ensemble of flux tubes results in clear morphological effects, which can be observed as the acoustic halos - the emission of high frequency acoustic waves that are usually observed as localized regions surrounding active regions and seen in some quite regions as well (Braun et al. 1992; Fernandez et al. 1992; Brown et al. 1992; Toner and Labonte 1993). Theory predicts the spreading of the energy absorption region over scales much larger than the size of the initial wave packet. The regions of an efficient energy input, their localization, and frequency shift signatures crucially depend on the characteristic properties of the wave packet as well as on the distribution of magnetic flux tubes in space and over their physical parameters (including noncollinearity). We can determine the "size" of wave packet being "large" or "small" with respect to characteristic distance between magnetic flux tubes (or with respect to filling factor of medium). "Large" wave packet with the size $D > \lambda/\alpha$ (more likely realized in the regions close to active regions with the moderate magnetic filling factor), wave packet interacting with an ensemble of flux tubes is damped away without a considerable displacement: all the resonant flux tubes are excited in the initial area of the wave packet. At a time which is larger than the Landau damping time and less than the time of the radiation of secondary acoustic waves

$$\nu_{L}^{-1} \ll t \ll \nu_{rad}^{-1} \quad (6)$$

the wave packet is damped away, but the excited flux tubes have not yet radiated secondary acoustic waves. In other words, its energy remains in the form of natural oscillations of resonant flux tubes imitating the initial area of wave packet. The excited perturbations (kink or sausage mode) propagate along the flux tubes carrying the accumulated wave packet energy to higher layers of the atmosphere. After a time

$$t \approx \nu_{rad}^{-1} \quad (7)$$

secondary acoustic waves are radiated. As mentioned earlier (see Fig.2), different flux tubes radiate in different times and at different heights. This fact leads to a significant spreading of the region where the energy of the initial wave packet is transferred to the medium. The location of emission of secondary acoustic waves differs, of course, from the expected position of the wave packet in the absence of flux tubes. Presence of noncollinear flux tubes results in formation
of large emission regions (the "acoustic halos") which extend beyond the original location of magnetic flux tubes and greatly exceed the size of the initial wave packet.

For a "short" wave-packet, with \( D < \lambda / \alpha \) (more likely realized in the quiet regions), qualitatively, the picture is similar to that for a "large" wave-packet, but is much more complicated. During the traveling of the "short" wave packet through the ensemble of flux tubes, both excited and nonexcited resonant flux tubes exist simultaneously: the wave packet excites resonant flux tubes on its way and propagates further, leaving a trace of excited flux tubes, which in turn radiate secondary acoustic waves; the first excited flux tubes can already radiate their energy before the wave packet is finally damped away. In this case secondary acoustic waves coexist with the initial wave packet. The particular scenario of wave packet dynamics and the final region of the energy emission depend on the specifics of flux-tube distribution.

3. ACTIVE REGIONS

Dispersion properties and the dynamics of the active regions are essentially different from those of quiet regions. In sunspots and plages where magnetic flux tubes and domains form dense fibril structures most pronounced effect is an enhanced absorption of acoustic wave power incident upon them. The effect was described by Ryutova and Persson (1984) in the paper, "Dispersion Properties and Enhanced Dissipation of MHD-Oscillations in a Plasma with Random Inhomogeneities", which contains two main results: 1) dispersion relation whose tensor coefficients carry the information on the statistical properties of medium (note, that acoustic waves entering the magnetized region become modified MHD-waves), and 2) the enhanced absorption in fibril magnetic regions. Strong absorption of p-mode oscillations were observed in sunspot and plage regions by Braun, Duvall and LaBonte (1987) and later many observed properties were found. LaBonte and Ryutova (1993) developed a theory of enhanced absorption of p-modes based on realistic inhomogeneous model of sunspot and plage regions, and found a good agreement with all the observed regularities. Nonlinear theory of the interaction of acoustic waves with active regions (Ryutova et al. 1991) shows that there are different scenarios for the process of the absorbed energy release: depending on the statistical properties of background inhomogeneities the absorbed energy can have different observational features like umbral dots, moving magnetic features in penumbra and acoustic halos at the edge of active region.

3.1. Dispersion Relation

Unlike the propagation of acoustic waves in a homogeneous plasma, in the case of a fibril medium the vortex component of the equations of motion is essential. For harmonic waves, these equations result in the anisotropic dispersion relation with renormalized phase velocity, which depends on the propagation angle in the xy-plane (magnetic field is directed along the z-axis):

\[
\omega^2 = 2 \left( \frac{1}{\rho (c_s^2 + \nu^2)} \right)^{-1} \left( \frac{1}{\rho} k^2 + Q_{\alpha \beta} \frac{k_{\alpha} k_{\beta}}{k^2} \right)^{1/2}
\]

\( Q_{\alpha \beta} \) is a tensor whose symmetry is determined by the statistical properties of medium; in other words, by the field of background density and magnetic field variations. This dispersion relation that significantly differs from that for homogeneously magnetized regions, can be directly used for diagnostic goals: the measured \( \omega(k_x k_y) \) diagram together with the equation (8) can give the morphological map of the observed region. Given that "time-distance helioseismology" proposed recently by Duval et al. (1993) contains as an important agent the group velocity of acoustic waves, for planning measurements it necessary to take into account the variation of the dispersion properties of medium along the ray trajectory of acoustic waves.

3.2. Enhanced Absorption

The physical mechanism responsible for the enhanced absorption of acoustic waves propagating in such an inhomogeneous media is easily understood. The perturbations of all parameters in a propagating wave are different inside the different flux tubes. In a dense conglomerate, flux tubes have common boundaries. Near those boundaries there appear strong local gradients with characteristic scale of the order of background inhomogeneities \( R \ll \lambda \). The presence of small-scale gradients results in the enhanced absorption of the wave energy with the enhancement factor \( f \propto \lambda^2 / R^2 \). There are two sub-regimes corresponding to higher and lower wavenumbers with respect to some critical value \( k_c \) which is determined by plasma parameters. In the viscosity dominant case, for example,

\[
k_c = \frac{\nu}{R^2 \text{cyc}}
\]

where \( \nu \) is the kinematic viscosity coefficient. The damping rate of p-modes with low wavenumbers \( k < k_c \) scales as \( k^2 \), that is, rises from zero, and reaches some value near the critical wavenumber:

\[
\text{Im} \ k \simeq k_c \left( \frac{7}{6} k_c^2 R^2 + \frac{1}{2} \frac{k_c}{k} \frac{\alpha}{p_m} < \frac{k}{\Pi^2} \right)
\]

Here \( p_m = \frac{\rho^2}{4\pi} \) and \( \Pi = p + p_m \) are magnetic and total pressures inside magnetic flux tube, the average is an ensemble average over the scale much larger than the radius of inhomogeneity and much less than the wavelength. The first term in Eq.(10) corresponds to
usual losses in homogeneous medium, while second term is provided by the presence of small scale inhomogeneities. Soon after critical value, at \( k \geq k_c \), the damping rate saturates, and becomes independent on the wavenumber:

\[
q_0 = Im \ k = k_c \frac{1}{2} \alpha \frac{\langle p_m^2 \rangle}{\Pi^2} > \]

and the enhancement factor \( f \) is quite large:

\[
f = \frac{3}{7} \frac{1}{k_c^2 R^2} \alpha \frac{\langle p_m^2 \rangle}{\Pi^2} \gg 1
\]

(12)

For a quantitative estimate, we use the observed quantities for \( k \) and \( R \), take the filling factor \( \alpha = 1 \) as appropriate to a sunspot, and assume that \( \frac{\langle p_m^2 \rangle}{\Pi^2} \approx 0.5 \) for a sunspot with a photospheric magnetic field of 2 kG (consistent with the models of Maltby et al., 1986). This gives an enhancement factor of

\[
f = 3 \times 10^4.
\]

(13)

Note that lower filling factor in plage compared to sunspots explains the reduced absorption seen in the observations, despite the larger size of the plage.

To unify the description of both limiting cases we derived the following interpolation formula for the total absorption coefficient \( \gamma_T \):

\[
\gamma_T = 1 - \exp \left( -q_0 L \frac{k^2}{k^2 + k_c^2} \right)
\]

(14)

here \( L \) is the sunspot dimension, and \( q_0 \) is the damping rate in saturation regime (Eq. (11)). This interpolation formula was used to match the theoretical curves with observational data for six sunspots. We summarize some of our results in Table 1. Examples for two sunspot groups are shown in Fig.4. The expressions (9)-(14) give a simple and natural explanations of regularities in the observed properties of the p-mode absorption in sunspot and plage regions:

1. The fraction of the incident p-mode power absorbed is zero at low wave numbers \( (k < 0.1 \text{Mm}^{-1}) \),

2. Theoretical value of \( k_c \) for all six sunspots coincides with the observed values of \( k \) corresponding to onset of saturation.

3. The absorption level increases with sunspot size. The local damping rate has no dependence on the sunspot size (cf. Eq.(11)), and the total absorption of a spot scales simply with the path length through the spot (cf. Eq.(14)).

4. The absorbed fraction is larger in spots than in plages. At the same time, the absorption per unit magnetic field appears to saturate in sunspots compared with plages. The dependence of the enhancement factor (12) on the magnetic field strength is \( f \propto B^4 \) for weak magnetic fields; at large values of \( B \), when \( \langle p_m \rangle \) becomes of the order of \( \langle \Pi = p + p_m \rangle \) the enhancement factor saturates because it becomes independent on the magnetic field strength. In plages, where magnetic filling factor is less than unity, the average magnetic pressure is always less than the sum of magnetic and gaseous kinetic pressure, which explains the observed difference.

5. According the observational data the dependence...
of the total absorption coefficient on the temporal frequency is different from those on wave number. This fact is in contradiction with the linear dispersion relation and indicates to more complicate $\omega - k$ relationship. We believe that this phenomena is quite natural and is provided by nonlinear processes and higher dispersion effects.

3.3. Nonlinear Stage

Observations of umbra, penumbra and plage regions show highly dynamic nature of these regions and quite uneven temperature distribution. The question is what role is played in these dynamic event by solar oscillations and where the absorbed fraction of acoustic energy goes. This study requires nonlinear consideration of the interaction of p-modes with dynamic magnetic structures. Ryutova, Kaisig, and Tajima (1991) studied analytically and numerically the propagation of nonlinear acoustic waves in closely packed ensembles of magnetic flux tubes and found that the energy input in such systems is determined by different scenarios which depend on the statistical properties of the medium. There is an interplay between dispersion, nonlinearity and dissipation, and so the energy of the primary acoustic wave can be transferred to the medium either by the formation of shocks, or by a preliminary storage of energy in a series of solitons and its subsequent release; or the process of enhanced absorption of wave energy can stop at the linear stage due to the usual dissipative effects. Fig. 4a shows the time variation and velocity field $V_x$ (1D-problem) and shock formation in the strongly nonuniform background medium. In Fig.4b time variation of thermal energy, $\Delta E_{th}$ for different wave length is shown; here $\Delta E_{th} = E(t) - E(0)$.

Nonlinear dispersion relation (Ryutova, Kaisig, and Tajima 1991) and corresponding equation show that the energy absorbed by sunspot and plage regions should have different observational signatures depending on the statistical properties of region. For example, there is a wide range of parameters characterizing the formation of standing shocks, which in the presence of inhomogeneities have a number of peculiarities (such as isothermal or isomagnetic shocks). Besides, the energy of a primary waves can be concentrated in some small regions, e.g. stored in the system of solitons which are later damped away. From observational point of view these effects can be seen as umbral and penumbral dots; under some conditions, namely when penumbral mass flows are involved the solitons may travel along the magnetic field lines appearing in a form of moving magnetic features.

REFERENCES