THE VELOCITY DIFFERENTIAL EMISSION MEASURE: DIAGNOSTIC OF BULK PLASMA MOTION IN SOLAR FLARES

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ABSTRACT

Mass motions are a ubiquitous product of solar flare energy release. A better understanding of the flare plasma's distribution—how much is moving and how fast—permits insight into the mechanisms of energy transport (and release) which lead to those motions. Observationally, mass motions during flares are often manifested in the shape and location of soft X-ray emission lines. Observed line profiles generally exhibit a width greater than the thermal Doppler width and a blue-wing asymmetry which has been cited as evidence for plasma motions along the line of sight. Past efforts to characterize this excess width and asymmetry have primarily involved the parametric fitting of a double-Gaussian form. In this paper we show, however, that simple two-component models are inconsistent with the observed evolution of spectral lines and hence serve as a poor diagnostic of plasma motions. We therefore generalize the synthesis of line profiles to the case of a continuum of Gaussian components, by introducing a quantity which we term the velocity differential emission measure (VDEM). The VDEM measures the distribution of emission from a volume of plasma as a function of its line-of-sight velocity. It can either be computed from theoretical model atmospheres, or recovered from observed line profiles using an inversion technique. We present the VDEMs of two model flare atmospheres and discuss the differences between them. We also deconvolve a VDEM from a representative Ca xix flare spectrum obtained by the Bragg Crystal Spectrometer instrument aboard Yohkoh.

Subject headings: Sun: corona — Sun: flares — Sun: particle emission — Sun: X-rays, gamma rays

1. INTRODUCTION

Various types of mass motions may occur during a solar flare, such as the expulsion of material from regions of imperfect magnetic confinement, the movement of an entire magnetic structure (flux tube), or the motion of plasma within a flux tube, driven by pressure gradients induced by the transport of energy throughout the structure. Because of the well-known radiative instability of plasma below 107 K (e.g., Raymond, Cox, & Smith 1977), material energized in flares rapidly rises to soft-X-ray-producing temperatures. Consequently, motions within a flux tube are frequently manifested in the shape and location of soft X-ray emission lines. Such lines have been observed with the Bragg Crystal Spectrometers (BCS) on both the Solar Maximum Mission (SMM) and Yohkoh satellites (Acton et al. 1980; Culhane et al. 1991).

These instruments generally have poor absolute wavelength calibration, so that determination of the line's absolute position is problematic and usually involves calibration against its position late in the flare, when motions are assumed to have subsided (e.g., Antonucci, Gabriel, & Dennis 1984). However, the shape of the line is generally well-determined; it usually exhibits (i) a width which significantly exceeds the thermal Doppler width appropriate to the emitting ion, and (ii) a blue-wing asymmetry, particularly early in the flare. Excess line broadening is generally greatest early in flares but is evident to some degree at all times during the flare (Doschek et al. 1980). Most interestingly, no center-to-limb dependence for the line broadening has been found: profiles of limb flares show excess widths similar to those produced by flares originating from the solar disk. Researchers have variously attributed the excess widths to processes which are independent of the flare's aspect angle, such as turbulence (e.g., Fludra et al. 1989), magnetic stochasticity (e.g., Antonucci, Rosner, & Tsinanos 1986), or magnetically driven motions such as Alfvén waves along twisted magnetic field lines. However, it is not necessary to introduce such mechanisms to explain the line broadening in limb flares, as the existence of differential plasma flows along the line of sight can adequately explain the symmetric broadening (e.g., Li, Emslie, & Mariska 1989; Alexander 1990).

The blue-wing asymmetry has been cited as observational evidence for plasma motions along the line of sight (e.g., Antonucci et al. 1987), as expected in the canonical picture of “chromospheric evaporation” (e.g., Antiochos & Sturrock 1978). In the evaporation scenario, the chromospheric plasma undergoes significant heating, such as by an electron beam or thermal conduction front. The resulting large temperature enhancements and concomitant pressure gradients (e.g., Mariska, Emslie, & Li 1989) drive the heated material upward into the lower pressure coronal portion of the flux tube. However, the exact cause of this upward motion has been debated at length (Doschek et al. 1986), and candidates other than evaporation (such as the collision of an emerging flux tube with an overlying structure) have been proposed. Furthermore, simple heating models (e.g., Li et al. 1989; Emslie, Li, & Mariska 1992) fail to account satisfactorily for observed spectral line profiles. Further progress therefore would seem to require more detailed information on plasma flows than can be provided by simple spectral fits to data.

Past efforts (e.g., Antonucci et al. 1984, 1985; Doschek 1990; Fludra et al. 1989) to characterize the widths and asymmetry of observed spectral lines have frequently involved the fitting of two Gaussian components to the profiles: one centered on the principal emission component of the ion; and the other centered on a displaced wavelength corresponding to upflow velocities ranging from 30 to 400 km s−1. Each Gaussian com-
ponent ostensibly represents the emission of a separate volume of plasma: one moving, one stationary. However, the widths of the fitted two-component Gaussians are generally not the thermal widths of the ion; in order to fit the observed line profile, they are artificially broadened by invoking “microwave turbulence” or other “nonthermal” broadening processes (e.g., Doschek 1990). In addition, an ad hoc relationship between the width of the “moving” component and (i) the width of the “stationary” component or (ii) the “moving” component’s velocity, must be postulated to reduce the number of free parameters in the fit from six to four and so obtain statistically significant values (e.g., Fludra et al. 1989; Antonucci et al. 1984).

On the basis of these fits, one derives the Doppler temperature T_D and the average line-of-sight velocity v of the “evaporated” plasma. However, it must be realized that such double Gaussian models are simply parametric fits to the data and not necessarily good tests of the presence of upflows (Antonucci et al. 1987); (i) lack a solid physical rationale for the assumed relationship between the widths of the two components or the width of the “moving” component and its velocity; (ii) are inadequate for comparison with numerical energy transport models.

Therefore, in the absence of any a priori reason why the flaring plasma should consist of only two components, as opposed to three, 10, or (more realistically), a continuous distribution of components, an investigation is clearly warranted to see whether two-component models are consistent with the shape and evolution of observed spectral lines and, if not, to propose a better method for obtaining the line-of-sight velocity distribution from soft X-ray spectra.

In this paper, we examine Ca xix line profiles observed by the Bragg Crystal Spectrometer (BCS) aboard the Yohkoh spacecraft. First, using a moment analysis (§ 2) we show explicitly that the evolution of observed soft X-ray profiles is inconsistent with simple two-component models, unless strong correlations exist among the parameters of the fit. In view of the two-component models’ shortcomings, we then develop a method which characterizes the flaring plasma’s motion with not just two, but instead a continuum of elementary Maxwellian components. The strength of the Maxwellian component with a line-of-sight velocity v is proportional to a quantity which we term the velocity differential emission measure (VDEM; § 3). VDEM is directly related to the flaring plasma’s properties. It can be either straightforwardly computed from model atmospheres, or alternatively, it can be inferred from observations by inverting an integral equation (§ 4) that determines the line profile for the plasma’s VDEM.

Using data from hydrodynamic simulations of an electron-heated solar flare (Mariska et al. 1989), we have computed VDEM functions and the resulting spectral line profiles (see Li et al. 1989). We test our spectral inversion procedure by applying it to those synthetic spectral lines, thereby recovering the model atmospheres’ VDEM. Having proved the utility of the VDEM concept and inversion technique, we then extract a VDEM from an observed flare spectrum and discuss (§ 5) the results obtained.

2. ANALYSIS OF YOHKOH CA XIX SPECTRA USING INTEGRAL MOMENTS

Integral moments, or weighted averages, provide a way to characterize the properties of a distribution function without resorting to parametric fits. The lowest normalized moments of I(λ) contain the “minimal” information on the profile, such as its centroid, width, and asymmetry (skewness). In our case, the distribution function is I(λ), the profile of the observed Ca xix flare spectral line.

We define the nth normalized moment about the resonance line peak as

\[ \langle \Delta \lambda^n \rangle = \frac{\sum_{i} (I(\lambda_i - \Lambda_{\text{max}}) \Delta \lambda_i)}{\sum_{i} I(\lambda_i) \Delta \lambda_i} \]

where \( \lambda_i \) is the wavelength at which the line peaks, \( \Delta \lambda_i \) is the detector bin width (\( \sim 0.3 \) mÅ for the Yohkoh BCS), and I(\( \lambda_i \)) is the observed intensity (photons cm\(^{-2}\) s\(^{-1}\) Å\(^{-1}\)) in the ith wavelength bin. Normalized moments do not depend on the flare’s absolute intensity; rather they characterize the shape of the line profile alone. They therefore provide a model-independent measure of changes in the width and blue-wing asymmetry of spectral line profiles and a means by which to compare different profiles.

Integral moments can be defined in a number of ways. Indeed, Mariska, Doschek, & Bentley (1993) utilized an alternative set of moments, \( \langle (\lambda - \lambda_0)^n \rangle \), defined about the line’s rest wavelength, \( \lambda_0 \), rather than the peak, \( \lambda_{\text{max}} \). However, our moments (about \( \lambda_{\text{max}} \)) provide a more intuitive and straightforward measure of the changes in the line’s shape, in that they quantify changes which are visually apparent. For example, the first and third moments about the peak characterize the development of any blue wing relative to the main component because of the weights that such features contribute to the moments. However, moments about \( \lambda_0 \) are dominated by the offset of the principal component from \( \lambda_0 \) and its changing intensity, rather than by changing emission in an extended wing. Hence the use of moments about an absolute reference wavelength makes it difficult to isolate the evolution of the (physically interesting) wing. Also, caution must be observed not to interpret moments about \( \lambda_0 \) as direct and unqualified evidence for upflows in the flare plasma (see Plunkett & Simnett 1994).

We examined three flares (1992 September 6, 1992 January 5, 1991 November 9) observed by the Yohkoh BCS (Culhane et al. 1991; Lang et al. 1992) and computed the moments for each 9 s accumulated spectrum. For a given spectrum the first moment measures the offset of the line’s centroid, \( \bar{\lambda} \), from the line’s maximum \( \lambda_{\text{max}} \). The second moment characterizes the line’s width, while the third reflects the asymmetry (skewness) in the profile.

The viability of a two-component fit can be tested by comparing the evolution of the observed moments with those of a series of double Gaussian profiles. Each of these synthetic profiles consisted of a principal Gaussian, of unit width and height, centered on the origin, added to a secondary Gaussian, of width \( w \) and height \( h \), with a peak located some distance \( \delta \lambda \)
from the stationary Gaussian. We computed, as functions of the three parameters, \( w, h, \) and \( \delta \lambda, \) the first and third moments of the synthetic profiles according to equation (1). We then reduced them to dimensionless form by dividing by the first and third powers, respectively, of the feature's standard deviation: 
\[ s = \left( \langle \Delta \lambda^2 \rangle - \langle \Delta \lambda \rangle^2 \right)^{1/2} \]
The evolution of the moments were calculated for two straightforward evolutionary scenarios, one in which the location and width of the secondary component were fixed and only its intensity varied, and the other in which the intensity and width were fixed and only the location varied.

In Figure 1, we plot as open diamonds the evolution of a model with \( w = 3, \delta \lambda = -2, \) and \( h \) varying over the range 0.1–1.5. The filled circles exhibit the phase space evolution of a model in which \( h = 0.5, w = 3, \) and \( \delta \lambda \) varies from –5 to 5. The evolution of the observed moments for a near disk-center flare, on 1992 January 5 (13:14 UT), is shown as crosses, the size of which represent the uncertainties associated with measurements of each 9 s accumulation of flare data. The wavelength range used to calculate the observed moments was limited to an interval that excluded most of the contribution from the d13 satellite feature (Seeley & Doschek 1989), so that the moments would not be contaminated by excess red-wing emission from this feature.

The poor fit between the phase space evolution of the actual flare, compared to those of the two-component models, is obvious. Other values of \( h, \) and \( w, \) as well as the other two flares examined, yield similarly poor results. Therefore, a double Gaussian model in which either: (1) a blue emission component with fixed velocity and width emerges, increases in intensity, and then subsides; or, (2) a secondary component, of fixed width and intensity, moves with varying velocities is inconsistent with the observed spectral data.

For the two-component models to match the flare's phase space evolution, the parameters \( \delta \lambda, h, \) and \( w \) must be somehow correlated and "conspire" to maintain the appropriate interrelationship between the nondimensionalized moments, a result which is difficult to understand a priori. Our moment analysis therefore strongly suggests that a continuum of emission components, rather than an artificial combination of a pair of "moving" and "stationary" components, is necessary to represent accurately the observed data.

![Figure 1](image_url)

**Figure 1.** The phase plots of \( \langle \Delta \lambda^2 \rangle / s^3 \) vs. \( \langle \Delta \lambda \rangle / s \) for double-component models: one in which only the secondary component's intensity varies (open diamonds); and one in which only the separation of the secondary Gaussian from the principal Gaussian varies (filled circles). The moments of 39 spectra from the 1992 January 5 are depicted as crosses.

### 3. THE VELOCITY DIFFERENTIAL EMISSION MEASURE (VDIM)

We seek a means to obtain the distribution of the flare plasma's line-of-sight velocity from observed spectral line profiles. It is known that any observed optically thin, collisionally excited spectral line \( I(\lambda) \) is a convolution of infinitesimal Gaussian line profiles, each produced by an elementary volume \( d^2 r \) of plasma with a specified electron number density \( n_e(r) \) (cm\(^{-3}\)), line emissivity \( G(T(r)) \) (photons cm\(^{-3}\) s\(^{-1}\)), and line-of-sight velocity \( v \) (cm s\(^{-1}\)) at position \( r \) in the source of volume \( V \). Hence, in the absence of contamination by other spectral features, the line profile observed at Earth can be expressed in units of photons cm\(^{-2}\) s\(^{-1}\) Å\(^{-1}\) as

\[
I(\lambda) = \frac{1}{4 \pi D^2} \int \int n_e(r) G(T(r)) \lambda_j(\lambda, v) d^2 r,
\]

where \( D \) is the Earth-Sun distance. The Doppler-displaced unit Maxwellian line profile, \( \lambda_j(\lambda, v) \), is given by

\[
\lambda_j(\lambda, v) = \frac{1}{\sqrt{2 \pi \sigma}} \exp \left\{ -\frac{[\lambda - \frac{\lambda_0}{c} (1 - v/c)]^2}{\sigma^2} \right\},
\]

where velocities toward the observer are positive and \( \sigma \) (Å) is the thermal Doppler width of the profile:

\[
\sigma = \frac{\lambda_0}{c} \sqrt{\frac{kT}{m}}.
\]

In equation (4), \( T \) is the electron temperature, \( k \) is Boltzmann's constant, and \( m \) is the mass of the ion in question. If we wish to take into account the possibility that line broadening does not arise solely from differential flows, but instead from a separate mechanism, we may write the profile's width as a combination of its thermal Doppler width and a nonthermal velocity, \( \xi \), excited by turbulence or some other process:

\[
\sigma = \frac{\lambda_0}{c} \left( \frac{kT}{m} + \frac{\xi^2}{2} \right)^{1/2}.
\]

In this paper, we consider line broadening to be due purely to line-of-sight plasma flows and therefore take \( \xi = 0 \).

Following the steps of Craig & Brown (1976), we transform equation (2)'s volume integral into an integral over the corresponding velocity domain, by writing

\[
d^2 r = |V(\mathbf{v} \cdot \hat{s})|^{-1} dS \; dv,
\]

where \( S \) is the element of surface of constant velocity in some region and \( \hat{s} \) is the unit vector normal to that surface. In general, there will be \( N(v) \) surfaces on which the line-of-sight velocity of the plasma is \( v \). Therefore, the total volume of material in the line-of-sight velocity range \( v \) to \( v + dv \) is

\[
dV = \sum_i \left( \int_{S_i} |V(\mathbf{v} \cdot \hat{s})|^{-1} dS \right) dv.
\]

Equation (2) for the intensity of the spectral line is consequently

\[
I(\lambda) = \frac{1}{4 \pi D^2} \int \lambda_j(\lambda, v) \sum_i \left( \int_{S_i} n_i^2 |V(\mathbf{v} \cdot \hat{s})|^{-1} dS \right) dv.
\]

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We write this in the form

$$I(\lambda) = \frac{1}{4\pi D^2} \int VDEM(\nu) \mathcal{I}(\lambda, \nu) d\nu,$$

where \( VDEM(\nu) \) is the emissivity-weighted measure of material in \( dV \) with line-of-sight velocity between \( \nu \) and \( \nu + d\nu \); i.e.,

$$VDEM(\nu) = \sum_{i} n_i G(T) \left( \iint_{S_i} n_i^2 \left| \mathbf{V} \cdot \hat{s} \right|^{-1} dS_i \right),$$

with the units of photons s^{-1} (cm s^{-1})^{-1}. Physically, \( VDEM \) determines the intensity of the line emission from material with line-of-sight velocity \( \nu \).

If the surfaces of constant velocity are also surfaces of constant \( n_\nu \), then \( VDEM(\nu) \) may be rewritten in the simpler form,

$$VDEM(\nu) = n_\nu^2 G(T) \frac{dV}{d\nu} = n_\nu^2 G(T) A(s) \frac{ds}{d\nu},$$

where \( A(s) \) is the total surface area, normal to the curvilinear coordinate \( s \), of the material with line-of-sight velocity \( \nu \). For example, in simple one-dimensional loop models, we may take \( s \) to be the distance along a flux tube’s length and \( A(s) \) to be the loop’s cross-sectional area with line-of-sight velocity \( \nu \). Alternatively, \( VDEM(\nu) \) may be written

$$VDEM(\nu) = n_\nu^2 G(T) \hat{A}(z) \frac{dz}{d\nu},$$

where \( \hat{A}(z) \) is the projected area (on the sky) of the loop at distance \( z \) from the observer. The equivalence of equations (10) and (11) essentially reflects the fact that how the volume of the emission region is sliced up is irrelevant to obtaining \( VDEM \).

In practice, the calculation of \( VDEM \) will be done using equation (10), while the inversion of \( VDEM \) profiles obtained from observed spectra could proceed, in the absence of a priori knowledge of the flare geometry, by using the volumetric definition, equation (9).

VDEM is analogous to the usual (temperature) differential emission measure, \( DEM \sim n_\nu^2 dV/dT \), which has been used for decades to characterize theoretical and empirical model atmospheres (e.g., Withbroe 1978). Like DEM, VDEM represents the “middle ground” between physical conditions in the atmosphere and observed data: just as DEMs deduced from model temperature and density profiles have been compared with DEMs deduced from EUV spectra (consisting of unresolved lines formed at different temperatures), so VDEMs deduced from dynamic flare models can be compared with those inferred from spectrally resolved lines formed at one characteristic temperature. We note that there already exists considerable confusion regarding the definition of “differential emission measure” in the literature. For example, all of \( n_\nu^2 dV/dT \), \( n_\nu^2 dV/d\ln T \), \( n_\nu^2 dz/dT \), \( n_\nu^2 dz/d\ln T \), have been used by various authors in the past, with the parameter \( z \) playing various roles (e.g., vertical distance, length along the loop, length along the line of sight, etc.). To set the record straight here, we term the VDEM(\nu) of equation (9) the volumetric VDEM (units of photons s^{-1} cm^{-1}), while VDEM(\nu)/A(s) or VDEM(\nu)/A(\lambda) is the specific, or one-dimensional, VDEM, with units of photons cm^{-2} s^{-1} cm^{-1}.

A specific VDEM can be computed directly from axisymmetric, one-dimensional (e.g., plane-parallel, toroidal) model atmospheres \( [n_\nu(s), T(s), \mathbf{v}(s)] \) where \( s \) is a suitable position parameter using its definition, equation (10) or (11).

This the “forward calculation.” Alternatively, the volumetric VDEM may be recovered from observed flare data by deconvolving the integral equation (8) (we term this the “backward calculation”). Estimating the appropriate area of emission then permits a calculation of the specific VDEM for comparison with those of model atmospheres. In this section, we present results of the forward calculation; then § 4 addresses the inversion or deconvolution problem.

Specific VDEM functions were computed from model atmospheres generated by simulating the response of the solar atmosphere to electron beam heating. We used the series of hydrodynamic models computed by Mariska, Emslie, & Li (1989) (hereafter MEL) and the emissivity function \( G(T) \) appropriate to the Ca xix resonance line (Mewe & Gronenschild 1981). VDEM curves were constructed using the physical properties, \( n_e(s), T(s), \) and \( \mathbf{v}(s) \), \( s \) = position along the loop) of the plasma specified by the MEL models. In these one-dimensional models, the surfaces of constant velocity are, naturally, also surfaces of constant \( n_\nu \), and so equation (10) was utilized for the computation. The flaring loop was assumed to be semicircular and at disk center; other geometries and observation angles (which relate plasma velocities to line-of-sight velocities) can be accommodated straightforwardly (cf. eq. [9]). The electron beam parameters are: total energy flux \( F = 5 \times 10^{10} \) ergs cm^{-2} s^{-1}, “knee” energy \( E_k = 15 \) keV, and spectral index \( \delta = 6 \), as described in MEL. We present in Figure 2 the specific VDEM curves (VDEM per loop cross-sectional area \( A(s) \)) for two heating models, chosen simply to illustrate VDEM’s power to discriminate different conditions: one model is characterized by a low-density preflare corona \( (n_e \approx 1 \times 10^{10} \text{ cm}^{-3}) \) and the other by a relatively high density preflare corona \( (n_e \approx 2 \times 10^{11} \text{ cm}^{-3}) \).

For the low-density model (e.g., early in the flare, before evaporation has filled the flux tube with hot plasma), the coronal column density is not sufficient to have much effect on the beam electrons’ energies. Hence a large fraction of the beam’s energy reaches the low-temperature chromosphere, and large velocities are generated in an “explosive” evaporation (Fisher 1987). The corresponding VDEM curve therefore has an almost bimodal shape: there is a preponderance of plasma with very low line-of-sight velocities (corresponding to heated, but not yet moving material—see Fig. 1 in MEL), an absence

![Fig. 2.—The specific VDEMs (VDEM per loop cross-sectional area, VDEM/A(s)) for two atmospheres: a low-density, preflare corona (dashed line) and a high-density, preflare corona (solid line). The specific VDEMs' units are 10^{11} photons cm^{-2} s^{-1} (km s^{-1})^{-1}.](image)
at intermediate speeds of \( \approx 100 \text{ km s}^{-1} \), and increasing amounts at higher line-of-sight velocities near 270–300 km s\(^{-1}\).

For the high-density model (which could represent the situation in the flare’s later stages), the corona is now a collisionally thick target and absorbs much of the beam’s energy. In addition, the large coronal overpressure effectively suppresses any upward motion. Consequently, the evaporation is more “gentle,” driven by a combination of very high energy electrons and thermal conduction (Fisher, Canfield, & McClymont 1985). There is again a substantial amount of plasma at near-stationary line-of-sight velocities, but the bulk of the plasma is moving with more moderate speeds of \( \approx 200 \text{ km s}^{-1} \).

In contrast to the bimodality observed for the low-density case, the VDEM of the high-density preflare corona exhibits a more gradual progression from low to high velocities. We see, then, that VDEM profiles, if they can be determined reliably from spectral line profiles, are quite capable of distinguishing between different physical conditions in flare plasmas.

Once the VDEM of a model atmosphere is known, synthetic line profiles can be generated by convolving the VDEM with the Gaussian kernels in the integral expression, equation (8). Exact line profile synthesis (see Li et al. 1989) requires that we take into account the different widths of the Maxwellians corresponding to each elemental volume of the flare plasma, as determined by their temperature \( T \). However, by binning all plasma elements with the same line-of-sight velocity together in the VDEM calculation, this information is lost; and we therefore tentatively assign each component a fixed temperature of \( 1.8 \times 10^7 \text{ K} \) (representative of the typical temperature, weighted by the \( G(T) \) function for Ca xix, found in flare coronae). We also assume an absence of turbulent broadening in these model atmospheres, such that the kernel’s width is merely the thermal Doppler width (i.e., \( \xi = 0 \)). Line profiles, per unit loop cross-sectional area and per unit detector area, were computed with the isothermal assumption and are shown with solid lines in Figure 3. The more exact line profiles (which take into account the effect of varying widths of the elementary Maxwellian profiles) are shown dotted for comparison. Figure 3 shows that the assumption of isothermality made in the kernels for the inversion process does not significantly alter the line profiles obtained.

As expected from the bimodal form of the low-density corona VDEM, the line profile has a double-peaked form, with the second peak offset from the rest wavelength by an amount corresponding to a velocity of 270 km s\(^{-1}\). The high-density corona VDEM instead generates a simpler line profile shifted to the blue of the rest wavelength by an amount corresponding to a velocity of approximately 200 km s\(^{-1}\).

4. THE INVERSION PROBLEM: OBTAINING VDEM FROM LINE SPECTRA

Having demonstrated the ability of VDEM to distinguish between different flare atmospheres in the “forward” calculation, we turn to the problem of obtaining VDEM from observed spectra. Equation (8) for the line profile, repeated here,

\[
I(\lambda) = \frac{1}{4\pi D^2} \int \text{VDEM}(v)j_q(\lambda, v)dv,
\]

is a Fredholm integral equation of the first kind. \( I(\lambda) \) is the observed profile, VDEM\((v) \) is the physical source function of interest, and \( j_q(\lambda, v) \) is known as the kernel. Properly discretized, equation (8) is a matrix equation, \( A \cdot f = g \), where \( f \) (the source function vector) represents VDEM, \( g \) (the data vector) represents \( 4\pi D^2 I(\lambda) \), and \( A \) is the discretized kernel function (\( j_q \)), including the weights from the quadrature method chosen for integration. Inversions of such equations are generally ill-conditioned and unstable (e.g., Craig & Brown 1976), but a number of techniques exist to overcome some of the difficulties inherent in the process.

We have implemented a linear regularization method for inverting line profiles to recover volumetric VDEMs. Linear regularization optimizes the trade-off between minimizing the residual (\( \chi^2 \)) error and constraining the source function with a smoothing condition (Twomey 1977; Craig & Brown 1986; Press et al. 1992, pp. 779–809):

\[
\| A \cdot f - g \|^2 + \mu \| K \cdot f \|^2 = \text{minimum}.
\]

The first term in equation (12) is the residual, while the second term is a measure of smoothness, expressed as a quadratic function of \( f \). The weighting quantity \( \mu \) is termed the smoothing parameter. If \( \mu \) is too small, the inversion is unstable, and the solution unpredictably oscillatory. On the other hand, if \( \mu \) is too large, the recovered solution is oversmoothed, and information is lost. In effect, some a priori information (smoothness) is necessarily incorporated into the inversion process to stabilize the results. For example, if one believes a constant function is a good approximation to \( f \), one expects its first derivative to vanish; and so one uses \( K \cdot f = 0 \). For a linear function, one uses \( K \cdot f = f' \), since the second derivative vanishes if \( f \) is perfectly linear. Given that a piecewise quadratic appears to be an adequate representation of the model atmospheres' VDEMs (Fig. 2), we choose to use the third derivative as our smoothing constraint, and represent it using the central difference formula:

\[
(K \cdot f)_m = f'''(v_m) = -f(v_{m-1/2}) + 3f(v_{m-1/2}) - 3f(v_{m+1/2}) + f(v_{m+1/2})/(\Delta v)^3
\]

(13)

The effect of including this norm in the minimization equation (12) is to force the solution toward a piecewise quadratic form.
Taking the derivative of equation (12) with respect to \(f\), the minimization principle reduces to a linear set of normal equations (Twomey 1977),

\[
(A^T \cdot A + \mu H) \cdot f = A^T \cdot g ,
\]

where \(H = K^T K\). We now solve equation (14) for \(f\), thus resolving the ill-posed problem with a stable minimization problem. (For other successful applications of this method, see Craig & Brown 1976; Tittering 1985; Jeffrey & Rosner 1986; Metcalf et al. 1990.)

To test our inversion process, we inverted the synthetic line profiles (generated by convolving model VDEMs with Gaussian kernels in eq. [8]) of Figure 3 to recover (hopefully) the corresponding VDEMs of Figure 2. The optimal smoothing parameter, \(\mu_{opt}\), was chosen by finding the value which minimized the \(\chi^2\) difference between the actual and recovered VDEMs. The results of the inversion are shown in Figure 4, where the recovered VDEMs are depicted by solid lines, while the dashed lines are the actual VDEMs from which the line profiles of Figure 3 were constructed. At higher velocities, the recovered VDEMs match the actual VDEMs very well. At low velocities, where the actual VDEMs are very spiky (and almost discontinuous in the low-density case), the inversion process smooths the recovered VDEMs. The fit at low velocities could be partially remedied by weighting the smoothing matrix to diminish the degree of smoothing applied in those regions. Nonetheless, the difference between the two recovered VDEMs is significant enough to reveal the physical differences between the atmospheres to which they correspond.

Having demonstrated the viability of the inversion technique on synthetic line profiles, we now turn to inverting actual flare spectra observed by the Yohkoh BCS. We selected a spectrum in the rise phase (UT 9:02:27) of the 1992 September 6 event and applied our inversion technique. Figure 5 shows the spectrum, as well as the specific VDEMs (obtained by dividing

the volumetric VDEM by an assumed projected flare area of \(10^{18}\) cm\(^2\); i.e., VDEM/\(\tilde{A}(z)\) [cf. eq. (11)], recovered using various degrees of smoothing. Again, we assume that broadening due to turbulence is negligible such that the kernel's width is simply the thermal Doppler width. For a small value of \(\mu\), the recovered VDEM (in the second panel) shows a great deal of structure. Larger values of \(\mu\), or a greater degree of smoothing, tend to wash out those features, as seen in the third and fourth panels of Figure 5.

We also applied a two-component model to this spectrum for comparison with the VDEM results. The two-component fit indicates the presence of a large volume of upwelling plasma, moving with a velocity \(v = 125\) km s\(^{-1}\) relative to the principal component, having a turbulent velocity width of \(44\) km s\(^{-1}\), and having an emission measure which is larger than the principal component's, \(EM = 1.04 \times 10^{48}\) cm\(^{-3}\) as compared to \(EM = 5.4 \times 10^{47}\) cm\(^{-3}\). However, the VDEMs show instead a broad dispersion of velocities, rather than the two discrete components used in such modeling efforts.

The large-scale features of recovered VDEMs are well-defined, even with the coarsest binning (\(\Delta v\)) and the strongest smoothing (large \(\mu\)) applied to the inversion. The finer (and most interesting) structure, however, may be unresolved when \(\mu\) and \(\Delta v\) are large. One's first inclination then is to make \(\mu\) and \(\Delta v\) as small as possible; but care must be exercised so that the solution highlights interesting, real features without enhancing artifacts arising from noise in the data or from the inversion process itself. In the absence of a priori criteria to apply to the choice of \(\Delta v\) and \(\mu\), we must seek, by trial and error, the values that yield acceptable solutions. But we must first define "acceptable."

As a first step, we demand that acceptable VDEM solutions be everywhere nonnegative. (VDEM is a measure of emission from plasma moving with a given velocity, and so a negative VDEM has no physical meaning.) This restriction on the recovered solutions essentially limits the degree of oscillatory behavior that is acceptable. We also note that the velocity-space resolution \(\Delta v\) of the VDEM solutions cannot be less than that which is supported by the spectral data, namely the instrument's wavelength bin size, which is \(0.3\) m\(\AA\) or \(30\) km s\(^{-1}\). On the other hand, \(\Delta v\) cannot exceed the half-width of the resonance line, \(0.8\) m\(\AA\) or \(80\) km s\(^{-1}\), or else the utility of VDEM is destroyed. (For such a large \(\Delta v\), the VDEM analysis would be essentially reduced to fitting a single component to the line. If the above conditions are satisfied, the solution is deemed to have sufficient physical basis to represent the distribution of the flare plasma's line-of-sight velocities. Further analysis is required to determine the optimal values of \(\Delta v\) and \(\mu\), based on multiple realizations of randomly perturbed data sets. Such a technique, employed by Metcalf et al. (1990) will permit estimates of the errors in the recovered VDEMs. In addition, the contamination of the resonance line by the d13 satellite must be factored into the inversion by using a kernel with a d13 feature added. Despite the need for further refinement, these results demonstrate the promise and power of the VDEM concept and inversion technique.

5. DISCUSSION

Knowledge of the plasma's line-of-sight velocity distribution is pivotal to understanding solar flare dynamics and, in turn, the energy release and transport processes that cause the plasma's motion. We have demonstrated that a two-component fit to observed line profiles cannot realistically rep-
resent the velocity distribution of the flaring plasma. Instead, we have developed and tested the concept of the VDEM. VDEM contains useful physical information about flaring plasmas and can accurately discern the evolution of the plasma's characteristics. It can also be used to calculate bulk plasma quantities such as the total momentum of the upflowing material:

$$p = \frac{m}{n_e G(T)} \int nVDEM dv$$  \hspace{1cm} (15)

for comparison with the downward motion evidenced by Hz observations (e.g., Ichimoto & Kurokawa 1985; Canfield et al. 1987).

As this paper demonstrates, it is possible to construct volumetric or specific VDEMs from model atmospheres using VDEM's definition. However, the deconvolution of observed spectra will yield solely the volumetric VDEM, as is also true with temperature emission measures (Craig & Brown 1976). To translate the volumetric VDEM into information about the emitting plasma's $n_e(r)$, $T(r)$, and $v(r)$ requires assumptions about the region's geometry and topology (e.g., plane parallel, toroidal, etc.) Nevertheless, VDEM is a promising diagnostic which has the potential to settle lingering questions about chromospheric evaporation.

For example, flare energy transport models predicting chromospheric evaporation have been criticized for their failure to generate spectral line profiles which are consistent with observations. In his review of the chromospheric evaporation debate, Doschek (1990) notes that, using the two-component formalism, strong stationary components are observed at flare onset and the blueshifted components are smaller than predicted (also noted by Feldman et al. 1994). Further, Antonucci et al. (1987) have demonstrated that even though chromospheric evaporation is present in a hydrodynamic simulation, the resulting blueshifted component can be undetectable in the application of a two-component fit to the synthetic line profile constructed for this model atmosphere. Thus, simplistic two-component models may lead to false conclusions about the presence and magnitude of evaporation.

The VDEM technique provides a much better measure of bulk plasma motion than the two-component formalism. It is evident in Figure 3 (and also Emslie & Alexander 1987 and Alexander 1990) that a simple distribution of line-of-sight velocities can result in profiles which are both broader than the thermal width of the Ca xix line and asymmetric. There is no need to assume ad hoc relationships between the width of a "moving" component and its velocity or the width of a "stationary" component to generate these features (Antonucci, Dodero, & Martin 1990; Fludra et al. 1989). Instead, VDEMs deconvolved from flare spectra indicate the existence of upflows or downflows without prejudicing the results toward the existence of certain plasma components. The analysis of recovered VDEMs should therefore make a significant contribution toward resolving the chromospheric evaporation debate.

The fact that a distribution of line-of-sight velocities can produce excess line widths also bolsters the view that differential flows can account for some, if not all, of the profile broadening observed in flare emission, without invoking a
separate mechanism (such as turbulence). Critics counter with the observation that excess line widths are observed in limb flares, as well as disk flares; but this argument rests on an oversimplified view of limb flare loops' being oriented entirely north–south, transverse to the line of sight. Limb flares rarely conform to this type of geometry; instead, one footpoint generally leads, creating positive velocity components along the line of sight, while negative velocity components arise in the following leg of the flare loop. The magnitudes of such components are roughly equal, creating a symmetric broadening of the line profile. It is conceivable then that differential flows can account for most of the excess width in all line profiles. Any residual broadening, due to turbulence or some other mechanism, is easily accommodated by including a nonthermal velocity component in the kernel's width, equation (4).

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