KINEMATIC MODELS OF SUPERGRANULAR DIFFUSION ON THE SUN

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ABSTRACT

We develop kinematic models of diffusion generated by supergranulation at the solar surface. These models use current observations for the size, horizontal velocity, and lifetime of supergranules. Because there is no observational description of the appearance and disappearance of supergranules, we investigate models using several plausible evolution processes, including the effect of different lifetime distribution functions for the cells. The results are quite insensitive to the methods chosen to replace old supergranules, the distribution of cell lifetimes, and even the cell lifetime itself, for mean lifetimes between 15 to 30 hr. Calculated diffusion coefficients range between 500 and 700 km² s⁻¹, in agreement with the best fit diffusion coefficients used by Sheeley and his collaborators to model the large-scale distribution of magnetic fields over the solar surface. However, our models do not explain the field distribution in plage, and they predict that virtually all the strong field in quiet Sun exists in relatively isolated clumps. We suggest possible mechanisms for the creation of plage and the bright network seen in quiet Sun.

Subject headings: diffusion — Sun: granulation

1. INTRODUCTION

The discovery of a large-scale flow pattern, supergranulation, nested inside the Ca ii network (Simon & Leighton 1964) and the idea that the bright network was evidence for the presence of magnetic fields led Leighton (1964) to suggest that surface diffusion of magnetic fields is generated by the random eruption of supergranules. Leighton (1969) created a two-dimensional surface diffusion model which took as input the observed birth rate of sunspots, their net equatorial tilt, and differential rotation. He showed that the solar magnetic field reversed in 11 years when the diffusion coefficient was on the order of 1500 km² s⁻¹. In his dynamo model, the reversed dipolar field was then wound up by differential rotation to produce an azimuthal field that erupted to form the active regions of the next cycle. The issue that concerns us here is not the field reversal, but whether supergranulation is sufficient to create diffusion rates in the range necessary to account for the large scale distribution of flux at the solar photosphere.

To test Leighton’s ideas, Smithson (1973) measured motions of small magnetic fragments on time sequences of magnetograms and obtained a diffusion coefficient of about 175 km² s⁻¹; as a consequence polar reversal using diffusion alone would require about 100 years. Mosher (1977) undertook a much more detailed set of measurements to determine the surface diffusion coefficient. He traced the spread of active region, plages, and unipolar and bipolar magnetic regions and obtained values between 100 and 300 km² s⁻¹, which bracketed Smithson’s result. Schrijver & Martin (1989) measured diffusion coefficients in plage and enhanced network by tracking magnetic elements, and found diffusion coefficients of 120 and 280 km² s⁻¹ for strong plage and enhanced network, respectively. Wang (1988a) has measured a diffusion coefficient via the width of magnetogram autocorrelation functions and obtained 150 km² s⁻¹.

Since his diffusion coefficient measurements were much too small to reverse the polar field in 11 years, Mosher suggested combining the observed diffusion rate with a meridional flow on the order of 3 m s⁻¹ in order to reproduce the observed duration of the magnetic cycle. Meridional flows of about 40 m s⁻¹ were discovered by Duvall (1979) while Ulrich et al. (1988) have observed 20 m s⁻¹. Both of these results are obtained by measuring the difference between limb to limb velocity profiles in the equatorial and polar directions on full disk Dopplergrams. Recently, Komm, Howard, & Harvey (1993) measured meridional flows of about 14 m s⁻¹ by tracking small magnetic features in Kitt Peak magnetograms. Mosher’s prediction of meridional flows can be regarded as a major success of the surface diffusion concept.

Sheeley and his collaborators have elaborated upon this concept (Wang, Nash, & Sheeley 1989a; Wang & Sheeley 1993) and have extensively examined the consequences of surface diffusion. They have found an excellent fit to observation using a meridional flow speed of 10 m s⁻¹, the solar differential rotation rate (Snodgrass 1983; Sheeley, Wang, & Nash 1992), and a diffusion coefficient of 600 km² s⁻¹, plus the assumptions that the fields are passively transported and that Lorentz forces can be neglected. Their models reproduce the observed distribution of flux with latitude, rigid rotation of coronal holes, and solar reversals in 11 years. Whether these models can accurately predict reversals of magnetic fields at the poles remains controversial (Wilson & McIntosh 1991).
Nevertheless, the agreement, over timescales of more than a solar cycle, between the synoptic magnetograph observations of the Wilcox Observatory, which have a resolution of 3 arcminutes, and the surface fields predicted from the observed sunspot data and the surface diffusion calculations strongly suggest that large-scale distribution of fields over the solar surface can be explained in large part by meridional flow, differential rotation, and diffusion. However, this excellent agreement makes the disagreement between the model's best-fit diffusion coefficient and observed diffusion coefficients a disturbing issue.

The flight of the Solar Optical Universal Polarimeter (SOUP) on Spacelab 2 (Title et al. 1986) allowed the collection of sequences of images of the solar surface completely free from the distortions and blurring introduced by the Earth's atmosphere. Using the SOUP movies, it has been possible to measure horizontal flow patterns near disk center by correlation tracking of the intensity patterns (November 1986; Tarbell et al. 1992). From these data and others produced from observations at Pic du Midi Observatory and the Vacuum Tower Telescope on La Palma, it was possible to measure the statistical properties of the flow (Title et al. 1989, 1992), properties of mesogranulation (November et al. 1981; Brandt et al. 1988; Muller et al. 1992), and the relations between magnetic field sites and large scale flow patterns (Simon et al. 1988). The primary technique for visualization of the flow fields and their evolution has been the study of "cork" motions.

Corks are passive test particles which are displaced along the surface by the horizontal component of the flow field. Using corks, Simon et al. (1988) showed that observed magnetic fields in the SOUP field of view were located at sinks in the flow field. These measurements suggested that the diffusion of magnetic fields over the solar surface can be estimated by studying the evolution of corks. Wang & Zirin (1988) have also used the cork technique to show that internetwork fields collect in supergranule boundaries. November (1989) has demonstrated that reliable flow maps can be made from ground based data. Bogart et al. (1988) have shown that seeing no worse than about 1.5 arcseconds is necessary to obtain accurate velocity estimates. Because of seeing limitations and day-night interruptions, the flow field data required for measuring directly the evolution of supergranulation have not yet been collected. However, recently several excellent 10 hr data sets have allowed an accurate measurement of the flow speeds in supergranules (Simon et al. 1994).

In this paper, we explore two-dimensional diffusion by supergranulation through kinematic modeling. Simon & Weiss (1989) used horizontal flow maps produced from SOUP images to demonstrate that motions of test particles in the SOUP flow fields were well matched by kinematic flow fields generated by suitably placed plumes, each of which was represented by an axisymmetric source. Simon, Title, & Weiss (1991) then used various plume patterns to demonstrate that mesogranules are real phenomena rather than just chance arrangements of exploding granules. The technique of kinematic modeling allows us to develop some insight into how magnetic fields are dispersed over many cell lifetimes, across a field of view that contains many cells. This cannot yet be done with a full convection calculation. We begin in § 2 with a largely analytical approach to follow the effects due to fixed cellular patterns: motion to cell boundaries, motions along boundaries to sinks, and accumulation in sinks. We use estimates of the mean square displacement in a single cell to infer the diffusion coefficient on the assumption that the entire pattern is replaced after each cell lifetime. The mathematical experiments in § 2 allow us to explore a wide range of parameter space and parameter sensitivities. In § 3, we turn to numerical simulations which allow velocity patterns to evolve as individual supergranules appear and disappear. This removes some of the limitations of the procedures used in § 2 and yields a fairly robust estimate for the solar diffusion coefficient. We are aware that the magnetic field is three-dimensional, that Lorentz forces may not be negligible, and that large-scale magnetic patterns may be linked to a deep-seated dynamo, but we shall not address those issues here. Rather, our approach is to determine which features of the large scale evolution of the magnetic field can and cannot be explained by the simple concept of diffusive spreading. In § 4, we discuss our results, emphasizing the tendency of quiet Sun fields to form isolated clumps and the consequent difficulty of explaining both quiet and active solar diffusion with the same model. Finally, we suggest areas for further investigation.

2. DIFFUSION BY FLOW CELLS

In this section, we estimate diffusion coefficients for test particles moving in prescribed velocity fields. These velocity fields do not evolve continuously in time. Instead, we suppose that the pattern of motion remain fixed for a time T (the cell lifetime) and that it is then replaced, instantaneously and everywhere, by a similar pattern, displaced and arbitrarily oriented, with the same lifetime. The whole process is then repeated many times, so that the test particles carry out a random walk. To calculate the diffusion coefficient D, we need to know the mean square displacement \( \langle d^2 \rangle \) over the time T for particles that were initially uniformly distributed. Then the diffusion coefficient

\[
D = \langle d^2 \rangle / (4T),
\]

where R is the cell radius.

Leighton assumed that magnetic fields were displaced to the boundaries of supergranules by their radial outflow, and then the fields remained stationary until a new supergranule was born in their vicinity. The distance that a particle inside the cell at a distance r from the center has to move to get to the cell boundary is \( (R - r) \). Therefore, for a circular cell

\[
\langle d^2 \rangle_c = \frac{1}{\pi R^2} \int_0^R 2\pi(R - r)^2r \, dr = \frac{R^2}{6}. \tag{2}
\]

The diffusion coefficient, \( D_c \), is

\[
D_c = R^2/(24T). \tag{3}
\]

The average supergranule has \( R = 20 \) arcseconds and \( T = 20 \) hr (Simon & Leighton 1964; Rogers 1970; Jansen 1970; Singh et al. 1994) which generates a diffusion coefficient, \( D_c \), of 122 km² s⁻¹. This value is much smaller than the 600 km² s⁻¹ used by Sheeley and his collaborators in their models of dispersal of magnetic flux over the solar surface. Therefore, it is appropriate to examine some more elaborate estimators of the diffusion coefficient.

It is worth noting at this point that an estimate of the lifetime of supergranulation cell is difficult because the lifetime is longer than an observing day and it is extremely difficult to recognize individual cells after an interruption of 10 to 14 hr due to the Sun passing below the horizon. Because of the day-night cycle and the difficulty of unbiased recognition of indi-
individual structures, almost all of the measurements of supergranulation lifetime are based on statistical techniques such as cross-correlation and are in the range of 20 to 30 hr. In plage some individual cells can be observed for several days. In quiet Sun it is difficult to recognize cells except from their Doppler signature near the limb. Wang & Zirin (1989) have estimated cell lifetimes by several different recognition techniques and have concluded the cells live 50 hr or longer.

The simple approximation of the diffusion coefficient made above neglects a number of effects, in particular the finite time required for fields to move to the cell boundaries, which is why it approaches infinity for short lifetimes. As an introduction to the effect of finite travel times, we first consider a uniform flow speed toward the cell boundaries. Then the mean square displacement is

$$\langle d^2 \rangle_e = (Vt)^2 \left[ 1 - \frac{4}{3R} VT + \frac{1}{2R^2} (VT)^2 \right] \quad \text{for } t \leq \frac{R}{V},$$

(4)

where $V$ is the velocity toward the boundary. When $t = R/V$, all the test particles are at the boundary and the mean square displacement is the same as that obtained neglecting the flow speed. The diffusion coefficient for the case of a uniform radial flow speed $V$, a cell lifetime $T$, and random eruption of cells is

$$D_e = \frac{V^2 T}{4} \left[ 1 - \frac{4}{3R} VT + \frac{1}{2R^2} (VT)^2 \right] \quad \text{for } T \leq \frac{R}{V},$$

(5)

$$D_e = \frac{R^2}{(24T)} \quad \text{for } T > \frac{R}{V}.$$  

(6)

Including a finite travel time causes the diffusion coefficient to grow initially like $V^2 T$ because the initial displacements are all proportional to $VT$. Since the diffusion coefficient becomes inversely proportional to $T$ once the particles reach the boundary, there is a maximum value for the diffusion coefficient.

$$D_e^{\text{max}} = 0.0575RV,$$

(7)

which is achieved when

$$T = T^{\text{max}} = 0.538R/V,$$

(8)

with $T$ in seconds, $R$ in km, and $V$ in km s$^{-1}$. Shown in Figure 1 are plots of the diffusion coefficient versus cell lifetime for cases with $V = 0.2, 0.3, \text{and } 0.4 \text{ km s}^{-1}$, and $R = 20$ arcseconds (14,500 km). The peak lifetime and maximum diffusion coefficient pairs, $(T_{\text{max}}, D_e^{\text{max}})$, that correspond to these drift velocities are $(10.8, 167), (7.2, 250), \text{and } (5.4, 333)$, where the peak lifetime is in hr and the diffusion coefficient is in km$^2$ s$^{-1}$.

For uniform flow to a boundary (see Fig. 1) the maximum diffusion coefficient occurs at smaller lifetimes than are observed for supergranulation. However, in an actual convection cell the horizontal component of the velocity is zero at the center and falls toward the outer boundary, so that in a more realistic model the maximum diffusion coefficient would be lower and occur at a later time than given by the estimates above. Nevertheless, the diffusion coefficients versus lifetime curve will always have a maximum value and the initial behavior will be proportional to the initial velocity squared times the lifetime. In the vicinity of the maximum value, the diffusion coefficient is relatively insensitive to the cell lifetime.

The idea that convection cells only displace magnetic fields to their boundaries is a serious oversimplification. In an array of interacting plumes the flow field will collect test particles into sinks (Simon & Weiss 1989). Each sink is located at a stagnation point of the flow, i.e., at a point with zero velocity, but not all stagnation points are sinks. The collection of test particles into sinks is also observed using solar flow patterns (Spruit, Nordlund, & Title 1990). A square lattice of flow plumes is a starting point for estimating the effect of sinks. Test particles move toward a square array of sinks. The mean square displacement, neglecting flow time, to the sink located in the center of four plumes in a square array is

$$\langle d^2 \rangle_s = \int_0^{\sqrt{3}kr} \int_0^{\sqrt{3}kr} \frac{(kr - x)^2 + (kr - y)^2}{(kr)^2} \, dx \, dy = \frac{2(kr)^2}{3},$$

(9)

where $kr$ is half of the separation between adjacent plumes. We have distinguished between the cell half spacing, $kr$, and the cell radius $R$ because there will be effects on the mean square displacement when a flow time in a cell is introduced.

When the plume centers form a hexagonal lattice, the unit cell is a rhombus with a 60° acute angle and there are two sinks per unit cell. The mean square displacement to sinks in a hexagonal lattice is

$$\langle d^2 \rangle_h = \frac{4}{\sqrt{3}(kR)^2} \int_0^{\sqrt{3}kR/2} \int_0^{\sqrt{3}kR/2} \left[ \left( \frac{\sqrt{3}kR}{2} - x \right)^2 + \left( \frac{kr}{2} - y \right)^2 \right] \, dx \, dy = \frac{(kr)^2}{3}.$$  

(10)

Hence, for square and hexagonal plume arrays the diffusion coefficients, neglecting the effect of finite flow times to reach the sinks, are

$$D_s = \frac{(kr)^2}{(6T)}.$$  

(11)

and

$$D_h = \frac{(kr)^2}{(12T)}.$$  

(12)

Thus, motion to sinks in the flow field increases the mean square displacement by factors of 4 and 2 for the square and hexagonal lattices, respectively, compared to displacement only to the cell boundary. For a pattern of 20 arcsecond radius supergranules, each of which is inscribed in a square unit cell with side length $2kR = 40$ arcseconds, the diffusion coefficient, $D_s$, is 487 km$^2$ s$^{-1}$ if $T = 20$ hr. Clearly, if $k > 1$, the diffusion coefficient will increase with $k$. A diffusion coefficient of about 600 km$^2$ s$^{-1}$ occurs when $k = 1.11$. By allowing flow into sinks,
we are able to generate diffusion coefficients as large as those used in Sheeley's surface diffusion models.

Since the distances test particles must move to reach a sink are greater than those to a cell boundary on a flow field with sinks, the diffusion coefficient reaches a maximum at a larger value of the lifetime than in the case of radial flow to a boundary. It is therefore useful to examine in more detail how the mean square displacement develops with time in a flow field generated by a square lattice of cells. Simon & Weiss (1989) fitted flow patterns observed from correlation tracking of the solar surface, and found that the horizontal velocity in meso-granulation is well fitted by the function of the form

$$v(r) = 2.338 V(r/R)e^{-r(R)^2},$$

where $V$ is the free peak velocity in an isolated plume which acts as a source. The parameter $R$, which we shall call the cell radius, is the radial decay constant in the exponential portion of this velocity function.

We consider now the flow fluid field in a square array of four plumes embedded in an infinite array of similar plumes. Because the velocity component contributed by each plume falls exponentially with distance from the plume center, the net velocity at any point is dominated by the contributions of the nearest plumes. Thus the flow field within a four-plume array can be accurately represented by the flow pattern in the central portion of a finite array of 16 plumes. The $x$ and $y$ components of the 16 plume velocity field at a point $(x, y)$ are

$$v_x = \sum_{i=0}^{3k} \sum_{j=0}^{3k} v(r_{ij})(x_i/r_{ij}),$$

and

$$v_y = \sum_{i=0}^{3k} \sum_{j=0}^{3k} v(r_{ij})(y_j/r_{ij}),$$

where $v_i = (3 - 2)k, v_j = (3 - 2)k$ denote the cell centers, $x_i = (x - \epsilon_i), y_j = (y - \epsilon_j)$, and $r_{ij} = (x_i^2 + y_j^2)^{1/2}$. For convenience we have set $R = 1$ but, because the decay radius enters the plume function as a ratio, the flow pattern is easily scaled. This arrangement has a sink at $(0, 0)$.

In Figure 2 we show a contour plot of the flow speed in the central unit cell for the case $k = 1$. There are stagnation points at the midpoints of the lines connecting cell centers. Half of these stagnation points are sinks and the other half are saddles, with flow along inward and outward separatrices. A test particle leaving a plume center along the $x$ or $y$ axis will move in a purely radial direction toward one of the saddle points. Similarly a test particle moving from a plume center along a diagonal will move in a purely radial direction toward a sink. Because of the greater distance between plumes centers in the diagonal directions, the peak speed along the diagonals is greater than the peak speed along the $x$ or $y$ direction. Figure 3a shows the speeds along a line connecting a plume center and a sink, and along a line from a saddle to a sink, for the case $k = 1$. The case with $k = 2^{1/2}$ is shown in Figure 3b.

Comparison of Figures 3a and 3b shows that, as the plume separation increases, the flow speed along the lines connecting saddles and sinks drops, while the speed along lines connecting plume centers and sinks increases. This trend is summarized in Figure 3c which shows plots of the peak flow speed along a line connecting a plume center and a sink, and the peak speed on a line connecting a saddle and sink, as functions of $k$. As the plume separation increases, the peak radial flow speed rapidly approaches that of the free plume, while the peak speed on lines connecting saddles and sinks rapidly approaches zero for separations larger than $k = 1.1$. When $k$ is only slightly greater than 1.1, the rate of collection into sinks drops drastically. Qualitatively, this behavior is expected: as the separation of the plumes increases, the flow field near any plume must approach that of a free plume. This happens quickly with increasing $k$ and suggests that the diffusion coefficient will be more dependent on the properties of plumes than on their spacing. This is important because it shows that the diffusion coefficient is more strongly dependent on the cell radius than on the cell spacing.

To measure the mean square displacement toward a sink as a function of time, we filled the unit cell centered at $(0, 0)$ with 10⁴ randomly located corks and calculated the path of each cork for 120 hr with 10 minute time resolution. Since we know the starting point and position of each cork at each time step, it is straightforward to determine the mean square displacement and diffusion coefficient as a function of cell lifetime. Shown in Figure 4 are calculations of the diffusion coefficient versus lifetime functions for peak flow speed and decay radius pairs $(V, R) = (\frac{1}{4}, 10), (\frac{1}{4}, 20), (\frac{1}{4}, 30)$ with $V$ in km s⁻¹ and $R$ in arcseconds. When $R$ and $V$ are scaled together $(R/V ≈ constant)$, the flow pattern does not change. However, $D$ should increase as $R^2$. The value of the diffusion coefficients at equal times in the $(\frac{1}{4}, 10)$ and $(\frac{1}{4}, 30)$ calculations are in the ratio 1 to 9 as expected.

Also shown in Figure 4 are fits to the data by the three parameter function

$$D(R, V, T) = \frac{k^2 R^2}{6T} \left[ 1 - e^{-a(V/R)^2} \right] + b_k V^2 T e^{-c_k(V/R)^2},$$

(16)
where $a_k$, $b_k$, $c_k$ are constants determined by the data. The subscripts on the fitting parameters indicate that they change with $k$. Only one set of dimensionless constants ($a_1$, $b_1$, and $c_1$) was used for all the curves in Figure 4. The functional form of equation (16) was selected such that, for short lifetimes, the diffusion coefficient is proportional to $V^2T$ and for large lifetimes it approaches $D_1$ (eq. [11]). The functional form was also required to have the property that, when $R$ and $V$ are scaled by the same factor, the diffusion coefficient increases by the square of that factor.

As discussed above and illustrated in Figure 3, the radial velocity increases and the nonradial velocity toward the sinks decreases, as the separation $2kR$ between plumes is increased. Therefore, as $k$ increases, at first the peak value of the function $D(T)$ increases but, at some separation, a maximum diffusion rate will occur. This behavior is illustrated in Figure 5 which shows $D(T)$ for $V = \frac{1}{2}$ km s$^{-1}$, $R = 20$ arcseconds, and plume separations of 48, 50, and 52 arcseconds ($k = 1.20, 1.25$, and 1.30). The peak diffusion coefficient occurs for a spacing of $k = 1.25$. This result suggests that, even if the spacing of the plumes is nonuniform, there will not be a large effect on the diffusion coefficient caused by regions in the pattern where there is a large spacing between cells.

Using $D$ we can estimate the value of the diffusion coefficient for any combination of cell radius, cell spacing, peak free veloc-

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Fig. 3.—Speed vs. distance along the line connecting a plume center and a sink point (thick) and flow speed along the line connecting a saddle point and sink (thin) when $V = 0.5$ km s$^{-1}$ for (a) the case $k = 1$ and (b) the case $k = 2^{1/2}$. In (c), peak speeds along the line connecting a plume center and a sink (thick) and on the line connecting a saddle point and a sink (thin), as a function of the half separation $k$, are shown for $V = 0.5$ km s$^{-1}$.

Fig. 4.—Diffusion coefficient vs. lifetime for the cases where the peak velocity and decay radius ($V$, $R$) pairs are ($\frac{1}{2}$, 10), ($\frac{1}{4}$, 20), and ($\frac{1}{6}$, 30), where $V$ in km s$^{-1}$ and $R$ is in arcseconds. The dots are obtained from the numerical evaluations and the solid curves from a single fitting function. All plots are for the case $k = 1$.

Fig. 5.—Diffusion coefficient versus lifetime for $V = 0.5$ km s$^{-1}$, $R = 20$ arcseconds, and $k = 1.20, 1.25$, and 1.30, which correspond to plume spacings of 48, 50, and 52 arcseconds. The dots are obtained from the numerical evaluations and the solid curves are from a single fitting function.
SUPERGRANULAR DIFFUSION

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Supergranular diffusion plays a role in the spread of properties in solar convection. We show in Figure 6 plots of diffusion coefficient versus lifetime for cells with \( R = 20 \) arcseconds and \( V = 0.2, 0.4, 0.6, 0.8, \) and \( 1.0 \) km s\(^{-1}\). Comparison of the left \(( k = 1)\) and right \(( k = 1.25)\) panels shows how the diffusion coefficient changes with \( k \). Since the case \( k = 1.25 \) has the maximum diffusion coefficient, these plots indicate the range of diffusion coefficients that can occur when the primary displacement mechanism is collection of corks into sinks on a square lattice.

From Figure 6 we see that the more realistic plume velocity profile used in the square lattice model causes the peak diffusion coefficient to occur at a later time than a simple model with uniform flow speed to a boundary. To interpret Figure 6, recall that the peak velocity in the cell is somewhat less than the peak free velocity. Wang & Zirin (1988) tracked internetwork fields and obtained a velocity of about 0.45 km s\(^{-1}\). In another study Wang (1988b) measured the Doppler signal of supergranules near the limb and obtained radial speeds of 0.5 km s\(^{-1}\). Recently Simon et al. (1994), using local correlation tracking on high-resolution continuum images near disk center, found peak speeds of 0.7 km s\(^{-1}\) for the supergranules. For the case \( k = 1 \), these measurements suggest that the appropriate peak free velocity is near 1.0 km s\(^{-1}\) which implies an estimated diffusion coefficient of about 520 km\(^2\) s\(^{-1}\); while for the case \( k = 1.25 \), the appropriate peak free velocity is 0.8 km s\(^{-1}\) and the diffusion coefficient is about 720 km\(^2\) s\(^{-1}\). For this range of velocities, the diffusion coefficient curves are relatively insensitive to the exact cell lifetime in the range between 10 and 30 hr. Thus, for our square array model the diffusion coefficient is relatively insensitive to both cell spacing and cell lifetime. Further, the estimated diffusion coefficients are consistent with the models of Sheeley and his collaborators.

Although the diffusion coefficients obtained by calculating the rate of displacement to the sinks of a square array of plumes are reasonably close to those used in the surface diffusion models, there is a serious question whether the square array is a good model for solar convection. Some idea of the validity of square array results is obtained by evaluating the magnitude of the changes in the diffusion coefficient versus lifetime function when the plume characteristics are varied. To do this we calculated changes caused by varying the properties of a single test plume at cell corner \((-k, -k)\) in the unit cell centered at \((0, 0)\). From numerical experiments on varying the properties of this single test plume, we find that for a reasonable mixture of variations in the peak velocity, decay radius, and position from those in a perfect square array, the diffusion coefficient will increase by 15% to 25% compared to a uniform array.

To simulate a plume pattern more like the Sun, we randomly placed plumes inside a square box whose side length was 10 times the plume diameter. The plumes were placed into the box subject to the condition that a new plume did not overlap a cell already in the box. Random nonoverlapping plumes have a density 0.71 ± 0.01 times that of a close packed square array, so that the mean center-to-center distance is 1.19 times larger. We also calculated the flow field generated by random nonoverlapping plumes in order to estimate the number of sinks points per plume. Evaluation of several random patterns yields 0.92 ± 0.02 sink per cell. Thus, the number of sinks per flow plume in a square array is similar to the number in a random pattern of plumes. This, together with the fact that spacing variations do not have a large impact on the value of \( D \), suggests that the diffusion coefficient is not very sensitive to the arrangement of cells over the surface.

3. KINEMATIC MODELS OF SURFACE FLOWS

In the previous section, the diffusion coefficient was calculated on the assumption that all cell lifetimes are identical, and that there is a completely random realization of the flow field at each time step. We now extend the model to allow supergranules to evolve individually. As a result, the overall velocity pattern changes on a timescale much shorter than the lifetime of any single cell. Moreover, the sinks located at nodes in the supergranulation network are displaced at irregular intervals in time and the diffusion coefficient is consequently affected. Since the models are significantly more complex than those described in the section above, numerical experiments are required to determine their evolution.

Unfortunately, there are no good observational data that serve as a guide to how supergranulation evolves. In order to develop some understanding of the effects of different evolu-

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Fig. 6.—Diffusion coefficient versus time for \( R = 20 \) arcseconds and \( V = 0.2, 0.4, 0.6, 0.8, \) and \( 1.0 \) km s\(^{-1}\) for the cases (a) \( k = 1 \) and (b) \( 1.25 \), respectively.

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tionary patterns on the value of the diffuson coefficient, it is
necessary to study evolutionary patterns that bracket the types
of behavior expected in solar convection. We have investigated
two classes of evolutionary patterns. In the first type, which we
call random-hole evolution, supergranules are assumed to be
nonoverlapping. When an old supergranule dies it is replaced
by a new supergranule, positioned randomly but subject to the
constraint that it also does not overlap any existing super-
granule. We define nonoverlapping to mean that each plume is
at least $3^{1/2} R$ from its nearest neighbor.

In the second, which we shall refer to as tethered evolution,
we assume that the supergranules are tethered to a square grid.
When an old supergranule dies it is replaced with another
whose center position can occur anywhere inside a circle of
radius $L$ from the fixed reference grid point. In the tethered
case, the new supergranule can significantly overlap existing
supergranules. Figures 7a and 7b show typical supergranule
distributions for the random-hole and tethered configurations.
The patterns are similar. However, tethered configurations
have a greater tendency to produce clumps and voids when
$L > R/2$, than the random-hole distributions. If the tether
length $L$ is too short ($L < R/2$), the regular grid becomes
apparent.

We have also evaluated two plausible but artificial rules for
replacing supergranules. We assume, in the first, that the super-
granule lifetimes are uniformly distributed over $\pm 40\%$ of
the mean lifetime, $T$. That is, a model with $T = 30$ hr will contain
supergranules that live between 18 and 42 hr. In the second, we
specify that the probability that a supergranule will die after a
time $t$ is proportional to $(t - T/2)$ for $t > T/2$ and 0 if $t < T/2$.
These two very different lifetime distributions are shown in
Figure 8.

A random-hole evolution kinematic model starts with a
random distribution of 144 equally sized supergranules
($R = 20$ arcseconds) in a square box 512 arcseconds on a side.
The number of cells in the box is 70% of the number of cells in
a close packed array and thus is close to the average number of
randomly placed nonoverlapping cells. Initially, it is assumed
that each supergranule has lived a random fraction of the mean
lifetime. Ten thousand (or sometimes 2500) corks are placed on
a square grid inside a box 432 arcseconds on each side. Each
supergranule has a prescribed peak flow speed $V$. It is straight-
forward to scale $R$ and $V$ in order to represent different cell
sizes. We do not assume a start-up or decay phase to an indi-
vidual supergranule flow plume. At every time step, we
compute the velocity vector at the position of each cork and
then displace that cork with that velocity for a time $\Delta t$ (in these
calculations $\Delta t$ is 10 minutes) which is short compared to the
life of the shortest-lived flow cell. When a cork diffuses out of
the 432 arcsecond box, it is repositioned randomly within a 240
arcsecond inner box; but, for each cork, the total displacement
is tracked with respect to its initial starting position.

To summarize the geometry: Supergranules are placed in
$512 \times 512$ arcsecond flow field box. Centered within this box is
a smaller $432 \times 432$ arcsecond box containing the corks we
track. We restrict corks to this smaller tracking box to mini-

![Figure 8](image-url)

**Figure 8.**—Lifetime distributions of model supergranules. The mean lifetime in both cases is $T_m = 30$ hr. The rectangle distribution (a) is uniform with $T = T_m/2$, 40%, so that here $T$ ranges from 18 to 42 hr. The second distribution function (b) assumes that the probability of cell death is proportional to the time $t$ it has already lived, with a minimum lifetime $t = T_m/2$, or 15 hr in this case.
Fig. 9.—Development of the cork pattern for a random-hole uniform-lifetime model with $R = 20$ arcseconds, $T = 30$ hr, $V = 0.5$ km s$^{-1}$. Patterns, which are shown at $t = 0, 15, 30, 60, 120$, and $480$ hr, evolve from a uniform random distribution to a network structure, and then to clumps at stagnation points in the flow field.
mize boundary effects caused because we do not calculate the flow field of an infinite array of supergranules. The size of the smaller box allows a boundary band of width 2R. When corks cross the boundary of the tracking box, they are placed into the central 240 x 240 arcsecond restarting box.

Since the calculations scale with the ratio of the cell radius to cell velocity, one arbitrary size cell is used in most of the model calculations. However, on the Sun there is a range in cell sizes arising from granules, mesogranules, and supergranules. Therefore, before starting on an extensive set of calculations using only supergranules, we experimented with models containing granules, mesogranules, and supergranules. The granules and mesogranules were randomly positioned. Although the initial behavior of the more complete models differed in the first 20 hr of their evolution, at later times the mean square displacement of corks and the cork patterns generated by models with and without granules and mesogranules are essentially indistinguishable when L > R/2. Shown in Figure 9 are the positions of corks and cells for a random evolution model with R = 20 arcseconds, V = 0.5 km s⁻¹, and T = 30 hr (uniform lifetime distribution) after 0, 15, 30, 60, 120, and 480 hr of evolution. The figures show that after 15 hr the supergranule centers have already been swept clear and the corks form a diffuse network. By 30 hr, the corks have collected in tight but intermittent linear groups and are beginning to accumulate at sinks. This accumulation is more apparent after 60 hr (2T). Thereafter, the corks remain near sinks in the flow field. Hence, the rate at which they diffuse is now determined by the motions of the sinks. The clumping phenomenon is not significantly modified by inclusion of granules and mesogranules into the models, and always occurs after several cell lifetimes. We also recall that collection of corks into clumps is seen in cork movies generated from correlation tracking of observed photospheric granulation.

We form an estimated diffusion coefficient by calculating the mean square displacement, ⟨d²⟩, of corks as a function of time at each time step and dividing by 4t, where t is the time elapsed at that step. This has been done for a set of models with a range of cell lifetimes, lifetime distributions, and evolution types. Shown in Figure 10 are plots of the estimated diffusion coefficient, D = ⟨d²⟩/(4t), versus time, for R = 20 arcseconds, V = 0.5 km s⁻¹, T = 30 hr and a uniform distribution of lifetimes for three cases where the evolution patterns are random and tethered with L = 14 and 16 arcseconds. In these examples, D₁ exhibits a rapid rise to a peak at about 20 hr and then drops to 350–500 km² s⁻¹ after 2 days and fluctuates about these values thereafter. Data generated from cases in which the lifetimes are proportional to the time the cell has lived yield quantitatively similar results.

The estimated diffusion coefficient versus time plots exhibited in Figure 10 are typical of our runs. Initially, there is a rapid rise because test particles are at first displaced directly toward a stagnation point. In most cases, the peak value of the estimated diffusion coefficient occurs between 20 and 30 hr. The peak's location depends principally on the cell velocity, V, and only weakly on the cell lifetime, because our estimate early in the calculation is an indicator of V²t rather than the diffusion coefficient. After several cell lifetimes the estimated diffusion coefficient fluctuates randomly about its average value, which we take as the diffusivity D. The amplitude of the fluctuations is large because by this stage the formation of cork clusters reduces the number of independent corks. Increasing the number of initial corks does not help to reduce the amplitude of these fluctuations. Reduction of the statistical fluctuations occurs only when the field of view, and hence the number of sinks, is increased. A stable estimate of the diffusion coefficient requires running a model with a large number of supergranules for many cell lifetimes. These kinematic models warn us that estimates of the diffusion coefficient based on observations that do not cover a field of view of 8 to 10 arcminutes and do not last for many days must be interpreted with considerable caution.

Figure 11a contains plots of diffusion coefficient versus cell lifetime for the cases of random-hole evolution and of tethered evolution with L = 14 and 16 arcsecond tethers, when V = 0.5 km s⁻¹. Each symbol in Figure 11a is determined from an estimated diffusion coefficient versus time calculation for a particular cell lifetime. This figure confirms that the diffusion coefficient is not unduly sensitive to the procedure adopted for replacing supergranules. The diffusion simulation results are well fitted by an expression of the form

$$D = \frac{m²R²}{6T} \left[1 - e^{-a₁(V/T/R)}\right] \left[1 - e^{-a₂(V/T/R)}\right],$$

(17)

where m varies between 1.5 and 1.9 and a₁ and a₂ are constants that vary slightly with the type of evolution. At short times the diffusion coefficient is proportional to V²T. The solid line in Figure 11a shows such a curve fitted to the points obtained from random hole evolution. Note that the functional form of equation (17) differs from equation (16). The slower decay of the diffusion coefficient indicated by equation (17) results from the more realistic description of pattern evolution.

The results for random hole evolution can be compared with the curves in Figure 5, in which D reaches its maximum value, 450 km² s⁻¹, at about T = 20 hr. This peak is comparable to that in Figure 11a and occurs at about the same time. However, note that the value of D in the kinematic calculations in which supergranules die at different times drops more slowly than in the analytic estimate where they all expire simultaneously. At T = 100 hr, for example, D = 150 km² s⁻¹ in Figure 5 but D = 300 km² s⁻¹ in Figure 11a. Thus, there is a significant difference in diffusion produced by continuous eruption of cells.
We express the results from the model runs of Figure 11a in dimensionless form in Figure 11b, where \( D^* \) is plotted against \( V^* \) for our standard random hole model. For \( V^* \) sufficiently small, \( D^* \) is proportional to \( V^{*2} \), as corks are displaced toward cell boundaries. This effect is demonstrated in the inset in Figure 11b. Once enough corks have reached the boundary, the slope of the curve in Figure 11b decreases, so there is an inflection point at \( V^* \approx 2 \). Thereafter, the slope decreases monotonically as \( V^* \) is increased, and \( D^* \) tends toward a limiting value \( D^* \approx 1.2 \) as \( V^* \) approaches infinity, just as the classical diffusion equation (1) has a constant value \( D^* = 0.25 \). This represents the effect of displacing the positions of clumps located at stagnation points. Since a sink occurs at a triple junction in the network, we expect its position to depend mainly on the three nearest supergranules. Thus the pattern will, on average, change after an interval \( T/3 \), at which time the clump is displaced by a distance \( aR \). Then we can estimate the diffusion coefficient as \( (3/4)(aR)^2/T \) and, setting \( \alpha \approx 1.2 \), we recover \( D^* \approx 1.2 \).

These results can be used to compute the effect of varying the velocity \( V \). The broken line in Figure 11a shows, as a function of \( T \), the ratio of the value of \( D \) derived from our dimensionless analysis with \( V = 0.7 \) km s\(^{-1} \) to that with \( V = 0.5 \) km s\(^{-1} \). As expected, this ratio declines monotonically from a value of 1.96 when \( T \) is very small (and \( D \approx V^2T \)) to unity when \( T \) is large (and the motion of clumps does not depend on \( V \)). For \( T = 30 \) hr the ratio has a value around 1.35, in agreement with the value obtained in the \( V = 0.7 \) km s\(^{-1} \) run discussed above.

In summary, the observed value of 0.7 km s\(^{-1} \) for the flow to the supergranule boundary and our kinematic modeling yield a quiet Sun diffusion coefficient between 500 and 700 km\(^2\) s\(^{-1} \). These results are robust. They are not very sensitive to model details such as pattern evolution, lifetime distribution, or even cell lifetime.

4. DISCUSSION AND CONCLUSIONS

In 1972, using the first video magnetograph, Smithson showed that in quiet Sun the magnetic field is contained in a few isolated “stable points” rather than spread either randomly or uniformly around supergranules. These stable points do not move continuously; rather, they occasionally jump to a new location. In his thesis summary, Mosher (1977) calculated a diffusion coefficient based on collection of the field into sinks. He showed that if sinks occur at random, then the mean square distance between sinks is

\[
\langle d^2 \rangle_{\text{sink}} = 1/(\pi n_0),
\]

where \( n_0 \) is the number of sinks per unit area. One sink point per supergranule yields a diffusion coefficient of 730 km\(^2\) s\(^{-1} \) for 20 arcsecond radius supergranules with a 20 hr lifetime. Mosher’s result assumes a new pattern every cell lifetime. He noted that the number of observed isolated magnetic features in quiet Sun was consistent with the estimated number of sinks per unit area. However, because all of his other measurements indicated a diffusion coefficient of 200 ± 100 km\(^2\) s\(^{-1} \). Mosher concluded that “the field is not completely confined to a few stable points, but only prefers to be there, and the lifetime of the velocity cells is a little longer than generally assumed.”

Our models have extended and refined Mosher’s original sink argument and they demonstrate that the diffusion coefficient is not very sensitive to cell lifetime, distribution of cell
lifetimes, or cell pattern evolution for cells with characteristics similar to those observed for solar supergranules. Our models also exhibit clumping of test particles very similar to the clumping of quiet Sun magnetic fields observed by Smithson. We conclude that the sink argument is fundamentally sound, and that the quiet Sun diffusion coefficient is in the neighborhood of 600 km$^2$ s$^{-1}$ as inferred by Wang, Nash, & Sheeley (1989b). The agreement between the diffusion coefficient obtained with our kinematic models and the values used by Wang and Sheeley to match the evolution of large scale magnetic patterns suggests to us that the surface diffusion together with differential rotation and meridional flow is sufficient to explain surface evolution of the magnetic field after its escape from plage and enhanced network areas.

The calculations described here indicate that the flow fields that disperse magnetic field in quiet Sun are different from those in plage and enhanced network. Most previous estimates of diffusion coefficients were based on observations of properties of active Sun and are in the vicinity of 200 km$^2$ s$^{-1}$. This has occurred because there is not much magnetic field in quiet Sun, and it is difficult, therefore, to identify and follow individual clumps for the many days required. The majority of Mosher's (1977) estimates come from the spread and lifetime of active regions. He noted that the rate of flux disappearance in active regions and the rate of growth of filament channels is not consistent with the predictions of diffusion. But, as we have mentioned above, when Mosher estimates a diffusion coefficient based on sinks he obtains values consistent with ours. Schrijver & Martin's (1989) estimates of the diffusion coefficient were made from plage and enhanced network data. Of course, it is essential to directly measure the diffusion coefficient in quiet Sun. We plan to do this in the not very distant future from the flow maps generated by the Michelson Doppler Imager that will be flown on the Solar and Heliospheric Observatory in 1995.

Smaller diffusion coefficients are not the only evidence that flow fields in plage and enhanced network are different than those in quiet Sun. Plage and enhanced network magnetic field distributions differ significantly from those expected to result from simple diffusion. Schrijver (1987) observed that plage, regardless of size or age, could be defined by the 50 G contour on Kitt Peak daily magnetograms. Inside that contour the average magnetic flux level has a value of 100 ± 22 G. Earlier, Mosher (1977) had shown that filament channels have much sharper boundaries that would be expected from diffusion. A diffusion process causes the magnetic field to spread continuously and therefore the mean field in a fixed area should drop monotonically with time. A diffusion process should produce neither a well defined boundary nor a uniform flux level that persists inside that boundary long after flux has stopped emerging. Schrijver (1989) modeled plage spreading assuming different diffusion coefficients inside and outside of plage. However, these models did not generate plages with the observed well defined boundaries. He was only able to produce a uniform magnetic field inside the plage when he assumed an escape coefficient for the magnetic field significantly less than 0.5 at the 80 G contour level. He argued that a reflection coefficient at the boundary was a reasonable idea given that cell sizes are smaller inside the plage than outside (Zwaan 1978) and magnetic fields tend to move along cell boundaries (Simon et al. 1988).

More recently, Lawrence & Schrijver (1993) have used data from Schrijver & Martin (1989) in plage and enhanced network to create displacement distribution functions. They argue that these indicate that the evolution of plage is subdiffusive; i.e., the mean square displacement of test particles grows more slowly than linearly with time. Lawrence, Ruzmaikin, & Cadavid (1993) have used the spatial distribution of magnetic field elements in plage, enhanced network, and quiet Sun to estimate the multifractal dimensions of the magnetic field patterns. They find evidence that plage and enhanced magnetic field spatial distributions are multifractal and that the field distributions differ in quiet and active Sun.

Thus there is considerable evidence that the evolution of plages and active regions may require a different explanation than the distribution of magnetic field over most of the solar surface. During our experimentation with tethered evolution, we noted that when the tether length is short ($L < R/2$), tightly tethered, the diffusion coefficient is very small. Further in these cases, the normal clumping observed with longer tethers and random patterns did not occur; rather, the corks clumped in an irregular square array which was relatively stable. We suspect that this result is more general and that if each new cell is brought up within half the radius of the one it replaces, then there will be minimal merging of sinks. When the sinks become stable the effects of granulation and mesogranulation can no longer be neglected. Work is now in progress on tightly tethered evolution which includes supergranules, random fields of granules and mesogranules, and rules that limit the amount of magnetic flux that can accumulate in a particular location. Such models may be able to mimic many features of plages and the enhanced network that surrounds them.

In our models, as on the quiet Sun, it is observed that magnetic field exists in isolated clumps that nestle between supergranules. Random clumps with bigger areas, comparable to that of a supergranule, have also been noted by Wang & Sheeley (1994) in large scale diffusion models which use a much simpler method of generating flux diffusion. What, then, is responsible for the net-like geometry seen in Ca II spectroheliograms? One possibility is that the bright Ca II network away from active regions is not caused by kilogauss fields. Rather, it may be that the quiet Sun network arises at least in part from weak field which emerges constantly over the surface in the form of bipoles (Spruit et al. 1988) which are small compared to the diameter of supergranules. We are now studying kinematic models in which a strong field component is allowed to interact with a constantly emerging field of weak bipoles. Details of these models will be discussed in a subsequent paper.

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REFERENCES