IS THE SOLAR CHROMOSPHERIC MAGNETIC FIELD FORCE-FREE?

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ABSTRACT

We use observations of the Na i λ5896 spectral line, made with the Stokes Polarimeter at Mees Solar Observatory, to measure the chromospheric vector magnetic field in NOAA active region 7216. We compute the magnetic field from observations of the Stokes parameters at six wavelengths within this spectral line using a derivative method and calculate the height dependence of the net Lorentz force in the photosphere and low chromosphere. We conclude that the magnetic field is not force-free in the photosphere, but becomes force-free roughly 400 km above the photosphere.

Subject headings: MHD — polarization — Sun: chromosphere — Sun: magnetic fields

1. INTRODUCTION

It is often assumed that the solar magnetic field is force-free. This assumption greatly simplifies the study of the three-dimensional structure of the magnetic field, for example in the extrapolation of photospheric vector magnetic field data into the corona using linear or nonlinear force-free models (e.g., Sakurai 1981; Gary 1989; Low & Lou 1990). It is clear that the field is likely not force-free in the photosphere due to the high plasma $\beta$, and it is equally clear that the field likely is force-free in the corona (except in current sheets) due to the low plasma $\beta$. However, since the magnetic field is routinely observed only in the photosphere, it is important to understand the transition from non-force-free field in the photosphere to force-free field in the corona and thereby quantify the limitations of the force-free assumption.

This understanding is particularly important now that the Yohkoh satellite is regularly providing us with soft X-ray images of the corona. These images provide us a view of those portions of the coronal magnetic field which happen to contain hot plasma. However, to completely understand the three-dimensional structure of the magnetic field, we must rely on extrapolations of the vector field observed in the photosphere. How valid is the force-free assumption made in these extrapolations? In this paper we present measurements of the chromospheric magnetic field and we use these data to test to what extent the field there is force-free.

We observed the Na i λ5896 spectral line in NOAA active region 7216 and use these data to measure the chromospheric vector magnetic field. We compute the magnetic field with the weak field derivative technique proposed by Jeffries, Lites, & Skumanich (1989, hereafter JLS) and subsequently tested by Metcalf, Canfield, & Mickey (1991), and Jeffries & Mickey (1991). In the appendix we show how this technique is applied specifically to the Na i line.

With measurements of the chromospheric magnetic field at several wavelengths within the Na i line, we compute the net Lorentz force as a function of wavelength using the equations of Molodenski (1969). This crudely demonstrates the height dependence of the force since the wings of the Na i line are formed deeper in the atmosphere than the core. However, this does not give a reliable estimate of the height at which the field becomes force free. Hence, we derive the height dependence of the force with a simple regularized inversion and the Na i contribution functions computed with the VAL-F model atmosphere (Vernazza, Avrett, & Loeser 1981).

2. OBSERVATIONS OF THE CHROMOSPHERIC MAGNETIC FIELD

We observed NOAA active region 7216 on 1992 July 3 (N15E16) with the Mees Solar Observatory Stokes Polarimeter (Mickey 1985). The Stokes Polarimeter is a scanning instrument which measures one set of four Stokes profiles, from a single spatial location, at a time. The Na i data were obtained between 17:41 UT and 22:25 UT, with the Polarimeter observing at each raster point for 6 seconds. We used a 6° pinhole (sampling aperture) and scanned the region in 5'7 steps in a 63 by 45 rectangular grid (terrestrial east-west by north-south). We observed the Na i λ5896 line with 128 wavelength pixels spaced 20 mA apart over a 2.5 Å range of the solar spectrum centered slightly to the red of the Na i line to ensure a good measure of the continuum.

We computed the magnetic field at six wavelengths within the Na i spectral line using the derivative method of JLS, in the special case of the non-normal Zeeman effect in the Na i line (Appendix A). To apply the JLS weak field equations to the Na i data, we computed the polarization signal by assuming that the line is symmetric in $Q$ and $U$ and anti-symmetric in $V$. We then averaged a linear interpolation on the red side of the line with a linear interpolation on the blue side of the line. We computed the derivative of the $I$ profile by fitting a Voigt function to the line ($\Delta I$ from $-0.6$ to $+0.3$ Å) rather than computing the smoothed derivative described by Metcalf et al. (1991). This is computationally simpler and does not affect the result in this case. The Doppler widths and damping parameters used in the JLS method were computed from the densities and microturbulent velocity field in the VAL-F model atmosphere (Vernazza, Avrett, & Loeser 1981), though the JLS technique is rather insensitive to the values used.

We selected six wavelengths used in the calculation of the vector magnetic field (0.121, 0.104, 0.092, 0.086, 0.075, 0.068 Å...
from line center) based on two criteria. We used a field strength of 1700 G and the conditions given by Jeffries & Mickey (1991) to determine how close to line center the JLS weak field equations are applicable. For the longitudinal field, the weak field approximation holds over virtually the entire line, since the Na i line is quite broad. However, for the transverse field, the weak field approximation breaks down where the $Q$ and $U$ profiles change sign (0.03 to 0.04 Å). Second, compared to the polarization noise, the values of the $Q$ and $U$ Stokes profiles become small in the wings, so we could not compute reliable magnetic fields further than $\sim 0.13$ Å from line center. Within these two constraints, we selected the six wavelengths to give us good coverage of the low chromosphere (see Fig. 6, below). Note that the observations at the six wavelengths selected are not all independent since the pixel spacing is 20 mÅ.

From our observations at these six wavelengths, we derive six vector magnetograms. Figure 1 shows the vector magnetogram derived from the data at 0.121 Å from line center. The underlying image shows the sunspots in AR 7216. The magnetic fields have been artificially rotated to the center of the solar disk so that the magnetogram shows the vertical and horizontal fields, rather than the observed longitudinal and transverse fields. We use the vertical and horizontal fields throughout the following analysis.

Figure 2 shows the wavelength dependence of the total magnetic field derived from the Na i data. The various curves show the field at several raster points within the magnetograms. In general the magnetic field increases with the wavelength from line center, implying that the field is decreasing with height.

**Fig. 1.—** Vector magnetogram derived from the Na i observations at 0.121 Å from line center. The contours show the vertical field strength with solid contours indicating upward field and dashed contours shown downward field. The contour levels are 50, 100, 200, 400, 800, and 1600 G. The line segments show the strength and direction of the horizontal field; the scale at the bottom shows the length of a 1000 G horizontal field. The underlying image is a continuum image showing the sunspots in AR 7216. Solar east is to the left and north is up in the figure and the axes are labeled in units of 57. The magnetogram was scanned from 17:41 UT through 22:25 UT on 1992 July 3.

**Fig. 2.—** Magnetic field as a function of Na i wavelength at several points in Fig. 1. The horizontal axis shows the wavelength from line center in Å. The wavelength increases to the right and, hence, the height in the atmosphere increases to the left.
3. THE CHROMOSPHERIC MAGNETIC FORCE

Molodenskii (1969, 1974) and Low (1984) pointed out that for an isolated magnetic structure located in the infinite half-space above the plane \( z = 0 \), the net Lorentz force in the volume \( z > 0 \) is just the Maxwell stress integrated over the plane \( z = 0 \), provided that the field falls off fast enough as \( z \) goes to infinity. From Low's paper, necessary conditions for the magnetic field to be force-free are

\[
F_x \approx F_0,
\]
\[
F_y \approx F_0,
\]
\[
F_z \ll F_0,
\]

where \( F_x, F_y, \) and \( F_z \) are the components of the net Lorentz force, and \( F_0 \) is a characteristic magnitude of the total Lorentz force that can be brought to bear on the atmosphere if the magnetic field is not force-free. Low sets \( F_0 \) to the magnetic pressure force, and we follow this suggestion.

\[
F_x, F_y, F_z, \text{ and } F_0 \text{ are simply computed from}
\]

\[
F_x = \frac{1}{4\pi} \int_{z=0} B_x B_z \, dx \, dy,
\]

\[
F_y = \frac{1}{4\pi} \int_{z=0} B_y B_z \, dx \, dy,
\]

\[
F_z = \frac{1}{8\pi} \int_{z=0} (B_x^2 - B_y^2 - B_z^2) \, dx \, dy,
\]

(2)

\[
F_0 = \frac{1}{8\pi} \int_{z=0} (B_x^2 + B_y^2 + B_z^2) \, dx \, dy.
\]

To test whether the observed chromospheric magnetic field is force-free, we compute the scaled net Lorentz forces, \( F_x/F_0, F_y/F_0, \) and \( F_z/F_0 \), as a function of wavelength and height. We can conclude that the field is force-free at a given height only if each of these quantities is small compared to unity.

For condition (1) to be applicable, the magnetic structure under consideration must be isolated. We verified this by computing the imbalance between the upward and downward vertical magnetic flux in our observations. In all six magnetograms, the upward and downward directed vertical
This energy must be greater than the energy of the potential field above the same height \( E_{p} \): 0 < \( E_{p} \) \( \leq \) \( E_{ff} \). Note that the force-free energy given by equation (3) depends on the origin of the coordinate system unless the horizontal Lorentz forces vanish.

We computed \( E_{p} \) from a potential field extrapolation of the vertical field measured at each of the six Na I wavelengths used above. We computed \( E_{ff} \) from the observed vertical and horizontal magnetic field at each wavelength. Since our \( F_{v}/F_{o} \) and \( F_{h}/F_{o} \) do not vanish, the force-free energy depends on the coordinate system. Hence, we computed \( E_{ff} \) five times at each wavelength, with the origin set at each corner and at the center of the magnetograms. Figure 5 shows the potential field energy (dashed line) and the mean force-free energy (solid line) as a function of wavelength. The error bars on the force-free energy at each wavelength are the standard deviation of the force-free energy computed in the five coordinate systems. Clearly, further from line center (deeper in the atmosphere), the field is not force-free since the mean force-free energy is less than the potential field energy and since the force-free energy is strongly dependent on the coordinate system. The observed magnetic field does not satisfy the force-free condition except at the

flux nearly balance, as seen qualitatively in Figure 1. In each magnetogram, the vertical flux imbalance is 0.5% and is within the uncertainty in the observations. Thus, there is little or no magnetic flux leaving the region and condition (1) is applicable.

We independently compute \( F_{v}/F_{o} \), \( F_{h}/F_{o} \), and \( F_{q}/F_{o} \) for each of the magnetograms observed at the six wavelengths in the Na I line. Since the \( F_{v} \) and \( F_{h} \) integrals use the square of the field strengths, the noise in the measurements will be magnified in the results. Hence we include only those raster points with magnetic field greater than 150 G (the average noise level in horizontal magnetic field) in equation (2). This simple calculation yields the scaled net Lorentz force as a function of wavelength (Fig. 4). Since the wavelengths closer to line center are formed higher in the atmosphere, it is clear that the net force is falling with height.

There are several other conditions which must be satisfied by a force-free magnetic field (Aly 1989). In particular, the virial theorem gives the energy of the force-free field above some height when nonmagnetic forces are neglected:

\[
E_{ff} = \frac{1}{4\pi} \int_{z=zo} (xB_{x} + yB_{y})B_{z} \, dx \, dy .
\]  

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4. THE EFFECT OF THE MAGNETIC FILLING FACTOR

With a pinhole of 6°, there is certainly unresolved magnetic flux within the field of view of the Polarimeter. Unfortunately, since we are utilizing the JLS weak field equations to compute the magnetic field, we cannot readily determine the magnetic filling factor at each raster point. If we neglect the filling factor, by assuming it is 1.0 everywhere, we underestimate the true transverse magnetic field by \((f)^{1/2}\) and the true longitudinal field by \(f\), and we overestimate the area covered by the magnetic field by \(f\) (e.g., Jefferies & Mickey 1991). In the integrals for the scaled force, setting the filling factor to 1.0 weights the weak field regions too little. Thus, if the magnitude of the force in the weak field regions is smaller than the magnitude of the force in the strong field regions, we typically overestimate the magnitude of the scaled force by setting \(f\) to 1.0.

To crudely account for the filling factor in our data, we assume that all magnetic field is of KG strength and that plage fields appear weaker because of the filling factor (Rabin 1992a, b). This gives

\[
f \approx \begin{cases} 
B_{\text{total}}/1200 \text{ G} & \text{if } B_{\text{total}} < 1200 \text{ G} \\
1.0 & \text{if } B_{\text{total}} > 1200 \text{ G}
\end{cases}
\]  

(4)

where \(B_{\text{total}}\) is \((B_x^2 + B_y^2 + B_z^2)^{1/2}\).

We computed the force twice for each magnetogram, once assuming the filling factor is 1.0 everywhere and again assuming the filling factor from equation (4). As expected, including the estimate of the filling factor reduces the magnitudes of the scaled forces. However, the effect is slight. Since neither equation (4) nor \(f = 1.0\) is exact, we show both cases throughout the paper.

5. THE MAGNETIC FORCE AS A FUNCTION OF HEIGHT

From Figure 4, the force is clearly falling with height. However, the radiation from each wavelength is sensitive to the atmospheric conditions over a rather broad range of heights, and it is unclear where in the atmosphere the field can be said to be force-free. We, therefore, invert the data to obtain the force as a function of height rather than wavelength. Ideally, we should invert the magnetic data at each spatial point independently, but we have simplified the problem to avoid many difficulties with this approach. We invert only the total force integrals over the magnetograms on the assumption that the force we measure at each wavelength is the average force in the region of the atmosphere in which the radiation formed, weighted by the Na I contribution functions.

To this end, we have computed the contribution functions to the line depression for the Na I line in the VAL-F model atmosphere. These contribution functions are shown in Figure 6 for several wavelengths at which we computed the magnetic field.

We used an 11 level plus continuum model Na I atom with 29 lines treated in detail and computed the contribution functions following the suggestions of Magain (1986).

The VAL-F contribution functions will not give a perfect inversion for two reasons. First, we assume a single atmospheric model for the entire active region. This is necessary since the force integrals (e.g., [2]) average over the whole region. We estimate the error incurred from this assumption by comparing the VAL-F contribution functions with the VAL-C contribution functions. In the wavelength regime we are using, the shift in the mean height of the contribution functions is about 70 km from the VAL-C to the VAL-F models, with the VAL-C contribution functions lower in the atmosphere.

Second, we have used the contribution functions for unpolarized light. In principle, we should use contribution functions for the various states of polarization since the force is derived from observations of the magnetic field (van Ballegooijen 1985; Grossmann-Doeth, Larsson, & Solanki 1988; Solanki & Bruls 1994). This presents a difficulty since we examine the active region as whole but the polarized contribution functions are dependent on the magnitude and direction of the magnetic field. We estimate the error incurred from this assumption with an expression for the separation between the left and right circularly polarized contribution functions given by Murphy (1990). His expression implies a separation of about 50 km for the Na I line.

From the above considerations, the height dependence of the force is accurate to about 100 km. Note that we use only the normalized contribution functions below and differences in magnitude between alternative contribution functions are not important; only a difference in shape or a shift in height will affect our results.

To derive the approximate height dependence of the net Lorentz force, we use second order regularization to invert the wavelength dependence (Craig & Brown 1986; Press et al. 1992; Metcalf et al. 1990). To apply the technique, we assume that the force as a function of wavelength is related to the force as a function of height by

\[
F_{\text{obs}}^{\text{obs}}(\lambda) = \frac{\int_{-\infty}^{\infty} C_{\lambda}(h) F_{\lambda}(h) dh}{\int_{-\infty}^{\infty} C_{\lambda}(h) dh},
\]

(5)

where \(h\) represents the height and \(\lambda\) the wavelength, the \(C_{\lambda}(h)\) are the contribution functions, \(F_{\text{obs}}^{\text{obs}}(\lambda)\) are the observed forces as a function of wavelength, and \(F_{\lambda}(h)\) are the forces as a function of height which we seek. In equation (5), we have assumed that the force we measure at each wavelength is the average force in the region of the atmosphere in which the radiation is formed, weighted by Na I contribution function.

Since we measure the net force at discrete wavelengths, we convert this integral equation into a matrix equation in which both \(F_{\text{obs}}^{\text{obs}}(\lambda)\) and \(F_{\lambda}(h)\) are discrete vectors. We also add the second-order regularization smoothing matrix to keep the inversion stable:

\[
(C^T C + \Delta H)F_{\lambda}(h) = C^T F_{\text{obs}}^{\text{obs}}(\lambda),
\]

(6)
Fig. 7.—Scaled net Lorentz force measured in AR 7216 as a function of height above the photosphere. Figs. 7a, 7b, and 7c show the x, y, and z components of the net force, respectively. Smaller magnitudes of the scaled net force indicate that the field is more force-free.

$C$ is a matrix computed from the contribution functions, with the superscript $T$ representing the transpose, $H$ is the second order smoothing matrix, $\alpha$ is a Lagrange multiplier used to control the degree of smoothing, $F_{x,y,z}$ is the vector representing the force as a function of height, and $F_{x,y,z}^{\text{obs}}$ is the vector representing the observed force as a function of wavelength. Equation (6) is easily inverted to yield the force as a function of height.

Before inverting, however, we must first select the value of $\alpha$, the smoothing parameter. We do this by balancing the error introduced by smoothing (larger $\alpha$) and the error due to the instability inherent in the inversion (smaller $\alpha$). We determine $\alpha$ independently for each component of the net Lorentz force by minimizing the sum of the error due to the instability, determined from the scatter in the solution when many random perturbations of the data are considered, and the residual error due to the smoothing (e.g., Metcalf et al. 1990). The solution is not strongly dependent on the exact value of $\alpha$; $\alpha$ can vary by a factor of 10 up or down without significantly affecting the solution.

The inversions for the x, y, and z components of the scaled net Lorentz force are shown in Figure 7. The error bars on the plots were computed by randomly perturbing the magnetic field observations many times and recomputing the inversion. The error bars show one standard deviation of the resulting set of inversions. In the perturbations, random noise of 150 G in the horizontal field and 100 G in the vertical field was added (the observed noise in the magnetograms). The error bars tend to be largest where the inversion is not well constrained by the contribution functions. This yields larger error bars at the highest and lowest heights where all the contribution functions are small.

From Figure 7, we see that the magnetic field in AR 7216 is not force-free in the photosphere, but the magnitude of the force decreases monotonically into the chromosphere until the field can be considered force-free above 400 km. Although sizable vertical forces are possible because of gravitational stratification (Molodenskii 1969), the non-negligible horizontal forces imply that the atmosphere is not in equilibrium since there is no way to balance the horizontal Lorentz force. In Table 1 we estimate the acceleration timescales implied by the observed horizontal force using the VALF model atmosphere.

In the table, $h$ is the height above the photosphere, $M$ is the column mass above $h$, $F/F_0$ is the scaled force, $a$ is the average acceleration of the material above $h$, $c$ is the sound speed, $t_1$ is the time to accelerate the atmosphere to the sound speed, and $t_2$ is the time to move through $10^4$ km when the material starts from rest. The timescales presented in Table 1 are unreasonable and the observed horizontal forces could exist, at least for short times. However, these accelerations obviously could not be present throughout the 5 hours of the magnetogram raster scan time. Since the timescales are shorter than the time to complete the magnetogram raster scan, the apparent horizontal force imbalance could well be due to evolution of the active region during the scan.

We used the same regularized inversion to compute the height dependence of the magnetic field. Inside the sunspot umbra and penumbra, this inversion must be considered approximate, since the VALF contribution functions are inappropriate. Figure 8 shows the magnetic field as a function of

<table>
<thead>
<tr>
<th>$h$ (km)</th>
<th>$M$ (g cm$^{-2}$)</th>
<th>$F/F_0$</th>
<th>$a$ (km s$^{-1}$ hr$^{-1}$)</th>
<th>$c$ (km s$^{-1}$)</th>
<th>$t_1$ (h)</th>
<th>$t_2$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.3</td>
<td>0.45</td>
<td>11.7</td>
<td>7.9</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>100</td>
<td>2.1</td>
<td>0.44</td>
<td>23.4</td>
<td>7.3</td>
<td>0.3</td>
<td>0.5</td>
</tr>
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<td>44.6</td>
<td>7.0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>400</td>
<td>0.2</td>
<td>0.17</td>
<td>94.9</td>
<td>6.6</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

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Fig. 8.—Approximate height dependence of the magnetic field computed from the Na i data at the same spatial points shown in Fig. 2. As expected from Figure 2, the field generally decreases with height. Note that the total field at 0–100 km in the leading sunspot corresponds well to the Fe i data from Figure 3. Bruls, Lites, & Murphy (1991) computed the height of formation of the Fe i λ6302 line as 200 km in sunspot umbrae, consistent with our result considering the uncertainty in the derived height dependence of the magnetic field.

6. CONCLUSIONS

We have shown that the Na i λ5896 line yields the vector magnetic field in the solar chromosphere using essentially the same weak field equations derived by JLS for normal Zeeman triplet lines. Further, we have used Na i data to demonstrate that the solar magnetic field in AR 7216 is not force-free in the photosphere, but does become force-free higher in the atmosphere. The field can be considered force-free above about 400 km in this active region.

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APPENDIX A

WEAK FIELD EQUATIONS

Landi Degl’Innocenti & Landi Degl’Innocenti (1973) give very general expressions for the treatment of the anomalous Zeeman effect. We follow their work closely here, but specialize to the Na i line. Substituting the appropriate atomic parameters into their equations yields the same results we find here, however, we wish to explicitly demonstrate that the JLS equations can be used to compute the vector field from the Na i λ5896 line.

We start from the transfer equations for anomalous Zeeman splitting given by Landi Degl’Innocenti & Landi Degl’Innocenti (1972) and expand the Voigt function as in JLS:

\[ H(a, v + v_\text{s}) \approx H(a, v) + v_\text{s} H'(a, v) + \frac{v_\text{s}^2}{2} H''(a, v). \]  

(7)

The primes represent the derivative of the Voigt function with respect to v.

For the anomalous Zeeman effect, the magnitude of the splitting, \( v_\text{s} \), is,

\[ v_\text{s} = \frac{v_\text{l}}{\Delta v_D} (M g_j - M' g_J). \]  

(8)

where the primes denote the lower level of the transition (Landi Degl’Innocenti & Landi Degl’Innocenti 1972). The different splittings for the \( \pi \) and \( \sigma \) components are seen by rewriting this equation in the form

\[ v_\text{s} = \frac{v_\text{l}}{\Delta v_D} [M (g_j - g_J) - (M - M') g_J]. \]  

(9)

Defining

\[ v_m = \frac{M (g_j - g_J) v_\text{l}}{\Delta v_D}, \]  

(10)

and

\[ v_{\text{m}''} = \frac{g_J v_\text{l}}{\Delta v_D}, \]  

(11)

the splitting of the \( \pi \) components is \( v_m \) and the splitting of the \( \sigma \) components is \( v_m \pm v_{\text{m}''} \), since \( \Delta M = 0, \pm 1 \).

Using the expressions given by Beckers (1969), the strengths of the Zeeman components for the Na i transition \( J \rightarrow J' (\frac{1}{2} \rightarrow \frac{1}{2}) \) are seen to be equal for both the \( \sigma \) components and the summation of the two \( \pi \) components. Thus the absorption coefficients are simply

\[ \eta_\pi = \eta_0 \left( H + \frac{v_\text{m}^2}{2} H'' \right). \]  

(12)
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for the $\pi$ components and

$$\eta_{r,l} = \eta_0 \left[ H \pm (v_m + v_m')H' + \frac{(v_m + v_m')^2}{2} H'' \right]$$ (13)

for the right and left $\sigma$ components. We have simplified $v_m$ to $|v_m|$ since the splitting of the two $\pi$ components is equal in magnitude but opposite in sign for the Na I line. Since

$$\eta_1 = \frac{1}{2} \left( \frac{\eta_r + \eta_l}{2} (1 + \cos \gamma) + \eta_\sigma \sin^2 \gamma \right),$$

$$\eta_2 = \frac{1}{2} \left( \eta_\sigma - \frac{\eta_r + \eta_l}{2} \right) \sin^2 \gamma \cos 2\chi,$$

$$\eta_3 = \frac{1}{2} \left( \eta_\sigma - \frac{\eta_r + \eta_l}{2} \right) \sin^2 \gamma \sin 2\chi,$$

$$\eta_4 = \left( \eta_r - \eta_l \right) \cos \gamma,$$

we have

$$\eta_1 = \eta_0 \left[ H + \frac{v_m^2}{4} + \left( \frac{v_m v_m'}{2} + \frac{1}{4} v_m^2 \right)(1 + \cos^2 \gamma) \right] H'' \right],$$

$$\eta_2 = -\eta_0 \frac{2v_m v_m'}{4} \sin^2 \gamma \cos 2\chi,$$

$$\eta_3 = -\eta_0 \frac{2v_m v_m'}{4} \sin^2 \gamma \sin 2\chi,$$

$$\eta_4 = \eta_0 (v_m + v_m') \cos \gamma.$$ (15)

Comparing these equations to the equivalent equations in JLS, we see that, to compute the chromospheric magnetic field from the Na I line, we should simply replace $g_L$ in the longitudinal field equation by

$$|M| (g_L - g_R) + g_R = \frac{3}{4}$$ (16)

and $g_L$ in the transverse field equation by

$$2 |M| (g_L - g_R) + g^2_r = \frac{3}{4},$$ (17)

since, for the Na I $\lambda 5896$ line, $g_R = 2$, $g_L = \frac{3}{4}$, and $|M| = \frac{1}{2}$.

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