HIGH-FREQUENCY OSCILLATIONS OF A POLYTROPIC LAYER

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ABSTRACT We study the high-frequency acoustic oscillations of a plane-parallel polytropic layer overlain by an isothermal atmosphere. The oscillations are both forced and damped at some depth within the polytrope. Significant peaks in the power spectrum are discernable at frequencies $\omega$ well beyond the atmospheric acoustic cutoff frequency $\omega_c$, although the amplitudes are much lower than those of the modes with $\omega < \omega_c$. The spacing between adjacent peaks decreases with increasing frequency as $\omega_c$ is approached, and then, as $\omega_c$ is crossed, there is a jump to higher values of frequency spacing and of line width. At still higher frequencies additional variations in widths and spacing reflect the depth of the driving and damping layer.

INTRODUCTION

The observed acoustic power spectrum of the Sun contains peaks well beyond the expected acoustic cutoff frequency $\omega_c$ of the solar atmosphere (e.g. Duvall et al. 1991). Theoretical discussion of their source has focussed on two mechanisms: partial reflection at the chromospheric-coronal temperature transition (Balmforth & Gough 1990) and observational filtering of traveling waves excited by a subphotospheric source (Kumar et al. 1990). Here we examine a simplified model consisting of a plane-parallel polytropic layer overlain by an isothermal atmosphere and attempt to understand its acoustical spectrum in terms of the underlying model properties. With such a model we replace observational filtering by wave interference, as do Kumar and Lu (1991). The oscillations are both forced and damped within the polytrope. The amplitudes, linewidths, and frequency spacing characterizing the resultant radial modes are determined by fitting Lorentzian profiles to the power-spectrum peaks. Peaks below $\omega_c$ are well approximated by such a fit, while those above $\omega_c$ are generally broader and flatter. Nevertheless, we characterize the lines by their Lorentzian parameters throughout the spectral range examined.

FORMULATION

Consider a plane-parallel polytrope [polytropic index $m$, density scale height...
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$H_1(z)$, and sound speed $c_1(z)$] overlain by an isothermal atmosphere [density scale height $H_0$, sound speed $c_0$, and acoustic cutoff frequency $\omega_c$]. Depth $z$ is measured downward, with the interface between the two layers occurring at $z = z_0$ and the bottom at $z = z_b$. The pressure is continuous across $z_0$, but the sound speed and density are not, and we characterize the discontinuity by the parameter $\epsilon_0 = c_0/c_1(z_0)$. The layer is subject to internal driving and damping, of amplitude $\psi_0$ and $a_c$ respectively, at depth $z = z_d$ within the polytrope. Lagrangian pressure perturbations $\psi$ obey the equation

$$\frac{d^2 \psi}{dz^2} - \frac{1}{H} \frac{d \psi}{dz} + \frac{\omega^2}{c^2} [1 + i a_c \delta(z - z_d)] \psi = \psi_0 \delta(z - z_d) .$$

(1)

The causal solution to this equation is found subject to the following boundary and matching conditions:

1. $\frac{d \psi}{dz} = 0$ at $z = z_b$ ,
2. $\psi$ continuous at $z = z_0$ ,
3. $\frac{1}{\rho} \frac{d \psi}{dz}$ continuous at $z = z_0$ ,
4. $\psi$ continuous at $z = z_d$ .

Also \( \left( \frac{d \psi}{dz} \right)_{z_d+\delta} - \left( \frac{d \psi}{dz} \right)_{z_d-\delta} = \psi_0 - i a_c \frac{\psi(z_d)}{4z_1} \frac{1}{z_d} \), with $z_1 = \frac{H_0\omega_c^2}{(m + 1)\omega^2}$.

The amplitude at $z = z_0$ is then given by

$$A = 2Q_0Q_1q_1\lambda_3 \left( \frac{z_0}{z_d} \right) \psi_0 \left[ Q_1 \left\{ Q_A J_m(\lambda_3) - Q_B Y_m(\lambda_3) \right\} + \left( Q_2 + i \frac{Q_1\lambda_3a_c}{2} \right) \left\{ Q_B Y_{m+1}(\lambda_3) - Q_A J_{m+1}(\lambda_3) \right\} \right]^{-1} ,$$

(2)

where $J_m$ and $Y_m$ are Bessel functions of the first and second kind, and

$$Q_0 = J_{m+1}(\lambda_1)Y_m(\lambda_1) - J_m(\lambda_1)Y_{m+1}(\lambda_1) ,$$

$$Q_1 = J_{m+1}(\lambda_3)Y_m(\lambda_2) - J_m(\lambda_2)Y_{m+1}(\lambda_3) ,$$

$$Q_2 = J_m(\lambda_3)Y_m(\lambda_2) - J_m(\lambda_2)Y_m(\lambda_3) ,$$

$$Q_A = 2z_1\lambda_1\kappa_0^2z_{m+1}(\lambda_1) - Y_m(\lambda_1) ,$$

$$Q_B = 2z_1\lambda_1\kappa_0^2J_{m+1}(\lambda_1) - J_m(\lambda_1) ,$$

$$\lambda_1 = \left( \frac{z_0}{z_1} \right)^{1/2} , \quad \lambda_2 = \left( \frac{z_b}{z_1} \right)^{1/2} , \quad \lambda_3 = \left( \frac{z_d}{z_1} \right)^{1/2} ,$$

$$\kappa_0 = \frac{1}{2H_0} \left[ 1 + \sqrt{1 - \frac{\omega^2}{\omega_c^2}} \right] \text{ for } \omega < \omega_c ,$$

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Fig. 1 Line widths. The values corresponding to \( z_d = z_0 + 2 \times 10^{-3} (z_b - z_0) \) are plotted correctly; the others have been multiplied by 10 and 10^2 for clarity.

\[
\kappa_0 = \frac{1}{2H_0} \left[ 1 - i \sqrt{\frac{\omega^2}{\omega_c^2} - 1} \right] \quad \text{for} \quad \omega > \omega_c ,
\]

and

\[
\omega_c = \frac{c_0}{2H_0} .
\]

For each value of \( \omega \) the solution is uniquely specified given the polytropic index \( m \), the atmospheric acoustic cutoff frequency \( \omega_c \), the sound-speed ratio \( \epsilon_0 = c_0/c_1(z_0) \), and the depths \( z_0, z_d, \) and \( z_b \). For the solutions presented here we take \( m = 1.5 \) and \( \epsilon_0 = 0.73 \). Depth is scaled such that \( z_b = 1.0 \), and frequency so that \( \omega_c = 1.0 \). We choose \( z_0 \) such that there are 14 peaks in the power spectrum between \( \omega_c/2 \) and \( \omega_c \).

RESULTS

Figures 1 and 2 display the mode line width \( \sigma \) (half width at half maximum measured in units of \( \omega_c \)) and frequency spacing \( \Delta \omega \) as a function of frequency for \( z_d \) equal to \( 5 \times 10^{-4}, 1 \times 10^{-3}, \) and \( 2 \times 10^{-3} \) of the total depth of the polytrope. Modes with \( \omega < \omega_c \) are trapped by the atmosphere and have relatively small line widths, determined predominantly by the damping coefficient \( a_c \). As \( n \) increases the frequency spacing approaches its asymptotic value \( \omega_0 = \pi \left( \int c^{-1} dz \right)^{-1} \), with the integral being evaluated from the top \( z = z_0 \) to the bottom \( z = z_b \) of the region of wave propagation. As \( \omega \) approaches \( \omega_c \), the atmosphere becomes less effective at confining the mode and the effective acoustical cavity is enlarged. Consequently, the frequency spacing diminishes.

Above \( \omega_c \) the modes are no longer contained and the energy leaks into the atmosphere. As a result the oscillation amplitude at \( z = z_0 \) is much smaller.
Fig. 2 Frequency spacing. Symbols defined as in Fig. 1. The values corresponding to \( z_d = z_0 + 2 \times 10^{-3}(z_b - z_0) \) are plotted correctly; the others are displaced upwards by 0.005 and 0.01 for clarity.

than it is when \( \omega < \omega_c \). Amplitude variation with frequency above \( \omega_c \) is essentially sinusoidal (Kumar et al. 1990), and the line widths (half width at half maximum) are approximately one quarter the frequency spacing. The frequency spacing is greater than the typical spacing of high-order modes beneath the acoustical cutoff, and corresponds to the asymptotic eigenfrequency spacing \( \omega_s = \pi \left( \int c^{-1}dz \right)^{-1} \) of a smaller cavity, \( z_d \leq z \leq z_b \), because the dominant interference is between outgoing waves generated directly at \( z = z_d \) and waves that have been reflected from the boundary at \( z = z_b \).

There is an additional contribution to both the frequency spacing and the line widths which results from interference with waves partially reflected at \( z = z_0 \). It is manifest as a modulation of \( \sigma \) and \( \Delta \omega \) as \( \omega \) increases and is most clear in Figures 1 and 2 for the example with the deepest driving layer. For that layer the difference between \( \omega_s \) and \( \omega_0 \) is the greatest. In the solar case, to which the example with the shallowest driving layer probably corresponds the most closely, the modulation frequency is too low for its signature to be recognized in these figures.

REFERENCES