TESTING THE STATISTICAL SIGNIFICANCE OF THE ASYMME- 
TRIES OF p-MODE LINE PROFILES: APPLICATION TO THE IPHIR 
DATA

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ABSTRACT We have used a statistical test for measuring the 
significance of low-degree p-mode line asymmetries obtained by the 
maximum likelihood technique. The test is based on the likelihood ratio 
test. We show theoretically that the test statistic is $\chi^2$ with 1 or 2 
degrees of freedom, depending on the number of parameters describing 
the asymmetry. The test is applied to synthetic data simulating a single 
p mode and to the IPHIR data. We report on which formula, from the 
many we have used, has the greatest statistical significance. We outline 
the future work that is planned.

INTRODUCTION

Low-degree p-mode lines are often assumed to be symmetrical. Various 
phenomena can create line asymmetries, such as frequency drift and mode 
excitation. The latter could give us information about the location of the source of 
the excitation of the modes (e.g. Duvall et al, 1993; Gabriel, 1993). Spectral 
lines of high-degree modes ($l = 157-221$) have been successfully determined with 
a high signal-to-noise ratio and power with a Gaussian distribution, to which 
line profiles can be fit with a least-squares technique (Duvall et al (1993)). For 
low-degree modes, the signal-to-noise ratio is not so high, and the non-gaussian 
statistics require the use of a differently tailored maximum likelihood technique 
(Toutain and Appourchaux, 1994, and references therein). Recently, Toutain 
(1994) inferred low-degree p-mode line asymmetries from the IPHIR data. 
However, it was clear from the dependence of the standard errors on frequency 
that the signal-to-noise ratio played an important role in the determination of 
the asymmetry. This single fact leads us to check the statistical significance of 
the values of the asymmetry he obtained. In the first section, we explain how 
one can assess, using the likelihood ratio test, the significance of line parameters 
derived using the maximum likelihood technique. In the second section, we apply 
the significance test to synthetic data simulating a single p mode, and we verify 
the test statistics. In the third section, we apply the test to the IPHIR data
and check the statistical significance of various for mulae. In the last section, we mention future work planned for investigating some aspects of the physics of the Sun.

THE LIKELIHOOD RATIO TEST

Low-degree p-mode lines are fit best using the maximum likelihood technique (Toutain and Appourchaux, 1994, and references therein). The method involves maximizing the likelihood $L(\omega_p)$ of a given occurrence for which $p$ parameters are used to described the line profile. If one wants to describe the same occurrence but with $p + n$ parameters, the likelihood $L(\Omega_{p+n})$ will have to be maximized. The likelihood ratio test concerns the ratio of the two likelihood estimates (Brownlee, 1965):

$$\lambda = \frac{L(\omega_p)}{L(\Omega_{p+n})}. \quad (1)$$

If $\lambda$ is close to 1, there is no improvement in the maximized likelihood, and the $n$ additional parameters are not significant. On the other hand, if $\lambda \ll 1$ the additional parameters are very significant. Wilks (1938) (see Brownlee, 1965) showed that for large sample size the distribution of $-2\ln \lambda$ tends to the $\chi^2(n)$ distribution. For least squares, we point out that a test similar to the likelihood ratio test can be used: the so-called $R$ test (Fried, 1983).

APPLICATION TO SYNTHETIC DATA

In order to verify the distribution of $\lambda$, we simulated a single p-mode spectrum assuming classically that each frequency bin has a $\chi^2$ probability distribution with 2 degrees of freedom. The simulated line profile was symmetrical and Lorentzian with a 1-\muHz linewidth. The frequency range over which the spectrum was fitted was 60 \muHz. Each spectrum was fitted with 2 different line profiles, one symmetrical, the other asymmetrical. Four different formulae describing the asymmetry were used:

$$S_s(x) = \frac{1}{1 + x^2} + \frac{\alpha x^3}{(1 + x^2)^2}, \quad (2)$$

$$S_d(x) = \frac{1}{1 + x^2} + \frac{\alpha x}{(1 + x^2)^2}, \quad (3)$$

$$S_g(x) = \frac{1}{(1 + x^2 + \beta x^4)} + \frac{\alpha x^3}{(1 + x^2 + \beta x^4)^2}, \quad (4)$$

where $x$ is the reduced frequency (frequency in units of half the linewidth), and $\alpha$ and $\beta$ characterize the asymmetry and a departure from the Lorentz profile respectively.
A fourth formula, after Duvall et al (1993), was also used:

\[
S_t(\nu) = \frac{1}{N(R, D)} \left| 1 + \frac{D e^{i(-2i(\nu - \delta\theta))}}{1 - R e^{i(-2i\hat{\theta}(\nu))}} \right|^2 \quad \text{with} \quad \theta(\nu) = \frac{\pi(\nu - \nu_0)}{\Delta\nu} + n\pi,
\]

where \( \nu \) is cyclic frequency, \( \nu_0 \) is the mode frequency, \( \Delta\nu \) is the (large) frequency separation with increasing radial order, \( D \) and \( R \) characterize the symmetric profile of the line, and \( \delta\theta \) characterizes the asymmetry; \( N(R, D) \) is a normalization factor determined by requiring that \( S_t(\nu) = 1 \) when \( \theta(\nu) = 0 \) and \( \delta\theta = 0 \).

\[
\begin{array}{c}
\text{Reference statistic} \\
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
\end{array}
\]

\[
\begin{array}{c}
-2\ln(\lambda) \\
\begin{array}{ccccccccc}
0.1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
\end{array}
\end{array}
\]

Fig. 1. Fractile diagram for the \( S_R(+) \). The continuous line represents the reference statistic, which is a \( \chi^2 \) with 2 degrees of freedom, with a mean of 2 and a standard deviation of 4.

For each formula, we computed a thousand realizations and fitted the spectra with the maximum likelihood technique. We checked that all the fitted parameters have either a normal or a log normal distribution; this is a prerequisite for applying the procedure of Wilks. We studied the statistics of \(-2\ln(\lambda)\) by computing the fractile diagram, i.e., by comparing the simulated cumulative probability to that of a reference statistic (Hald, 1952). Depending on the number of additional parameters describing the asymmetry, the reference statistic is \( \chi^2 \) distributed with either 1 or 2 degrees of freedom. From Fig. 1 it is clear that the simulated distribution behaves as the reference statistic. The same conclusion was obtained for the 3 other formulae. We concluded that the procedure of Wilks could safely be used for our purpose.

APPLICATION TO IPHIIR DATA

At this stage, it is worth noting that the application of the maximum likelihood technique to any data is carried out with the implicit assumption that the data are independent. When there are gaps in the data, this assumption is no longer valid. However, the IPHIIR data suffered only 10% data gaps, so the condition for validity is nearly satisfied. Consequently, the same fitting procedure as that described above was applied to the IPHIIR data. Figure 2 shows the statistical...
significance of the asymmetry for the $l = 1$ modes as a function of frequency for the different formulae.

DISCUSSION

The various formulae we have used do not exactly reflect the physics in the Sun. More realistic formulae have been derived whose asymmetry are related to the location of the excitation source; they resemble the formula of Duvall et al (1993). Additional efforts will be needed to apply these to the IPHIR data, and in the future to GONG and SOHO data. The likelihood ratio test that has been used for the asymmetry can be applied to any other parameter whose significance needs to be assessed, such as mode splitting.

\begin{figure}[ht]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Statistical significance for $l = 1$ modes as a function of frequency for the formulae $S_s$ ($\circ$), $S_d$ ($\diamond$), $S_g$ ($\triangle$) and $S_t$ ($+$).}
\end{figure}

REFERENCES


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