SPECTRA OF SOLAR MAGNETIC FIELDS AND DIFFUSION

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ABSTRACT The scaling of a time series of magnetic and Doppler images obtained in good seeing conditions at San Fernando Observatory are studied. It is shown that the distribution of scaling exponents (the singularity spectrum) is stable within the time period used (several hours). Two scaling parameters of the space-time power spectrum were found to characterize the turbulent diffusion of solar magnetic fields.

INTRODUCTION
Helioseismology in its present form is aimed at the study of oscillations on the solar surface. In Fourier language it is interested in discrete modes. The continuous (background) time spectrum of the solar velocity field has been estimated (Harvey, 1985; Pallè et al., 1994) but it was treated only as noise. Meanwhile some advances have been made in techniques of studying turbulence which can be applied to solar problems, such as turbulent diffusion or chaotic excitation of solar oscillations. This makes use of the space-time background spectrum, i.e. the distribution of energy over various scales and time intervals. This spectrum defines the character of turbulent diffusion and completely describes Gaussian turbulence. However, the actual distribution of the solar velocity field is very intermittent, i.e. highly non-Gaussian. Thus high-order spectra are also needed to complete the characterization of the turbulence. A good representation of the high-order spectra is given by the “singularity spectrum,” which describes the distribution of exponents of all high-order statistical moments. The singularity spectra of solar active regions and quiet sun network at a fixed time have been found previously using images obtained at San Fernando Observatory and at La Palma Observatory (Lawrence et al., 1993). In this paper we present the results of calculations of the singularity spectra for a time series of San Fernando images. We also present some results of our first attempts to estimate the parameters of the second-order space-time spectrum of the magnetic and velocity fields in order to characterize solar turbulent diffusion.
ON THE TIME DEPENDENCE OF THE SINGULARITY SPECTRA

We consider an image (magnetogram) of the line-of-sight photospheric magnetic field, which is equivalent to the net magnetic flux integrated over the size of an image pixel. Similarly, we consider Dopplergrams, which give line-of-sight velocity.

A natural measure on the magnetogram is the local magnetic flux $\Phi$ (in a box of size $s$) normalized by the total flux through the magnetogram. Let us assume that this quantity is self-similar, i.e. that it scales with a power law dependence on a dimensionless scale $\epsilon = s/L$: $\Phi/\Phi_0 = \epsilon^\alpha$, where $L$ is a characteristic size of the magnetogram. Then the line-of-sight magnetic field, which is the flux density $\Phi/f^2$, must scale like $\epsilon^{\alpha-2}$. Thus if $\alpha < 2$, the field strength is singular when $\epsilon \rightarrow 0$. The distribution function of the exponents for each coarse-graining scale $\epsilon$ can be found by counting the number $n(\alpha, \epsilon)d\alpha$ of boxes with a value of $\alpha$ in bins of width $d\alpha$. The density $n(\alpha, \epsilon)$ is scale-dependent; however, it may be presented in the form $\epsilon^{-f(\alpha, \epsilon)}$, where the function $f(\alpha, \epsilon) \rightarrow f(\alpha)$ in the limit of small $\epsilon$, i.e. is a scale-independent characteristic of the distribution of the scaling exponents, the so-called "singularity spectrum" (Meneveau and Sreenivasan, 1989; Evertz and Mandelbrot, 1992). The function $f(\alpha)$ was found for the observed magnetic field in some solar active regions and in quiet sun (Lawrence et al., 1993) and for some convection and dynamo models (Brandenburg et al., 1993; Cadavid et al., 1994).

In the present work we study the scaling of a time series of magnetic and Doppler images of a quiet sun area obtained in good seeing conditions on 9 September 1993. The data were recorded using the San Fernando Observatory 28 cm vacuum telescope and vacuum spectroheliograph operated in video spectroheliograph mode (Chapman and Walton, 1989). Twenty-four line-of-sight magnetic images were taken at ten minute intervals over a nearly four hour span. Each image is the difference of two magnetograms taken 75 seconds apart with reversed quarter wave plate settings. The images are composed of $L_1 \times L_2 = 470 \times 311$ pixels at 0.46 arcsec/pixel. The images are studied at variable coarse-graining sizes, $s \equiv \sqrt{L_1 L_2}$ with $\epsilon = 1/256, 1/128, \ldots, 1/2$. We find that the singularity spectrum is stable during the time studied (Figure 1).

![Fig. 1. A time series of singularity spectra for a quite region. The first curve is $f(\alpha)$ for the Gaussian noise.](image-url)
WHAT TYPE IS THE SOLAR TURBULENT DIFFUSION?

Turbulent diffusion is an essential concept in the study of the transport of scalar quantities, such as temperature, and vector quantities, such as the large-scale solar magnetic field. It is a necessary element of the solar dynamo, which is the driving mechanism of solar activity. Turbulent diffusion results from the presence of a wide spectrum of scales of motion. If the advected quantity is passive, i.e. does not affect the turbulence, then the character of the turbulent diffusion is defined by this spectrum. Recently Avellaneda and Majda (1992) show that the diffusion law for a passive scalar in a given turbulent velocity field is defined by only two numbers: the exponent $\epsilon$ in the spatial power spectra $E(k) \propto k^{1-\epsilon}$, and the exponent $\tau$ of the spatial scale dependence of correlation times in the non-linear cascade: $\tau \propto k^{-\tau}$. In the intermediate asymptotic regime these two numbers define the character of turbulent diffusion, i.e. the exponent in the dependence of the squared distance on time $(d^2) \propto t^h$. For the normal diffusion $h = 1/2$; $h > 1/2$ for the superdiffusion; and $h < 1/2$ for the subdiffusion. Note that on very long run, i.e. when $t \to \infty$, all regimes converge to the normal diffusion.

The solar case is more complicated than this in at least two aspects. First, the solar turbulence is not hydrodynamic, it is magnetohydrodynamic. Second, the large-scale magnetic field, the vector transported by the turbulence, might essentially affect the turbulent motions. Thus, formally we have no right to use the results for the passive scalar transport. However, the passive scalar diffusion can be a good first approximation to the diffusion of the line-of-sight component of the solar magnetic field. In this paper we make an attempt to estimate the character of the solar magnetic diffusion by fitting our data to the Avellaneda and Majda diffusion diagram. The two parameters $\epsilon$ and $\tau$ defining this diagram are taken from the correlation function in the inertial interval of turbulence according to

$$\langle v(x, s, t + \tau)v(x, t) \rangle = \langle v \rangle^2 \int k^{1-\epsilon}e^{-ak^\tau}e^{ikx}dk.$$ 

the Fourier transform of which gives the space-time energy spectrum. The Fourier transform of the spatial correlations at fixed time define the usual power spectrum $E(k) \propto k^{1-\epsilon}$. The exponent $\tau \geq 0$ defines the dependence of decorrelation times on the spatial scale. In particular, the long-wavelength modes have longer decorrelation times. In Kolmogorov's classical case of homogeneous fluid turbulence $\epsilon = 5/3$ and $\tau = 2/3$. Another example, perhaps more relevant to the solar case, is the random-phased Alfvén wave turbulence with spectral exponent $\epsilon = 5/2$ (Kraichnan, 1965). For this magnetohydrodynamic turbulence model $\tau \propto 1/(V_A k)$, i.e. $\tau = 1$. Another useful concept which we can borrow from this model is the equality of the velocity and magnetic energy densities. Thus one can use the magnetic correlations in place of the velocity correlations.

The energy spectrum of the magnetic fields for the September 9, 1993 image was calculated as $E(k) = \int \langle k < b(k, t)b^*(k, t) > dt$, where $b(k, t)$ is the spatial fast Fourier transform of the magnetic field, and the integral is taken over whole time interval (4 hours). The fit to the straight line in a self-similar part of the spectrum gives the exponent $\epsilon$. An estimate for the decorrelation time exponent $\tau$ was found by use of the relation $E(k)\tau(k) \propto k < b(k, 0)b^*(k, 0) > \propto k^{1-\epsilon-\tau}$. 

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Figure 2 shows the diffusion diagram by Avellaneda and Majda (1992) on which the Kolomogorov point (K), a point corresponding to the random-phased Alfvén turbulence (A), and one point each from our magnetogram (M) and our Dopplerogram (D) are shown. Both observational points are in the region III where time-decorrelation effects are negligible (i.e. the parameter \( z \) is inessential). Hence this is the regime of a "frozen-in" turbulence in which time and spatial spectra are equivalent.

The relation of the singularity spectra to the diffusion problem and the limits of the intermediate asymptotic will be discussed in a separate paper.

Fig. 2. The diffusion diagram by Avellaneda and Majda (1992). The region I correspond to the normal diffusion with a mean squared distance changing as \( l \propto t^{1/2}. \) In the region II the diffusion law has the form \( l \propto t^{4-\epsilon}. \) In the region III the diffusion law has the form \( l \propto t^{4-\epsilon}. \)

REFERENCES