SOLAR CYCLE DEPENDENCE OF P-MODE FREQUENCIES AT INTERMEDIATE AND HIGH DEGREES

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ABSTRACT - The dependence of low- and intermediate-degree p-mode frequencies upon varying levels of solar activity is now well established. Since a review of the frequency shifts of the low-degree modes is given in the following article (Palle, 1994), we concentrate here on a review of the status of such frequency shifts for the intermediate-degree modes. We also present the first results of solar cycle-dependent frequency shifts for high-degree p-modes. These results have recently been obtained from observations which we obtained at the 60-Foot Solar Tower of the Mt. Wilson Observatory during the summer of 1990. In particular, the solar cycle-dependent frequency shifts we present here range up to degrees of 600. We also discuss the temporal evolution of different indicators of solar activity during our Mt. Wilson observing run and we illustrate how the inclusion of the high-degree modal frequency shifts may allow us to better discriminate among these different indices in our search for the cause of these solar cycle dependencies.

INTRODUCTION

In 1988 we attempted to search for temporal changes in intermediate-degree p-mode frequencies using observations obtained in 1984 at the Mt. Wilson Observatory (MWO) and at the South Pole by Duvall, Harvey, and Pomerantz (1987) in 1981 (Rhodes et al., 1988). This search failed due to an error in the 1981 frequencies which was described by Jefferies et al. (1990). Subsequently, the first definitive observations of solar cycle-dependent frequency shifts of the intermediate-degree p-modes were presented by Libbrecht and Woodard (1990a and 1990b). The first of these two papers presented the differences of the p-mode frequencies for 5 ≤ ℓ ≤ 63 as observed at the Big Bear Solar Observatory (BBSO) during the summers of 1986 and 1988. Figure 1 is taken from Libbrecht and Woodard (1990a). It shows the frequency dependence of the mean frequency differences between the 1988 and 1986 BBSO datasets averaged over 5 ≤ ℓ ≤ 60. Also shown in this figure is the inverse mode mass for ℓ = 20, where the mode mass is defined as the ratio of the energy in a given p-mode to the square of that mode's surface velocity. Libbrecht and Woodard (1990a) pointed out that the similarity between the frequency differences and the inverse mode masses would be expected for a sound speed perturbation that is
localized close to the solar surface. They also pointed out that their 1986 observations had been obtained during the last solar activity minimum, while their 1988 observations were obtained at a considerably higher level of solar activity.

**Figure 1.** P-mode frequency differences between 1988 and 1986, as a function of frequency, after averaging over $5 \leq l \leq 60$. Also plotted is the inverse mode mass for $l = 20$, $M_{20}^{-1}(v)$. From Libbrecht and Woodard (1990a).

In addition to showing the frequency dependence of the frequency shifts between 1988 and 1986, Libbrecht and Woodard (1990a) also illustrated the degree dependence of these frequency shifts. Figure 2 is a replotting of this degree dependence of these shifts for $5 \leq l \leq 63$. This figure is taken from an analysis of the degree-dependence of frequency shifts by Shibahashi (1991). The points plotted are the observed points of Libbrecht and Woodard (1990a), while the solid line is an expected fit that Shibahashi generated from a theoretical analysis which assumed that the perturbation which caused the frequency shifts was located above a depth of 490 km beneath the photosphere. The dashed straight line which appears to fit the six data points better than this solid line is the least squares fit to those points. Because of the difference between the solid and dashed lines, Shibahashi (1991) concluded that it was only possible to localize the source of the frequency shifts to that portion of the sun that is located above the second helium ionization zone (i.e. within the outermost 14,000 km of the solar interior).

Very shortly after the appearance of the Libbrecht and Woodard (1990a) paper, Libbrecht and Woodard published a second comparison of their 1988 and 1986 BBSO p-mode frequencies (Libbrecht and Woodard, 1990b). In this second paper they
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Figure 2. The frequency change at $\nu = 3$ mHz as a function of degree $l$. The squares are Libbrecht and Woodard's (1990a) observational data. The solid line shows the theoretical expectation of Shibahashi (1991). The dashed straight line is the least-squares fit to the observational data points. From Shibahashi (1991).

extended their comparison to include modes having degrees up to 140. Figure 3 is from this second paper. It shows fits to the observed frequency differences as a function of degree. After comparing Figure 3 with Figure 2 we noticed that the inclusion of p-modes having degrees between 63 and 140 changed the apparent slope of the degree dependence of the frequency shifts. In order to test whether or not the inclusion of these additional modes resulted in a better agreement with Shibahashi's theoretical predictions (i.e. with the solid line in Figure 2), we graphically superimposed those predictions upon the frequency shifts shown in Figure 3. When we did so, we found that Shibahashi's predictions fit the frequency shifts in Figure 3 much more closely than they fit the six observed differences in Figure 2. In particular, we found that Shibahashi's (1991) theoretical predictions fit the points in Figure 3 so well that it now seems reasonable to us to suggest that the source of the frequency shifts is indeed located much closer to the photosphere than the depth of the second helium ionization zone.

Both the Libbrecht and Woodard (1990a) and (1990b) papers came from only two summers of observations at BBSO. Therefore, we were interested in seeing if similar intermediate degree p-mode frequency shifts could also be seen in the datatsets obtained at other observatories. We presented the first result of this study at the July, 1990, COSPAR meeting held in the Hague, Netherlands (Rhodes, Cacciani, and Korzennik, 1991).

In this paper we demonstrated that similar frequency shifts could be seen for all intermediate-degree p-mode datasets which had been obtained between late-1981 and June of 1989. Specifically, we presented two linear regression analyses in that paper in which the different p-mode shifts were regressed upon differences in solar irradiance and against differences in the mean international sunspot numbers. These
two regression analyses are shown here in Figure 4. In the frequency shift-sunspot number difference analysis, Rhodes, Cacciani and Korzennik (1991) found a slope which was nearly identical to the slope obtained from a comparison of low-degree (0 < $\ell$ < 2) p-modes observed between 1977 and 1988 by Elsworth et al. (1990). This similarity in regression slopes suggested to us that a common mechanism causes the shifts in p-mode frequencies for all degrees up to 120.

The next advance in the study of the solar cycle dependence of intermediate-degree p-mode frequencies was made by Woodard et al. (1991). These authors divided the 1986, 1988 and 1989 BBSO observing campaigns into 22 different intervals, each of which was 22.8 days long, and they then computed short-term frequency shifts for the restricted degree range of 18 < $\ell$ < 62 for all 22 intervals. They also computed the average of the absolute value of the magnetic field as measured on daily Kitt Peak full-disk magnetograms from March of 1986 through July of 1990. The comparison that these authors obtained between their short-term frequency shifts and this magnetic activity index is shown here in Figure 5. The 1986 BBSO frequency differences are shown as the triangles, while the 1988 shifts are shown as the squares, and the 1990 shifts are shown as the circles.

Woodard et al. (1991) also compared their 22 frequency shifts with the Sacramento Peak Ca K index and obtained a somewhat weaker correlation with that index than they did with their mean magnetic field index. They concluded that the
mean p-mode frequency shift at any time depends on at most several months of magnetic activity variations. They also concluded that the dominant mechanism of the frequency shifts has to be correlated with the overall level of surface magnetic activity.

Shortly after the publication of this paper Woodard and Libbrecht (1991) returned to the frequency dependence of the p-mode frequency shifts. In this paper they compared both 1989 and 1988 BBSO frequencies with their 1986 BBSO frequencies and pointed out that in both comparisons the size of the frequency shifts increased with increasing frequency only up to a frequency of 3.9 mHz, while for frequencies above the 3.9 mHz the magnitude of the frequency shifts dropped off dramatically. This sharp change in the slope of the frequency dependence of the BBSO frequency shifts is illustrated here in Figure 6.
Figure 5. Frequency shift-magnetic field plot. Seasons are indicated by different symbols: 1986, triangles; 1988, squares; 1989, circles. Single standard deviation errors are shown for frequency shift only. From Woodard et al. (1991).

Woodard and Libbrecht (1991) also presented a power spectrum which they had obtained from a time series of quiet sun Ca K-line filtergrams and they went on to point out that they could not find any evidence in that spectrum for the type of chromospheric resonance which Goldreich et al. (1991) had said should be partially responsible for the observed frequency shifts. They explained the absence of this predicted resonance by noting that the exact frequency and shape of any such resonance would be uncertain due to the inhomogeneous structure of the chromosphere. Hence, they stated that additional observational studies would have to be undertaken before such a chromospheric resonance either could be demonstrated to exist or could be eliminated more conclusively.

A more extensive confirmation that the intermediate-degree p-mode frequencies varied over the entire time interval of 1981 through 1989 was presented by Rhodes et al. (1993). In this paper the differences in all seven different intermediate-degree ($5 < \ell < 120$) p-mode frequency datasets obtained during that time period were analyzed as functions of raw and smoothed sunspot areas as well as irradiance residuals and sunspot numbers. Rhodes et al. (1993) found that the highest correlation coefficient for any of these regression analyses came from the comparison of the frequency shifts with differences in irradiance residuals measured by the ACRIM instrument on the Solar Maximum Mission (Foukal and Lean, 1990).

Shortly after the publication of the Rhodes et al. (1993) paper, Bachmann and Brown (1993) presented a similar study using observations obtained solely with the HAO/NSO Fourier Tachometer between October 1984 and November, 1990. Bachmann and Brown restricted the degree range of
their study to $2 < \ell < 60$ and they divided their data into 18 separate epochs having durations between 18 and 45 days. Figure 7 is taken from the Bachmann and Brown (1993) paper. It compares their average p-mode frequency shifts with three different indices of solar activity: (a) MgII 280 nm core-to-wing ratios, (b) 10.7 cm radio fluxes, and (c) He I 1083 nm equivalent widths. The mean frequency shifts between Fourier Tachometer data and the 1988 BBSO data are shown as the individual crosses, while the solid lines show 27-day filtered activity indices, and the solid circles in (a) are the frequency shifts from the 1986, 1988, and 1989 BBSO observations. Bachmann and Brown made similar comparisons with three additional indicators of solar activity: the Mt. Wilson Magnetic Plage Strength Index (MPSI), the Kitt Peak Magnetic Index, and the solar EUV flux. They found the highest correlations with the Mg II core-to-wing ratio and with the 10.7 cm radio flux. The other four indices all showed significantly lower correlation coefficients, with the MPSI ranking in third place, the HE I 1083 nm equivalent width in fourth place, the EUV flux in fifth place, and the KP magnetic index in last place.

The relatively low correlation coefficient ($r = 0.67$) of the Kitt Peak magnetic index that Bachmann and Brown obtained is quite surprising in view of the high correlation ($r = 0.96$) that Woodard et al. (1991) found with only a slight modification of the same KP magnetic index. Since the degree ranges employed by both groups were almost identical, the difference in correlation cannot be attributed to differences in the degrees of the p-modes employed by each of them. Bachmann and Brown (1993) carried out an additional analysis in which they eliminated all pixels for which the average absolute magnetic flux was less than about 25 Gauss. When they eliminated these low flux points they obtained a modified magnetic index which declined after early 1989, a behavior which they said was more similar to the decreases shown by their frequency shifts for the same period of time. This changed
behavior in the magnetic index led Bachmann and Brown (1993) to conclude that only the magnetic field contained in flux tubes is responsible for the variation in the activity indicators. It is clear that, even for the restricted degree range of \( l < 60 \), more comparisons will have to be made before the different correlation properties of the different indicators can be completely explained.

![Graphs](image)

**Figure 7.** Comparison of average p-mode frequency shifts versus three indices of solar activity from late 1984 through 1990. These are (a) Mg II 2796 nm core-to-wing ratio, (b) 10.7 cm radio flux, (c) He I 1083 nm equivalent width. Plotted frequency shifts show the average frequency differences between a given epoch of FTACH data and the 1988 BBSO data. The solid circles in (a) are frequency shifts from 1986, 1988, and 1989 BBSO data for comparison. From Bachmann and Brown (1993).
FIRST HIGH-DEGREE P-MODE FREQUENCY SHIFTS

Until now it has not been possible to study the possible solar cycle dependence of the high-degree (i.e. \( l > 140 \)) p-mode frequencies because no published sets of high-degree frequencies have come from more than a few days of observations. Even Korzennik's (1990) table of high-degree frequencies were obtained from only twenty consecutive days of data from the MWO 60-Foot Tower. In order to extend the intermediate-degree frequency shift studies of Woodard et al. (1991), Rhodes et al. (1993), and Bachmann and Brown (1993) to the higher-degree p-modes, we have been concentrating on the determination of frequencies from a 142-day long subset of the 60-Foot Tower's 1990 observing campaign.

In particular, we divided this 142-day run into seven 20-day long intervals. We then computed separate power spectra for every even azimuthal order p-mode for all degrees up to and including 600. We then employed a cross-correlation analysis to remove the frequency splitting introduced into the power spectra for each degree by solar rotation. In the process of removing these rotationally-induced frequency splittings we also generated an average (i.e. \( \nu < m = 0 \)) power spectrum for each degree. We then employed a non-linear least squares fitting routine which performed a Lorentzian fit to the individual p-mode peaks in each \( \nu < m = 0 \) spectrum. Hence, at the conclusion of this work, we generated seven sets of p-mode frequencies ranging up to \( l = 600 \). Once we had generated all seven sets of p-mode frequencies in this manner, we simply looked through those seven frequency datasets to find those modes where the fitting procedure converged in more than one of the seven intervals. We then subtracted the frequencies of those modes which appeared in more than one of the different datasets.

Examples of the frequency differences which resulted from this work are shown here in Figures 8 and 10. To generate these Figures the frequencies of p-modes computed from the second of our 20-day intervals were subtracted from the frequencies of the same modes as observed during our third 20-day interval. The frequency dependence of these frequency differences is plotted in here in Figure 8. The fact that the majority of the frequency differences shown in Figure 8 are less than zero means that most of the modal frequencies were higher in the second interval than they were in the third interval. This fact is also consistent with the average level of solar activity being higher during our second 20-day interval than it was during our third interval, as can be seen here in Figure 9 where we show the average Ottawa 10.7 cm flux (corrected to 1 AU) (Tapping, 1987) as a function of interval number (i.e. time) during our 1990 MWO campaign. Furthermore, the absolute magnitude of these negative frequency differences can be seen to increase with frequency up to a frequency of at least 3800 \( \mu \text{Hz} \). This behavior is similar to that exhibited in Figure 6 for the 1988-1986 and 1989-1986 BBSO frequencies differences, except for the sign change. However, in Figure 8 there are also some positive frequency differences at most frequencies. These positive frequency differences resulted from the inclusion of the higher-degree modes in our analysis of our 1990 MWO observations. When we re-plotted the frequency dependence of the frequency differences shown in Figure 8, but only included modes having \( l \leq 140 \), as did Libbrecht and Woodard (1990b), we found that a much smaller portion of the differences were positive. Clearly, the incorporation of the higher-degree frequencies has complicated the relatively simple
picture given in Figure 1 for the intermediate-degree modes alone.

The added complexity introduced by including the higher-degree frequencies is even more apparent in Figure 10, where we have illustrated the degree dependence of our 1990 frequency shifts. Here we see that the absolute value of these shifts increased with increasing degree as in Figures 2 and 3 but only for modes having \( \ell \leq 200 \). For \( 200 \leq \ell \leq 350 \) the absolute value of the frequency shifts decreased until they reached zero at roughly \( \ell = 350 \). Even more surprisingly, for \( 350 \leq \ell \leq 600 \) the frequency shifts became positive, thus implying that for these modes the frequencies were higher in our third 20-day interval than they were in the second interval, in direct contrast to the situation described above for the intermediate-degree frequencies.

These changes in the behavior of the frequency shifts for degrees above \( \ell = 200 \) imply that the p-modes for these higher degrees are affected in a different manner by the near surface solar magnetic fields than are the intermediate- and low-degree p-modes having \( \ell < 160 \). We are now in the process of evaluating similar frequency shifts for all of the other five 20-day subsets of our 1990 MWO observing run. A preliminary inspection of these additional frequency differences suggests that global solar activity indicators alone will not be able to explain the variation of both the intermediate- and high-degree p-mode frequencies.
Figure 9. Average 10.7 cm flux (corrected to 1 AU) plotted as a function of the number of the 20-day interval over which flux was averaged. The flux was higher in the second interval than it was in the third interval. All six other indicators of solar activity which we studied showed similar behavior during these two intervals.

Figure 10. Degree dependence of frequency differences shown in Figure 8.
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