RING DIAGRAM ANALYSIS OF MT. WILSON DATA: VELOCITY FIELDS WITHIN THE SOLAR CONVECTION ZONE

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ABSTRACT A ring diagram analysis has been applied to a set of 1024 × 1024-pixel Dopplergrams obtained at Mt. Wilson. A model fit to the 3-dimensional power spectrum at 9 different positions on the solar disk provides an estimate of the weighted average over depth of the two horizontal components of the velocity flow beneath the solar surface. The depth dependence of the flows is obtained via inversion for the upper 60 Mm of the convection zone. The inferred velocity field appears to execute a spiral as a function of depth with an organized character at different locations in the solar surface. The spatial coherence of the flows disappears below about 30 Mm.

INTRODUCTION
Detailed information about the horizontal velocity field \( U \) as a function of depth \( r \) and horizontal position \( x, y \) within the Sun is of great interest for solar physics. The spatial structure and temporal evolution of flows in the solar convection zone beneath the photosphere play an important role in solar convection zone physics and the dynamo problem. Here we briefly present the results of an analysis using solar oscillation ring diagrams to infer \( U(x, y, r) \).

Solar oscillations have been studied using the well-known 2-dimensional \( k - \omega \) diagrams in which discrete ridges of power are present. Here, \( \omega \) is the temporal frequency and \( k \) is the total horizontal wave number. For high-degree modes
(\ell \geq 190), the ridges are observed to be continuous structures rather than a series of individual peaks, indicating that a local plane-wave rather than a global spherical harmonic decomposition can be used. We then obtain information not only for \( k \) but also for its two horizontal components, \( k_x \) and \( k_y \), producing a 3-dimensional distribution of power corresponding to ridges oriented in any radial direction in a \( k_x - k_y \) plane. The final surface has a shape reminiscent of the bell of a trumpet, and a horizontal slice across these shapes at a given \( \omega \) displays a set of nested rings (Hill 1988, 1994). From these ring diagrams we can infer \( U_x \) and \( U_y \), the two horizontal components of \( \mathbf{U} \), by measuring the position of the rings in \( k_x \) and \( k_y \) and applying an inverse method.

**DATA AND ANALYSIS**

The data comprise a time series of \( 1024 \times 1024 \) full-disk Doppler images in the Na D line obtained with the 60-Foot Tower and the Magneto-Optical Filter at Mt. Wilson Observatory during 3–5 July, 1988. A Dopplergram was obtained every 60 s with a nominal spatial sampling of 2.2\arcsec. A description of the observing system can be found in Rhodes, Cacciani, & Korzennik (1990).

The 3-dimensional power spectra of these images is constructed essentially as described in Hill (1988). The data are remapped onto an equally-spaced longitude-latitude grid, by a cubic convolution interpolation of the original images. The sampling interval in this heliographic map is \( \Delta \theta = 0.11^\circ \) in both latitude and longitude. A \( 3 \times 3 \) mosaic of subrasters centered on the disk is constructed. Each subraster subtends an area \( 14.85^\circ \) square, contains \( 135 \times 135 \) pixels, and is tracked to remove solar rotation. The subrasters are temporally filtered by subtracting a simple 21-point running mean. Finally, the data are Fourier transformed in three dimensions: longitude, latitude, and time, and the power spectrum as a function of \( k_x \), \( k_y \), and \( \omega \) is constructed. The resulting \( k \) resolution is 0.035 Mm\(^{-1}\) and the \( \omega \) resolution is \( 3.03 \times 10^{-5} \) sec\(^{-1}\), or 4.82 \( \mu \)Hz.

We use a modified version of the maximum likelihood method of Anderson et al. (1990) to fit the ring structure and estimate \( U_x \) and \( U_y \). As in that paper, the power spectrum of a mode as a function of \( \omega \) is modeled by a Lorentzian, but for the rings the profile must also be a function of \( k_x \) and \( k_y \). The dependence on \( k_x \) and \( k_y \) is given by the effective Doppler shift of the unperturbed frequency:

\[
\Delta \omega = k \cdot \mathbf{U} = k_x U_x + k_y U_y.
\]

The resulting shifted frequency will be the position of the maximum of the Lorentzian. The model for the power in one ring at the coordinates \( \omega \), \( k_x \), and \( k_y \) is:

\[
P(\omega, k_x, k_y) = \frac{A^2}{[\omega - (ck^{1/2} + U_x k_x + U_y k_y)]^2 + \Gamma^2} + b_1 k^{-3} + b_2 k^{-4}.
\]

We approximate the unperturbed frequency by the quantity \( ck^{1/2} \), describing the dispersion relation. The term \( U_x k_x + U_y k_y \) is the Doppler shift of the frequency. The amplitude and half-width at half-maximum of the Lorentzian are given by \( A \) and \( \Gamma \), respectively. The background is parameterized as a function of \( k \) by \( b_1 \) and \( b_2 \).
The fitted values of $U_x$ and $U_y$ really represent weighted averages over depth of $U$. An inversion technique must be applied to infer the actual velocity field. Here we used a least-squares with second derivative smoothing method. Approximately 1700 modes, with $0 \leq n \leq 7$, and $175 \leq \ell \leq 800$ were included for each subraster. A total of 51 dissections covering the depth range $0.9 \leq r/R_\odot \leq 1.0$ and 10 values of the smoothness parameter $\lambda$ were chosen, with $0.1 \leq \lambda \leq 5.0$. In this paper we consider the smoothest solutions with the lowest formal errors, but the poorest depth resolution.

RESULTS

The inversion provides inferred curves of $U_x(r)$ and $U_y(r)$ for each subraster. The limited space in these proceedings precludes detailed display of the results; the interested reader should consult the more complete report (Patrón et al. 1994). Here, we note that there are similarities between the velocity curves at different spatial positions. For example, $U_x$ typically reaches a value on the order of 200 ms$^{-1}$ at depths of 30 to 300 km, drops to about $-200$ ms$^{-1}$ at depth of around 800 km, and virtually vanishes just above the hydrogen ionization zone at a depth of about 1.5 Mm. The north-south component $U_y$ has a qualitatively similar behavior. Vector plots of the velocity show that the vectors in most of the substrasters are pointing westward near the surface and eastward at the deeper depth, creating two strong shear layers in opposite directions. These shear layers may be the source of instabilities that could cause granulation and might also contribute to the excitation of the $p$ modes. This source has been estimated to be near the surface at depths of 60 (Duvall et al. 1993) to 150 km (Kumar 1993).

Another shear layer is apparently oriented westward at depths of about 16 Mm, near the He$^{++}$ zone. There is thus a close association in depth between two east-west shear layers with the H$^+$ and He$^{++}$ ionization zones in the outer 20 Mm. This is in agreement with anelastic compressible convective models, which predict that giant cell vertical flows would be deflected into strong horizontal flows near the ionization zones (Latour, Toomre & Zahn 1983). Indeed, the vector velocity plots suggest that there may be a convective roll oriented east-west at a latitude of about $+10^\circ$, and at depths of 5–8 Mm.

The most striking feature of the flows is the spatially coherent organized rotation of the velocity vectors in depth through about 360$^\circ$. The vector velocity field thus appears to perform a spiral in depth, as can be seen in the pseudo-perspective view of the vectors in Figure 1. Below about 30 Mm, the spatial coherence nature of the flow field disappears, the flows become chaotic, and there are no apparent changes in the orientation of the vectors with depth. The inferred spiral character of the flow could be related to the observed helicity of the solar magnetic field. A spiral flow field has also been seen in numerical convection models (Brumme, Hurlbut & Toomre 1993) and may be a solar analog of the so-called Ekman layer seen in the Earth’s oceans.

These results are physically plausible, although the inferred amplitudes are high. The reliability of these results needs to be thoroughly tested, requiring more measurements, more efficient data analysis, and a realistic set of simulations.
Fig. 1. Pseudo-perspective view of the velocity field. Lines parallel to the lat-long plane are proportional to the velocity vectors. The shading illustrates the orientation of the flows. Numbers in the lat-long plane are degrees, and the depth units are in Mm.

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REFERENCES

Hill, F., 1994, these proceedings.