FREQUENCIES OF LOW-DEGREE MODES AND THE STRUCTURE OF THE SOLAR CORE

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ABSTRACT  An attempt is reported to reduce uncertainties of p-mode parameter estimation by incorporating constraints imposed by smooth behaviour of some of the parameters over a group of lines. We give some examples of the procedure for whole-disk measurements by the IPHIR space experiment. It is shown that the additional constraints do not result in significant changes in the frequency estimates. Inversions of the IPHIR datasets are compared with corresponding inversions of data from the Birmingham Solar Oscillation Network (BISON). The difference between the IPHIR and BISON inversions is significant, preventing any definite conclusion about the deviation of the solar core structure from that of the model.

INTRODUCTION

Owing to the relatively weak sensitivity of p-mode frequencies to the core structure, relatively small systematic errors in the frequencies can result in significant errors in the structure inversions. The main difficulty in estimating the powerspectrum parameters, such as the frequency, is caused by corruption of the spectral line profiles by the stochastic forcing of the oscillations which makes the fitting of individual lines by a smooth model (e.g. Lorentzian) statistically poor. But, estimation of the parameters using several lines, for which the parameters are similar, reduces the number of the estimated parameters and improves the statistics. One of our main interests in this paper is the stability of the inferred solar core structure, estimated by inverting IPHIR frequency datasets which have been obtained by imposing various constraints on the power spectrum models. Such stability is important evidence of reliability of the current
inversions. Also in the paper, we present results of inversion of the BISON dataset, with corrected error estimates.

FREQUENCY DETERMINATION UNDER ADDITIONAL CONSTRAINTS

In Table I we list characteristics of six different datasets. Two of them, IPHIR0 and BISON, are the original data published by Toutain & Fröhlich (1992) and by Elsworth et al. (1991). The other four sets, IPHIR1–IPHIR4, are the new frequency data obtained under various constraints imposed upon the model of the power spectrum from the IPHIR ‘green’ channel. These IPHIR frequencies have been determined by performing simultaneous maximum-likelihood fits of the oscillation models to all the significant peaks in the power spectra. The BISON dataset comprises mean frequencies over the years 1981–88, obtained by averaging bimonthly spectra. The \( l = 1 \) and \( l = 2 \) multiplets were fitted by only single Lorentzians. This pinpoints, perhaps, the most significant difference between the BISON and IPHIR datasets. Four new frequency datasets, IPHIR1–IPHIR4, have been obtained from the same power spectrum as was IPHIR0. The additional constraints applied to the data were chosen to reflect the expected smooth behaviour in the observed frequency interval of the linewidths, the asymmetry parameters, the values of rotational splitting and the background noise level. In the four examples considered in this paper, the constraints are taken in their simplest forms, with a small number of parameters. The functional forms of the constraints reflect observationally determined trends in the solar oscillation data, rather than theoretical opinions. The constrained parameters are assumed to be either constants or smooth functions of frequency (see Table 1). Of course, these examples do not cover all possible combinations. Thus, these examples just demonstrate stability of the procedure. They also illustrate possible uncertainties in the inversions for the solar structure. In Fig. 1 we show the frequency differences between the IPHIR0 and BISON datasets. Only the frequencies between 2.5 and 3.0 mHz, the range adopted in our inversion analysis (cf Gough & Kosovichev, 1993), are plotted. The most prominent

<table>
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<th>Dataset</th>
<th>Linewidths</th>
<th>asymmetry</th>
<th>splitting</th>
<th>noise</th>
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</table>
Fig. 1 Frequency differences between the IPHIR0 (Toutain & Fröhlich, 1992) and the BISON (Elsworth et al. 1991) data sets, plotted against cyclic frequency

difference between these datasets is at low frequencies. In principle, it could be due to variations in the structure of the solar core (cf Gough & Kosovichev, 1993) or could also have been produced by some unknown systematic artefact of the data analysis procedure. Frequency differences between the four new datasets, IPHIR1–IPHIR4, and the old IPHIR0, are within two standard deviations.

INVERSION RESULTS

For the data inversion we use the linearized integral equations (Gough and Kosovichev (1993)) relating the frequency difference between the eigenfrequencies of a solar model (model 1 of Christensen-Dalsgaard et al. (1993)) and the corresponding frequencies of the Sun, to deviations of $\delta \ln u$ and $\delta Y$ of solar structure, where $u \equiv p/\rho$ is the ratio of the pressure to the density and $Y$ is the helium abundance in the convection zone. The inverted data are combinations of the 16 frequencies of the low-degree modes, taken from each of the datasets (Table 1), and 598 frequencies of intermediate-degree modes ($l = 4 - 140$, $\nu = 1.5 - 3.0$ mHz), observed at BBSO in 1988 (Libbrecht et al. 1990), near the time of the IPHIR space experiment. The relative deviations of the parameter $u$ from the reference model, obtained by inverting BISON data with both the old and the new error estimates, are shown in Fig. 2. Included in the right panel is a similar inversion of the original IPHIR0 data. The results of the BISON and IPHIR data inversions are clearly inconsistent near the centre of the Sun. It is interesting to note that when the BISON data are taken with the new smaller errors, thus providing better spatial resolution, there is a hint of increasing $\delta \ln u$ towards the centre. However, the discrepancy between the BISON and IPHIR data inversions remains significant and unexplained. The results of inversions of the IPHIR1–IPHIR4 datasets are all generally consistent with the original IPHIR0 inversion (see Fig. 2). The parameter $u$, which is approximately proportional to the ratio $T/\mu$, decreases towards the centre because of an increased abundance of helium produced in the nuclear reactions, which increases the molecular weight $\mu$. However, the inversions show more gradual decrease than in the model, thus indicating that helium could be less abundant at the solar centre than in the model. This might be caused by redistribution of the core material due to processes that are not included in the standard evolutionary model.
LOW-DEGREE MODE FREQUENCIES

Fig. 2  Optimally localized averages of the difference \( \delta \ln u \) between the Sun and the reference solar model, inferred from a combination of BBSO intermediate-degree mode frequencies and the low-degree mode frequencies from: a) BISON with the originally published error estimates, b) BISON with the corrected errors, and c) IPHIRE0.

SUMMARY AND CONCLUSION

We have considered some examples in which one or several parameters of the power spectrum obtained from the IPHIRE space experiment are approximated by smooth functions of frequency for all the lines in the frequency range of 2.5 – 3.0 mHz. The oscillation frequencies so determined are generally in agreement with those obtained previously by Toutain & Fröhlich (1992). We have performed inversions of datasets obtained by combining the low-degree data with intermediate-degree mode frequencies obtained by Libbrecht et al. (1990), and have found a smaller depression of \( u \) in the central region of the Sun than in the reference model. This would suggest that the chemical composition of the core might have been modified by a process of material redistribution. Our experiments provide evidence for a certain robustness of the structure determination from the IPHIRE0–IPHIRE4 data. However, a disagreement remains with the corresponding inversions of BISON frequencies, and is unresolved.

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REFERENCES