APPLICATIONS OF MASSIVELY-PARALLEL COMPUTING IN SOLAR MODELING

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ABSTRACT The reduction of helioseismological data requires models of
the Sun which are as accurate as possible in both their physical
and numerical aspects. The solar model codes currently in use in the
astronomical community do not have a built-in accuracy check, and they do
not allow the convection zone boundaries to be located exactly within the
model. Hence, their numerical precision is inherently limited. The objective
of this paper is to point out how these drawbacks can be overcome by
resorting to a newly developed numerical technique which is based on
a shooting approach. The features of this code are described, and it is
shown that it is well suited for an implementation on the new generation
of massively-parallel supercomputers. Also, some additional improvements
in the model physics are briefly discussed.

INTRODUCTION
Within the next few years two large projects will allow uninterrupted observa-
tions of solar oscillations:
- The GONG (Global Oscillations Network Group) project, to be established
  in 1994;
- The SOHO (Solar and Heliospheric Observatory) satellite, to be launched
  in 1995.
Both experiments will provide extremely precise frequencies for a large number of modes of solar oscillations. Hence, a very detailed study of many aspects of the solar interior will become feasible.

The comparison of observed solar oscillation frequencies with theoretical predictions (the so-called "forward problem") and the generation of theoretical kernels for the inversion of helioseismic frequencies and frequency splittings (the "inverse problem") require models of the Sun which are as accurate as possible in both their physical and numerical aspects. During the past few years at the Mathematical Institute of the Technical University in Munich a new numerical method has been developed for solving the system of partial differential equations which describe the evolution of a spherically symmetric star (Reiter et al. 1994). Models computed with this new code (referred to as the RBP code) have been compared with models of Christensen-Dalsgaard in the framework of the GONG solar model project (Christensen-Dalsgaard and Reiter 1994). To date the RBP code has been run on traditional supercomputers (Cray Y/MPs in particular); however, several features of this new method suggest that it could be sped up by substantial factors on the new generation of massively-parallel supercomputers such as the Intel Touchstone Delta and Paragon supercomputers and the Cray T3D Massively-Parallel Processor.

ASPECTS OF NUMERICAL CODE IN SOLAR MODELING

Problems with Standard Solar Model Codes
As is well known, modeling the Sun in the standard framework gives rise to a nonlinear system of coupled partial differential equations with one spatial and one temporal independent variable. The canonic approach for solving these equations is to separate them into a spatial (stellar structure) and a temporal (chemical evolution) problem. Usually, the spatial problem is solved either by a finite difference technique (e.g., Christensen-Dalsgaard 1982) or by collocation (Morel et al. 1990). However, both approaches suffer from the following disadvantages:

- The accuracy of the spatial solution can be only estimated by varying the number of mesh points used since the codes do not have a built-in accuracy check.
- It is true that increasing the number of mesh points diminishes the spatial discretization error but, at the same time, the round-off errors are increasing. Beyond a certain number of mesh points depending on details of the discretization, round-off errors become the dominant source of error. Hence, the precision of the spatial solution is limited.
- Neither finite differencing nor collocation allows the convection zone boundaries to be located exactly within the model.

To overcome these drawbacks we recommend the use of the RBP code. Also, the high intrinsic numerical accuracy of this new code offers the important possibility of improving the tracing of differences between theory and observation back to uncertainties in the physics which was adopted for the model.

Features of Reiter, Bulirsch and Pfeiferer (RBP) Code
As usual, in the RBP code the stellar-evolution equations are separated into a spatial (stellar structure) and a temporal (chemical evolution) problem. In
numerical analysis such approach is known as the method of lines. By the
discretization of time, at each discrete time step, a boundary-value problem
(BVP) for the spatial structure and an initial-value problem (IVP) for the
changing chemical abundances must be solved. Contrary to the codes currently
in use in the astronomical community the RBP code solves the BVP cast in the
Eulerian formalism, i.e. the radius \( r \) is used as the independent spatial variable.
Numerically, this seems to be superior, for the fractional mass \( M_r \) becomes
almost constant near the surface of the stellar model while at the same time the
pressure and temperature gradients become very large.

The time derivatives are discretized implicitly to second order accuracy.
By reformulating the equations of stellar structure and stellar atmosphere as
a multipoint BVP the rather clumsy iterative matching procedure of stellar
atmosphere and interior is avoided. The multipoint BVP is solved by the multiple
shooting method. This not only ensures a high accuracy of the stellar models
calculated at each time step but also allows the core and outer convection zone
boundaries to be located exactly within the model. The accuracy of the solution
of the spatial problem is controlled by the prescribed precision of both the
numerical integrator and the multiple shooting method.

The IVP for the chemical composition is solved, at each time step, by a
sophisticated iterative procedure ensuring a high precision with respect to the
integration in time. Hence, the accuracy of the solution of the temporal problem
is mainly determined by the number of time steps used, and is routinely checked
by testing how good the first law of thermodynamics is fulfilled.

For more details of the RBP code we refer to Reiter et al. (1994).

Parallelization of the RBP Code
The RBP code is a perfect example of a parallel algorithm. Various levels of
parallelism can be identified:

- The spatial boundary-value problem (BVP) is solved by the multiple-
  shooting method which proceeds by iteratively solving initial-value prob-
  lems (IVPs). These IVPs are independent from one another, and hence can
  be solved in parallel.

- The IVPs are numerically solved by an extrapolation technique which is
  parallelizable.

- Before the spatial BVP can be solved at each time step, the chemical
  composition \( X_i \) must be known as a function of the spatial independent
  variable. This at first requires, at given mesh points, the numerical
  integration of the IVP describing the temporal evolution of the \( X_i \)’s, and
  then the spatial interpolation of the \( X_i \)’s over the grid. Both the integrations
  at the grid points as well as the interpolations of the different \( X_i \)’s can be
done in parallel.

- Each time step is completed by interpolating the dependent variables for
  mass, luminosity, pressure and temperature. Much of the work necessary
  for constructing these interpolants can be parallelized.

We intend to run the parallelized RBP code on the Paragon and the Intel
Touchstone Delta supercomputers located at Caltech and the new Cray T3D
located at JPL.
ASPECTS OF MODEL PHYSICS

In modeling the physics of the solar models to be constructed we will particularly focus our attention on the following issues:

- Implementation of a detailed nuclear network including the species $^1\text{H}$, $^3\text{He}$, $^4\text{He}$, $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{N}$, $^{15}\text{N}$, $^{16}\text{O}$, $^{17}\text{O}$;
- Use of the MHD equation of state (e.g., Mihalas et al. 1988);
- Use of the most recent opacities;
- Consideration of microscopic diffusion of different types of elements;
- Improved treatment of the boundaries of the convection zones beyond the canonic Schwarzschild criterion;
- Incorporation of a sophisticated model atmosphere;
- Use of a more accurate description of convection, perhaps from a three-dimensional numerical convection simulation code;
- Consideration of mass loss.

To further increase the accuracy of the computed solar models the interpolation in tables for the equation of state and for the opacity should be avoided. We therefore intend to evaluate both on-line in the code. This venture will certainly boost the computational time for a solar model into the region of hundreds of hours even on the fastest computers, but nevertheless will not really be a severe obstacle. We hope that parallelizing the RBP code may somewhat compensate for such drastic increase of the computational time.

REFERENCES

Christensen-Dalsgaard, J., Reiter, J. 1994, these Proceedings.