SEISMIC EFFECTS OF NORTH–SOUTH ASYMMETRY OF SUN’S ROTATION

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ABSTRACT Observations of rotation of large-scale magnetic structures on the solar surface show that the angular velocities of the northern and southern hemispheres may differ by about 5%. The asymmetry varies with time and correlates with the solar cycle (Antonucci, Hoeksema & Scherrer, 1990). The antisymmetric component appears only in a second-order correction to the oscillation frequencies, and it has to be separated from the second-order terms of symmetric rotation and from contributions of non-spherical structure perturbations. We estimate the effect of the north-south asymmetry and discuss an approach to its measurement from the even component of the frequency splitting, using asymptotic properties of seismic averages of the angular velocity, and from perturbations of oscillation eigenfunctions. The measurements are likely to be restricted to p modes of high degree \((l \gtrsim 300)\), implying that the asymmetry could be studied perhaps only in the subsurface layers.

THE EFFECTS OF THE ASYMMETRY

A seismic effect of asymmetry of rotation appears in the second-order corrections to the rotational splitting, which is even in angular degree \(m\). However, the even component contains also contributions from the symmetric component of rotation, the internal magnetic field and possibly from other departures of the Sun’s structure from spherical symmetry.

According to the theory for calculating of the second-order effects of stellar rotation, which has been developed recently by Gough & Thompson (1990) and by Dziembowski & Goode (1992), the even-\(m\) frequency splitting \(\Delta \omega_2\) can be divided into several components, the most significant being due to centrifugal force which causes distortion of the Sun’s equilibrium structure \(\Delta \omega_{\text{distor}}\) and also affects the oscillations directly \(\Delta \omega_{\text{inert}}\),

\[
\Delta \omega_2 = \Delta \omega_{\text{distor}} + \Delta \omega_{\text{inert}}. \tag{1}
\]

The north-south rotational asymmetry occurs in the second, inertia, term which
can be approximately estimated in terms of seismic averages of the angular velocity \( \Omega \):

\[
\Delta \omega_{\text{inert}} \approx \frac{m^2}{2 \omega_0} \left[ (\bar{\Omega})^2 - \langle \Omega^2 \rangle \right],
\]

where \( \omega_0 \) is the multiplet frequency, the bar means an appropriate radial average, and the angular brackets denote the angular integral, e.g.

\[
\bar{\Omega} = \int_0^R \Omega(r, \theta) K_{nl}(r) \, dr, \quad \langle \Omega \rangle = \int_0^\pi \Omega(r, \theta) (P_l^m)^2 \sin \theta \, d\theta;
\]

\( P_l^m(\theta) \) is a normalized Legendre function. Dividing \( \Omega \) into symmetrical and antisymmetrical parts, \( \Omega = \Omega_{\text{sym}} + \Omega_{\text{antisym}} \), and taking into account that \( \langle \Omega_{\text{antisym}} \rangle = 0 \), we obtain

\[
\Delta \omega_{\text{inert}} \equiv \Delta \omega_{\text{inert-sym}} + \Delta \omega_{\text{inert-antisym}} \approx \frac{m^2}{2 \omega_0} \left[ \langle \Omega_{\text{sym}} \rangle^2 - \langle \Omega_{\text{sym}}^2 \rangle \right] - \frac{m^2}{2 \omega_0} \langle \Omega_{\text{antisym}}^2 \rangle.
\]

Therefore, the asymmetric contribution of rotation appears in the frequency splitting as the seismic average of the square of \( \Omega_{\text{antisym}} \). If we assume that the seismic average is of the order of the observed antisymmetric surface angular velocity, which is about 5% of the equatorial angular velocity, then for \( p \) modes in the 5-minute range

\[
\Delta \omega_{\text{inert-antisym}} \approx 0.9 \left( \frac{m}{100} \right)^2 \text{ nHz}.
\]

Thus the effect is probably measurable only for modes of high degree \( (l > 100) \), the lower turning points of which are at \( r > 0.85 R_\oplus \), meaning that it would be impossible to determine the asymmetric rotation in the deep interior using \( p \)-mode frequencies.

**MEASURING THE ANTISYMMETRIC COMPONENT OF ROTATION**

The antisymmetric component of rotation has to be separated from the effects of distortion and from the contribution from \( \Omega_{\text{sym}} \). The distortion effect, which is determined mainly by the spherical component of rotation, can be calculated by using the inversion of the odd-\( m \) splitting component. The two terms in (4) can also be separated by applying a standard technique of expanding the data in Legendre polynomials \( P \left( \frac{m}{L} \right) \) and considering the functional dependences of the expansion coefficients on mode parameters; whilst the coefficients for \( \Delta \omega_{\text{distor}} \) are almost independent of the angular degree \( l \), the coefficients for \( \Delta \omega_{\text{inert}} \) grow as \( l^2 \) (Dziembowski & Goode, 1992). The symmetric-rotation contribution to this term can also be calculated from theory once \( \Omega_{\text{sym}}(r, \theta) \) is determined.

The direct contribution of the centrifugal force to \( \Delta \omega_{\text{inert}} \) from \( \Omega_{\text{sym}} \) is essentially cancelled by the quadratic correction to the first-order seismic average (Eq. 4). In the example of the solar rotation law considered by Dziembowski & Goode (1992), the first term in (4) can be approximated as

\[
\Delta \omega_{\text{inert-sym}} \approx 0.07 \left( \frac{m}{100} \right)^2 \text{ nHz},
\]
and is significantly less the antisymmetric contribution $\Delta \omega_{\text{inert,antisym}}$ (5). The effect of the cancellation in $\Delta \omega_{\text{inert,sym}}$ depends primarily on the radial gradient of the spherical component of the angular velocity in the subphotospheric layers. Without the gradient, the cancellation would be exact.

If the Sun’s rotation is considered in the parametric form

$$\Omega(r, \theta) = \sum \Omega_k(r) P_k(\cos \theta),$$

in which the symmetric and antisymmetric parts are represented by the even and odd terms respectively, then the seismic average is

$$\overline{\Omega} = \sum_{k \text{ even}} \Omega_k \lambda_k P_k \left( \frac{m}{L} \right),$$

where $L = \sqrt{l(l+1)}$, $\lambda_k = \frac{(-1)^{k/2} \Gamma \left( \frac{k}{2} + 1 \right) \Gamma \left( \frac{1}{2} \right)}{\Gamma \left( \frac{k+1}{2} \right)}$, $\Gamma$ being the gamma-function, and $\overline{\Omega_k}$ is the radial seismic average (2) of $\Omega_k(r)$. Substituting (7) and (8) into (4) yields

$$\Delta \omega_{\text{inert,sym}} = m^2 \sum_{j \text{ even}} c_j P_j \left( \frac{m}{L} \right),$$

where

$$c_j = \frac{1}{2 \omega_0} \sum_{i,k \text{ even}} G_{ijk} \left( \lambda_i \lambda_k \overline{\Omega_i} \overline{\Omega_k} - \lambda_j \overline{\Omega_j} \overline{\Omega_k} \right),$$

and $G_{ijk} = \int_0^\pi P_j P_i P_k \sin \theta d\theta$ is the Gaunt integral. Assuming $\Omega(r)$ to vary roughly linearly with acoustical depth $\tau$, the first-order coefficient $c_0$ can be estimated as

$$c_0 = \frac{1}{2 \omega_0} \left( \overline{\Omega_0^2} - \overline{\Omega_0}^2 \right) \approx - (\Omega_0')^2 \frac{\tau_e^2}{24 \omega_0} \approx - \frac{\Omega_0(R) - \Omega_0(r_e)^2}{24 \omega_0},$$

where $\Omega_0'$ is the first derivative of $\Omega_0$ with respect to $\tau$, and $r_e$ is the radius of the lower turning point, and $\tau_e$ is the acoustical depth of this point. Thus the contribution of the symmetric component of $\Omega$ is proportional to the squared overall variation of $\Omega_0$ in the region of mode propagation beneath the photosphere. Similarly,

$$\Delta \omega_{\text{inert,antisym}} = m^2 \sum_{j \text{ even}} d_j P_j \left( \frac{m}{L} \right),$$

where

$$d_j = - \frac{1}{2 \omega_0} \sum_{i,k \text{ odd}} G_{ijk} \lambda_j \overline{\Omega_i} \overline{\Omega_k},$$

and the first-order coefficient is

$$d_0 = - \frac{\Omega_0^2}{3 \omega_0}.$$

Figure 1 shows typical values of $\Delta \omega_{\text{inert,antisym}}$ as a function of the angular degree $l$ in comparison with $\Delta \omega_{\text{distor}}$ and $\Delta \omega_{\text{inert,sym}}$ calculated by Dziembowski & Goode (1992) for a solar rotation law inferred from the observed data.
CONCLUSION AND DISCUSSION

In principle, the north-south asymmetry of the Sun’s rotation could be measured seismologically in the subphotospheric layers from the even-$m$ component of rotational frequency splitting of high-degree $p$ modes.

Another possibility for measuring the asymmetry is to observe distortion of the mode eigenfunctions. An effect of latitudinal shear associated with a diminishing angular velocity towards the poles is to expand the eigenfunctions of prograde modes away from the equator and to compress the retrograde modes. The distortion can be computed in terms of the parameters $\Omega_k$ as a simple coordinate stretching. By projecting a Doppler signal of the Sun onto those eigenfunctions, one might try to seek those coefficients $\Omega_k$ that minimize the sidelobes in the temporal power spectrum. In that way one would be extracting the nearest approximation to a pure mode. It should be realized that the distortion is frequency dependent, so even for a given $l$ and $m$ it would be necessary, in principle, to consider a different distortion for each peak in the power spectrum. In practice, some kind of averaging would need to be carried out.

REFERENCES