CONSTRAINTS ON OBLIQUE ROTATION OF THE SOLAR CORE FROM LOW-DEGREE MODES

D.O. GOUGH
Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, U.K.
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, U.K.

A.G. KOSEVICH
Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, U.K.
Crimean Astrophysical Observatory, Nauchny, 334413 Crimea, Ukraine

T. TOUTAIN
Space Science Department, ESA, ESTEC, 2200 AG Noordwijk, The Netherlands

ABSTRACT We study seismic consequences of the suggestion by Bai and Sturrock (1993) that the core of the Sun rotates about an axis that is inclined to the axis of rotation of the envelope. We demonstrate that the IPHIR data (Toutain & Fröhlich, 1992) impose constraints on the rotation rate and the size of the obliquely rotating core which according to the data is unlikely to be larger than 20% of the solar radius.

INTRODUCTION

Rotating stars are normally presumed to rotate about a unique axis. For instance, this assumption is usually used in helioseismic determinations of the Sun's internal rotation. However, it is not known whether or not this assumption is correct. Both observational and theoretical arguments suggest that the rotation of stellar interiors could be very different from the rotation of envelopes. Indeed, Bai & Sturrock (1993) have recently suggested from their analysis of the longitudinal distribution of major solar flares that the core of the Sun rotates about a fixed axis which is inclined by 40° to the axis of rotation of the envelope.

A variation with radius of the direction of the rotation axis would modify the form of rotational splitting of oscillation eigenfrequencies. But so too does a variation with depth and latitude in the magnitude of the angular velocity. One type of variation can mimic the other, and so frequency information alone cannot differentiate between them. What is different, however, is the structure of the eigenfunctions. Therefore, in principle, one might hope to untangle the two phenomena using information about both the frequencies and the amplitudes of the oscillations. The problem has been discussed by Goode & Thompson (1992) and by Gough & Kosovichev (1993) who used observations of intermediate-
degree acoustic modes, carried out at BBSO by Woodard & Libbrecht (1993). These data have led to a conclusion that the obliquely rotating core, if it exists, is unlikely to be bigger than $0.5R_\odot$. In this paper we extend the helioseismic analysis to low-degree modes which probe the central regions of the Sun.

THE TRIANGLE RULE

Rotation lifts the degeneracy and splits the eigenfrequencies of the modes of the same values of angular degree $l$ and radial order $n$ but different azimuthal degree $m$, thus producing rotational multiplets in oscillation power spectra. For a star rotating about a unique axis the perturbation is axisymmetrical, and therefore the eigenfunctions are essentially the same as the form usually adopted without rotation if the polar axis of the coordinate frame coincides with the rotation axis. However, if the direction of the rotation axis varies with radius (Fig. 1a), then there is no immediately obvious choice of the coordinate frame of the oscillations. In this case, according to degenerate perturbation theory, the eigenfunctions are described by a superposition of the usual spherical-harmonic eigenfunctions, and rotational frequency splitting is determined by a secular equation (e.g. Gough et al., 1994).

If both the core and the envelope rotate spherically, that is, if the angular velocity is separately a function of radius alone both in the core and in the envelope, then the seismic integrals of the angular velocity in the core $\Omega_{c,m}$ and in the envelope $\Omega_{e,m}$ are independent of the local azimuthal number $m$, and the solution of the eigenvalue problem in a frame in which $\Omega$ is independent of time is given by a triangle rule (Fig. 1b): the seismic integrals $\Omega_c$ and $\Omega_e$, plotted along their corresponding rotation axes, constitute two sides of the triangle. The length of the third side gives the combined seismic average $\Omega$ for the two regions, and the direction of this side determines the inclination of the pulsation axis. In the coordinate frame whose polar axis coincides with this axis, the oscillation eigenfunctions are represented by separable spherical harmonic eigenfunctions. The angle $\alpha$ between the pulsation axis and the axis of rotation of the envelope
can be determined from a triangle relation, e.g.,

$$\bar{\Omega} \sin \alpha = \bar{\Omega}_e \sin \beta,$$

where

$$\bar{\Omega} = \sqrt{\bar{\Omega}_c^2 + \bar{\Omega}_e^2 + 2\bar{\Omega}_c \bar{\Omega}_e \cos \beta},$$

and $\beta$ is the angle between the rotation axes of the core and the envelope. Rotational frequency splitting

$$\omega_1^m = m\bar{\Omega}$$

has exactly the same form as in the case of rotation about a unique axis. Formally, when the core rotates obliquely ($\beta \neq 0$) rotational splitting is smaller because $\bar{\Omega} < \bar{\Omega}_c + \bar{\Omega}_e$. However, this depression of splitting could be compensated by an increase of the angular velocity of the core, which makes it impossible to detect oblique rotation inside a star from measurements of frequency splitting alone. However, an obliquely rotating core also results in a deviation of the axis of pulsation from the axis of rotation of the envelope. This phenomenon could be seen in the relative amplitudes of the rotational-multiplet components observed from Earth.

**ANALYSIS OF SOLAR DIPOLE MODES**

Rotational multiplets of modes of degree $l = 1$ each consist of three components $(Y_{1}^{-1}, Y_{1}^{0}, Y_{1}^{1})$ with azimuthal order $m = -1, 0, +1$, which correspond to the separable spherical harmonics. The eigenfunction of the central ($m = 0$) mode is antisymmetric with respect to the equator ($\theta = \pi/2$) of the coordinate frame of the oscillations, and the other two are symmetric. If there are no obliquely rotating layers inside the Sun, the pulsation axis coincides with the axis of surface rotation, which is almost perpendicular to the ecliptic, and the angle $\alpha$ is zero. In this case the antisymmetric mode is almost invisible in whole-disk measurements, such as those of the IPHIR space experiment (Toutain & Fröhlich, 1992). An obliquely rotating core results in inclination of the pulsation axis to the axis of the surface rotation by the angle $\alpha$ (Eq.1); and, therefore, measurements of the relative amplitude of the central component could give evidence of oblique rotation in the core. Table 1 shows the seismic averages in units of the angular velocity of the envelope, and the amplitudes of the central peak relative to the amplitudes of modes of $m = \pm 1$, computed for a solar model using asymptotic high-frequency p-mode eigenfunctions. The seismic averages and, hence, rotational frequency splitting do not change significantly from the case of rotation about a unique axis, if the radius of the obliquely rotating core is less than $1/2$ of the solar radius and if its rotation rate does not exceed the rate of surface rotation by a factor larger than three. The value of $\bar{\Omega}/\Omega_e$ estimated from the IPHIR data is $1.00 \pm 0.05$ (Toutain & Kosovichev, 1994), where $\Omega_e$ is the angular velocity of the upper radiative zone, which rotates almost spherically. Therefore, from Table 1 we see that the core could be of any radius between 0 and $0.5R_\odot$ provided it rotates somewhat faster than the surface.

We have made an attempt to measure the relative amplitudes $\tilde{c}_0/\tilde{c}_{\pm 1}$ by fitting a superposition of three Lorentzian profiles, instead of the conventional
TABLE I  The seismic averages for dipole multiplets and the relative amplitudes of the central components of the multiplets for the Bai-Sturrock model of solar rotation with $\beta = 40^\circ$

| $r_c/R_\odot$ | $\Omega_c/\Omega_e$ | $\bar{\Omega}(40^\circ)/\Omega_e$ | $|\tilde{c}_0/\tilde{c}_{\pm1}|$ |
|---------------|-------------------|---------------------------------|-------------------------------|
| 0.2           | 1                 | 0.97                            | 0.04                          |
|               | 2                 | 1.03                            | 0.08                          |
|               | 3                 | 1.10                            | 0.11                          |
| 0.3           | 1                 | 0.96                            | 0.06                          |
|               | 2                 | 1.05                            | 0.11                          |
|               | 3                 | 1.16                            | 0.15                          |
| 0.4           | 1                 | 0.95                            | 0.08                          |
|               | 2                 | 1.07                            | 0.14                          |
|               | 3                 | 1.21                            | 0.19                          |
| 0.5           | 1                 | 0.93                            | 0.11                          |
|               | 2                 | 1.12                            | 0.19                          |
|               | 3                 | 1.32                            | 0.24                          |

two, to peaks of the modes of $l = 1$ in the oscillation power spectrum from IPHIR; and we have found that the ratio is very small, with an upper limit less than 0.03. This result, considered together with the measurements of rotational splitting by Toutain & Kosovichev (1994), makes the obliquely rotating core unlikely to be larger than $0.2R_\odot$.

The small amplitude of the central peak of the IPHIR power spectrum prompts us to consider the influence of the angle of $7^\circ$ between the rotation axis of the solar surface and the normal to the plane of the ecliptic (the so-called $B$-angle). At the time of the IPHIR observations, the average $B$-angle of the Sun was $5^\circ$, which gives about 0.12 for the amplitude ratio $\tilde{c}_0/\tilde{c}_{\pm1}$. If the observed ratio is indeed significantly lower than this value then probably there is a variation of the rotation axis inside the Sun, which could compensate the effect of the $B$-angle and perhaps even indicate that the net angular moment of the Sun is in the same direction as the angular momentum of the planetary system.

REFERENCES