LINEAR VISCO-RESISTIVE COMPUTATIONS OF MAGNETOHYDRODYNAMIC WAVES

I. The Code and Test Cases

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Abstract. The stationary state of resonant absorption of linear, MHD waves in cylindrical magnetic flux tubes is studied in viscous, compressible MHD with a numerical code using finite element discretization. The full viscosity tensor with the five viscosity coefficients as given by Braginskii is included in the analysis. Our computations reproduce the absorption rates obtained by Lou in scalar viscous MHD and Goossens and Poedts in resistive MHD, which guarantee the numerical accuracy of the tensorial viscous MHD code.

Key words: Magnetohydrodynamics – Numerical Methods

1. Introduction

Numerical studies of resonant absorption in dissipative MHD have been carried out recently by, e.g., Poedts et al. (1990), Lou (1990), and by Goossens and Poedts (1992). Of particular importance for the present paper are the computations by Lou (1990) and by Goossens and Poedts (1992) since they will be used as test cases.

Viscosity is a prime candidate for providing dissipation in the analysis of resonant absorption. Especially since it has been suggested (Hollweg, 1985, 1986) that viscosity could be a more important dissipative term than finite electrical resistivity. The present note discusses a numerical code for computing the stationary state of resonant absorption in linear viscous MHD. Viscosity is described by the full Braginskii’s viscosity stress tensor.

Resonant absorption is studied in 1D cylindrical flux tubes. The dimensionless equations for the linear perturbations of MHD waves around a static equilibrium are

given in GP. In the momentum equation there is an additional term due to the full viscous stress tensor,
\[
\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p_1 + \left( \nabla \times \mathbf{B}_0 \right) \times \mathbf{B}_1 + \left( \nabla \times \mathbf{B}_1 \right) \times \mathbf{B}_0 - \nabla \cdot \mathbf{\tau},
\]
(1)

In eq. (1) \( \mathbf{\tau} \) denotes the viscous stress tensor given by Braginskii (1965), viz.,
\[
\tau_{\alpha\beta} = -\eta_0 W_{0\alpha\beta} - \eta_1 W_{1\alpha\beta} - \eta_2 W_{2\alpha\beta} + \eta_3 W_{3\alpha\beta} + \eta_4 W_{4\alpha\beta}.
\]
(2)

The five viscosity coefficients are assumed to be constant. In a strong magnetic field we can use the approximation \( \eta_2 \approx 4\eta_1 \) and \( \eta_4 \approx 2\eta_3 \) for the viscosity coefficients.

In addition to the boundary conditions given by GP, there are new conditions for the total pressure, which take the form at \( r = 1 \),
\[
p_1 + \mathbf{B}_0 \cdot \mathbf{B}_1 + \pi_{ij} - B_{0\rho}^2 \xi_\rho = p_{1e}, \quad i = j = 1, 2, 3
\]
(3)
\[
\pi_{ij} = 0, \quad i \neq j, \quad i, j = 1, 2, 3
\]
(4)

2. The Numerical Method and Results

Fourier-decomposition and prescribed time dependence, \( \exp(-i\omega t) \), enable us to convert the linear MHD PDE's into a set of linear ODE's. We solve this set of linear ODE's by means of a combination of the Galerkin procedure and the finite-element method. A combination of cubic and quadratic Hermite polynomials as finite elements are used for the spatial discretization.

The correct implementation and the numerical accuracy will now be scrutinized by computing the resonant absorption of acoustic oscillations in cylindrical flux tubes with the viscous MHD code and comparing the results with those obtained by Lou and GP. The remarkable agreement between the absorption rates obtained in resistive and in tensorial viscous MHD is illustrated in Fig. 1. This remarkable agreement is a strong indication of the correct implementation of the viscous terms in the numerical code. It also proves that the magnitude of the actual dissipation mechanism is independent as long as the dissipation coefficient is small (see also GP).

3. Conclusions

We have presented a numerical code using finite elements discretization for computing the stationary state of resonant absorption of MHD waves, in linear, compressible, and viscous MHD. The correct implementation and the numerical accuracy are verified by reproducing the results for the rate of the resonant absorption found in resistive MHD (see GP). We have also found fast convergence behaviour of the code.
Figure 1. Absorption coefficient $\alpha$ vs. the horizontal wave number (left) and vs. the frequency of the oscillation (right). $m$ indicates the different poloidal wave numbers. Dots refer to resistive and crosses to viscous computations.

Our results also provide an additional proof that the amount of the absorbed energy is independent of the specific dissipation mechanism.

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References

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