CORONAL HEATING BY THE RESONANT ABSORPTION OF ALFVÉN WAVES

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Abstract. Nonlinear evolution and stability of the resonant absorption layer is considered by solving the time-dependent 3D, low-\(\beta\), resistive MHD equations with the Lax-Wendroff explicit method. The calculations are initiated with the solutions of the linearized version of the MHD equations (Steinolfson and Davila, 1993, Ofman et al., 1994), with inhomogeneous background density, and a constant magnetic field. The narrow resonant heating layers are deformed by the self-consistent shear flow. When the driver amplitude is small compared to the average Alfén speed the dissipation layer appears to be stable and the driver-period-averaged ohmic heating rate saturates at a slightly higher than the linear rate. When the driver amplitude is large \((F_d \approx 1)\) the resonant heating may become unstable.

Key words: Sun – MHD

1. Introduction

It is well known from observations that the solar corona is much hotter \((T \approx 2 \times 10^6K)\) than the photosphere \((T = 5 \times 10^3K)\) and the chromosphere \((T \approx 10^4K)\) beneath it, however the exact heating mechanism is unknown. The heated corona is highly structured and inhomogeneous, containing a large number of discreet magnetic loops, and there is observational evidence of MHD waves propagating in the loops. The UV spectrum suggests non-thermal velocities of approximately 10-20 km/s (Cheng et al., 1979). Non-thermal soft X-ray line broadening, that may correspond to velocities of the order 50 km/s was observed above active regions with the X-Ray Polychromator (XRP) aboard the SMM (Acton et al., 1981). As recent study of soft X-ray lines from the XRP indicates non thermal motions of 30-40 km/s above active regions were suggested as a signature of MHD wave heating (Saba and Strong, 1986, 1991a, 1991b).

\textsuperscript{1} IAU Colloq. 144 "Solar Coronal Structures", V. Ru"{s}in, P. Heinzel, J.-C. Vial (eds.), 473–477
\textsuperscript{2} ©1994 VEDA Publishing Company, Bratislava – Printed in Slovakia
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Resonant absorption of Alfvén waves in coronal loops was first suggested by Ionson (1978; 1982; 1983) as a non-thermal heating mechanism of the corona and since then studied by many authors in the linear regime (e.g., Davila, 1987, Hollweg, 1987). Linear studies indicate that with the classical solar dissipation parameters \( S \approx R \approx 10^{14} \) the velocities at the resonant layer are several orders of magnitude larger than the observed velocities (e.g., Steinolfson and Davila, 1993, Ofman et al., 1994).

Nonlinear effects at the dissipation layer might become important when the velocity shear there becomes large. Kelvin-Helmholtz instability was suggested as a possible mechanism that enhance the effective dissipation (e.g., Heyvaerts and Priest, 1983, Hollweg, 1987, Hollweg and Yang, 1988, Uchimoto et al., 1991, Karpen et al., 1993)

In the present study we consider the nonlinear evolution of the resonant absorption of Alfvén waves via 3D low-\( \beta \) resistive MHD simulation, and investigate the stability of the resonant layer.

2. MHD Equations and Model

The normalized low-\( \beta \) MHD equations in flux conservative form

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ (\mathbf{v} \times \mathbf{B}) - \frac{1}{S} \nabla \times \mathbf{B} \right],
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \frac{1}{2} B^2 \mathbf{I}) + \mathbf{F}_d,
\]

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}),
\]

where \( \mathbf{F}_d = F_0(x) \sin \omega t \mathbf{e}_x \) is the driving force, \( F_0(x) = F_d e^{-x^2/w^2} \) with \( w^2 \ll 1 \), and \( F_d \) is the amplitude of the driver. Note that in the low-\( \beta \) approximation the thermal pressure is neglected (compared to the magnetic pressure), thus the energy equation uncouples from the rest of the MHD equations and can be omitted.

In the above equations the physical parameters are the Landquist number \( S = \tau_r/\tau_A \), where \( \tau_r = 4\pi a^2/\nu c^2 \) is the resistive time scale, and \( \tau_A = a/v_A \) is the Alfvén time scale. In the above definitions \( a \) is the thickness of the loop, \( \rho_0(0) \) is the density at the center of the loop, \( \nu \) is the resistivity, and \( v_A = B_0/[4\pi \rho_0(0)]^{1/2} \) is the Alfvén speed. The variables in the equations above are made dimensionless by \( \rho_0 \rightarrow \rho_0/\rho_0(0), \mathbf{v} \rightarrow \mathbf{v}/v_A, \mathbf{B} \rightarrow \mathbf{B}/B_0, t \rightarrow t/\tau_A \).

The initial density profile of the coronal loop in our model is given by

\[
\rho_0(x) = \rho_r + (1 - \rho_r) e^{-x^4},
\]
where $\rho_r = \rho_0 (x \to \infty) / \rho_0 (0)$. The resistive heating rate is given by

$$\dot{H}_r (t) = S^{-1} \int_V j^2 \, d^3 x,$$

(5)

where $j = j(x, y, z, t)$ is the magnitude of the current, and $V$ is the volume of the loop.

3. Numerical Results

In Figs. 1–3 we present the results of a run in a model loop with the normalized dimensions $6 \times 4\pi \times 20\pi$ and with the parameters $S = 10^3$, and $F_d = 0.1$. The resolution is $62 \times 41 \times 41$ grid points. The MHD equations were integrated with the Lax-Wendroff method that was found to be considerably more efficient than the ADI method. The nonlinear 3D MHD simulations were initiated with the linear solutions (Steinolfson and Davila, 1993, Ofman et al., 1994) with $S = 10^3$, $k_y = 0.5$ and $k_z = 0.1$ (where $k_y$ and $k_z$ are the normalized wavenumbers). An extensive parametric study of the nonlinear evolution is currently under way (Ofman et al., 1994). The

![Figure 1. The temporal evolution of the resonant heating rate.](image)

temporal evolution of the ohmic heating is shown in Fig. 1. At $t \approx 100 \tau_h$ the heating reaches a quasi-steady state. The temporal oscillations are due to the driver with long term variations due to the nonlinear effects.

In Fig. 2 the temporal evolution of the variables at $x = 1.25$, $y = 12.25$, and $z = 16.11$ is shown. It is evident that due to the nonlinear effects the $B_y$ component oscillates at a frequency that is double the drivers frequency, and that additional harmonics appear in $\rho$ and $v_x$ (these features do not appear if the run is performed with the linearized version of the 3D MHD code). The spatial distribution of the ohmic
heating is shown in Fig. 3. The heating occurs in two thin layer that are located near the ideal Alfvén resonance surfaces given by $\omega^2 \rho(x) - k_x^2 = 0$. The width of the dissipation layers agrees with the linear theory, however contrary to the linear case the heating layers are not symmetrical with respect to $x = 0$, and are deformed by the local time-dependent self-consistent shear flow. In the linear approximation the effect of the self-consistent shear on the location of the resonance layers can be modeled by a Doppler-shift in the Alfvén resonance condition (Ofman et al., 1994).

4. Summary and Discussion

The nonlinear evolution of the resonant absorption was investigated with the 3-D low-β resistive MHD simulations. For $S = 10^2$, $10^3$, and $10^4$ the dissipation layer is stable when the self-consistent velocity shear at the resonance layer is $< 0.1v_A$. In the stable parameter range the time-averaged nonlinear heating rate is 10-20% higher than the linear heating rate. When the driver’s amplitude is increased ($F_d \approx 1$), the resonance layer becomes unstable. Longer runs with higher resolution are needed to study the evolution of the unstable resonance layers.

Acknowledgements. This work was supported by NASA Solar Maximum Mission Guest Investigator Grant 370-04-01 to Goddard Space Flight Center. RSS was supported by the
Figure 3. The 3D spatial distribution of the ohmic heating regions at $t = 320\tau_h$.

National Science Foundation under grant ATM-90-15705.

References


