ENERGY RELEASE IN STELLAR MAGNETOSPHERES

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Abstract. The interaction of a stellar magnetosphere with a thin accretion disk is considered. Specifically, I consider a model in which (1) the accretion disk is magnetically linked to the star over a large range of radii and (2) the magnetic diffusivity of the disk is sufficiently small that there is little slippage of field lines within the disk on the rotation time scale. In this case the magnetic energy built up as a result of differential rotation between the star and the disk is released in quasi-periodic reconnection events occurring in the magnetosphere (Aly and Kuipjers 1990). The radial transport of magnetic flux in such an accretion disk is considered. It is show that the magnetic flux distribution is stationary on the accretion time scale, provided the time average of the radial component of the field just above the disk vanishes. A simple model of the time-dependent structure of the magnetosphere is presented. It is shown that energy release in the magnetosphere must take place for (differential) rotation angles less than about 3 radians. The magnetic flux distribution in the disk depends on the precise value of the rotation angle.

Key words: accretion disks – magnetic field

1. Introduction

Accretion onto stellar magnetospheres occurs in a variety of astrophysical systems. For example, in pulsating X-ray sources the X-rays are generated by accretion of matter onto a strongly magnetized neutron star \((B \sim 10^{12} \text{ G})\). In this case the accretion flow is dominated by the magnetic field within a relatively large distance from the neutron star \((\sim 100R_s)\). In low-mass X-ray binaries (LMXB) the neutron star accretes matter from a disk; the neutron-star magnetic field is believed to be somewhat lower \((\sim 10^9 \text{ G})\) and the inner edge of the disk is relatively close to the stellar surface. Magnetospheric accretion has also been proposed for pre-main sequence stars (Konigl 1991; Hartmann 1993).

Ghosh and Lamb (1978, 1979a,b) consider the interaction between an accretion disk and the magnetic field from a rotating neutron star. They argue that the stellar magnetic field invades the disk over a wide range of radii. They find that the radius of the inner edge of the disk depends mainly on the magnetic moment of the star and the mass accretion rate. Differential rotation between the disk and the star causes the magnetospheric field to be wound up in the azimuthal direction. Ghosh and Lamb assumed that the magnetic field is axisymmetric and stationary in time. This implies that the field lines slip across the disk at the rate imposed by the differential rotation, which is possible only for relatively large values of the magnetic diffusivity in the disk (the magnetosphere is assumed to be highly conducting). Ghosh and Lamb (1979b) demonstrate that magnetic coupling between the star and
the disk plays an important role in controlling the spin rate of the neutron star. For a recent review of the physics of accretion onto neutron stars, see Ghosh and Lamb (1991).

Aly (1980) and Aly and Kuijpers (1990) propose a somewhat different model of the magnetosphere/disk interaction. They argue that the stellar magnetic field is excluded from the disk most of the time, and that magnetic links between the disk and the star are formed only occasionally and at specific locations. When a link is formed, the linked field is stretched (without much slip across the disk), until a reconnection event ("flare") occurs in the magnetosphere and the link is severed. An important difference with the Ghosh and Lamb (1979a) model is that the magnetic field structure is essentially time dependent. Aly and Kuijpers (1990) suggest that flaring interactions are perhaps the explanation for the quasi periodic oscillations (QPO) observed in some LMXBs.

In this paper I consider a model which combines some of the features of the models of Ghosh and Lamb (1979a) and Aly and Kuijpers (1990). Specifically, the disk is assumed to be magnetically linked to the star over a wide range of radii, as suggested by Ghosh and Lamb (1979a). However, the resistivity of the disk is assumed to be small, so that there is virtually no slip of magnetic field lines across the disk on the rotation time scale. Then the stellar field will be continually stretched and a series of flare-like events will occur, as proposed by Aly and Kuijpers (1990). Hence, there are large variations in magnetic structure occurring on the rotation time scale. On the (much longer) accretion time scale the effects of finite resistivity cannot be neglected: it allows the disk plasma to move radially inward across the magnetic field lines.

Two aspects of this model will be discussed. Section 2 considers the radial transport of magnetic flux within the disk. It is shown that, for the flux distribution to be stationary on the accretion time scale, the time average of the radial component of the field just above the disk must vanish. Section 3 considers the force balance in the magnetosphere. A self-similar, force free model of the magnetosphere is developed. Strictly speaking, this model is applicable only for radii large compared to the inner radius of the disk. However, the model provides some insight into the nature the interaction between the accretion disk and the stellar magnetic field.

2. Magnetic Flux Transport

I consider the radial transport of a large-scale magnetic field in an accretion disk. The magnetic field $B(r, t)$ and velocity field $v(r, t)$ are symmetric with respect to the rotation axis of the star; close to the star the magnetic field is an aligned dipole. Let $B_{z,\text{disk}}(r, t)$ be the component of the magnetic field perpendicular to the disk. Using a cylindrical coordinate system, the
magnetic induction equation reads:

$$\frac{\partial B_{z,\text{disk}}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( v_z B_r - v_r B_{z,\text{disk}} \right) + \eta \left( \frac{\partial B_{z,\text{disk}}}{\partial r} - \frac{\partial B_r}{\partial z} \right) \right]. \quad (1)$$

where the first and second terms describe the transport of $B_{z,\text{disk}}$ by the accretion flow, while the third and fourth terms describe the effects of magnetic diffusion [$\eta(r)$ is the magnetic diffusivity]. If the disk thickness $H(r)$ is small compared to $r$, $v_z/v_r \sim dH/dr \ll 1$, so that the term with $v_z$ in Eq. (1) can be neglected. I assume that $B_r$ within the disk varies approximately linearly with $z$, so that $\partial B_r/\partial z \approx B_{r,\text{disk}}/H$, where $B_{r,\text{disk}}(r,t)$ is the radial component of magnetic field at the disk surface ($z = +H$). The accretion velocity is given by $v_r(r) = -k\nu/r$, where $\nu(r)$ is the turbulent viscosity within the disk and $k(r)$ is assumed to be of order unity. The requirement that the magnetic field is stationary on the accretion time scale implies that there is no net radial transport of $B_z$. Hence the time average of the quantity in square brackets in Eq. (1) must vanish:

$$k\nu \bar{B}_{z,\text{disk}} + \eta r \frac{\partial \bar{B}_{z,\text{disk}}}{\partial r} - \eta \frac{r}{H} \bar{B}_{r,\text{disk}} \approx 0, \quad (2)$$

where the bar denotes a time average.

The small-scale turbulent motions within the disk are responsible for both "viscous" angular momentum transport and magnetic diffusion of the large-scale field. As a first approximation I assume that $\nu$ and $\eta$ are of the same order of magnitude. Then the first term in Eq. (2) is of order $\eta \bar{B}_{z,\text{disk}}$, and assuming that $\bar{B}_{z,\text{disk}}(r)$ varies on the scale of $r$ the second term in Eq. (2) is also of that order. It follows that

$$|\bar{B}_{r,\text{disk}}| \sim \frac{H}{r} |\bar{B}_{z,\text{disk}}|. \quad (3)$$

In the thin disk limit $\bar{B}_{r,\text{disk}} \approx 0$. Hence, in order for the radial distribution of $B_{z,\text{disk}}$ to be stationary on the accretion time scale, the time average of $\bar{B}_{r,\text{disk}}$ must vanish.

The difference in angular velocity between the disk and the star causes magnetic field lines to be sheared in the azimuthal direction at a rate given by the beat frequency, $\Omega_B(r_0) = |\Omega_S - \Omega_K(r_0)|$, where $\Omega_S$ is the stellar rotation rate and $\Omega_K(r_0)$ is the Kepler rate at the radius $r_0$ where a field line is tied to the disk. The magnitude of the variation of $B_{z,\text{disk}}(r,t)$ on the rotation time scale can be estimated from Eq. (1):

$$\delta B_{z,\text{disk}} \sim \frac{\eta}{r H \Omega_B} \delta B_{r,\text{disk}}. \quad (4)$$
The magnetic diffusivity may be written as $\eta \sim \nu = \alpha_{visc}\Omega_K H^2$, where $\alpha_{visc} \leq 1$. Inserting this into Eq. (5) and assuming $\delta B_{r,disk} \leq B_{z,disk}$ and $H(r) \ll r$ yields

$$\delta B_{z,disk} \sim \alpha_{visc} \frac{H\Omega_K}{r\Omega_B} \delta B_{r,disk} \ll B_{z,disk}. \quad (5)$$

Hence, for a thin disk the variations of $B_{z,disk}(r,t)$ on the rotation time scale can be neglected.

### 3. Self-Similar Model of Magnetosphere

The plasma in the magnetosphere is subject to gravitational-, centrifugal- and magnetic forces. Assuming that the Alfvén velocity $v_A$ in the magnetosphere is large compared to $r\Omega_S$ and $r\Omega_K$, the force balance requires that the magnetic field $\mathbf{B}(r,t)$ is nearly force free:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B}, \quad (6)$$

where $\alpha(r,t)$ is a scalar function. The magnetic field at time $t = 0$ is assumed to be a potential field.

In the following I describe a simplified model of the magnetosphere in which the beat frequency $\Omega_B$ is taken to be constant (i.e., independent of radius). Then the azimuthal angle by which the star rotates relative to the disk is given by

$$\Delta \phi \approx \Omega_B t, \quad (7)$$

the same for all field lines. The magnetic field at the disk is taken to be a power law function of $r$:

$$B_{\theta,disk}(r) = -B_{z,disk}(r) = \frac{C}{r^{n+2}}, \quad (8)$$

where $C$ and $n$ are constants. Then Eq. (6) has a self-similar solution:

$$\mathbf{B} = [B_r, B_\theta, B_\phi] = \frac{C}{r^{n+2}} \left[ f(\theta), \frac{g(\theta)}{\sin \theta}, h(\theta) \right], \quad (9)$$

where $(r, \theta, \phi)$ is a spherical coordinate system and the functions $f(\theta)$, $g(\theta)$ and $h(\theta)$ are also functions of time. The boundary condition (8) requires $g(\pi/2) = 1$. From the condition $\nabla \cdot \mathbf{B} = 0$:

$$f(\theta) = \frac{1}{n \sin \theta} \frac{dg}{d\theta}. \quad (10)$$

By integrating the equation $dr/d\theta = rB_r/B_\theta$ the following expression for the shape of the field lines is obtained:

$$r(\theta) = r_0[g(\theta)]^{1/n}. \quad (11)$$
Here \( r_0 \) is the radius where the field line intersects the disk. The scalar \( \alpha(r) \) in Eq. (6) is given by:

\[
\alpha(r, \theta) = \frac{a(\theta)}{r},
\]

(12)

and the requirement that \( \alpha \) is constant along field lines yields:

\[
a(\theta) = a_0 [g(\theta)]^{1/n},
\]

(13)

where \( a_0 \) is a function of time only. The \( \theta \)-component of Eq. (6) yields

\[
h(\theta) = \frac{a_0}{(n + 1) \sin \theta} [g(\theta)]^{(n+1)/n}.
\]

(14)

Integration of the equation \( \sin \theta d\phi/d\theta = B_\phi/B_\theta \) yields the following expression for the azimuthal angle by which the magnetic field lines are rotated:

\[
\phi(\theta) = \frac{a_0}{n + 1} \int_0^\theta \frac{d\theta}{\sin \theta} [g(\theta)]^{1/n}.
\]

(15)

The rotation angle between the disk and the star is given by \( \Delta \phi = \phi(\pi/2) \). The \( \phi \)-component of Eq. (6) yields the following non-linear, second-order differential equation for \( g(\theta) \):

\[
\sin \theta \frac{d}{d\theta} \left( \frac{1}{\sin \theta} \frac{dg}{d\theta} \right) + n(n + 1)g(\theta) + \frac{n}{n + 1} a_0^2 [g(\theta)]^{(n+2)/n} = 0.
\]

(16)

Note that this equation is singular at \( \theta = 0 \). Analysis shows that solutions which are regular at \( \theta = 0 \) must have \( g(\theta) \propto [1 - \cos \theta] \) for \( \theta \to 0 \). Therefore, I define a parameter \( \gamma \) such that

\[
g(\theta) \approx \gamma [1 - \cos \theta] \approx \frac{1}{2} \gamma \theta^2 \text{ for } \theta \to 0.
\]

(17)

Equation (16) can be rewritten as a set of two coupled, first-order differential equations for \( g(\theta) \) and its derivative. These equations are solved using the second-order Runge-Kutta method, starting at a small value of \( \theta \) and ending at \( \theta = \pi/2 \); \( a_0 \) is treated as a free parameter. The starting values of \( g(\theta) \) and \( dg/d\theta \) are derived from Eq. (17).

First consider potential fields (\( a_0 = 0 \)). In this case Eq. (16) is linear and there is only one value of \( \gamma \) for which \( g(\pi/2) = 1 \). Figure 1 shows the resulting potential fields for three values of the parameter \( n \). For \( n = 0.1 \) the field is nearly a split monopole and the radial component just above the disk has the same sign as the stellar field in that hemisphere: \( B_{r,\text{disk}} > 0 \).

For \( n = 1.0 \) the field is a dipole, \( B_{r,\text{disk}} = 0 \), and for \( n = 1.9 \) is field is nearly a quadrupole, \( B_{r,\text{disk}} < 0 \). Therefore, the sign of \( B_{r,\text{disk}} \) depends on the exponent \( n \) describing the flux distribution at the disk [see Eq. (8)].
Now consider force-free fields, \( a_0 \neq 0 \), so that Eq. (16) is nonlinear. To solve this equation I perform a series of calculations with fixed \( a_0 \) but different values of \( \gamma \), and for each case I determine \( g_{0}(\gamma) = g(\pi / 2) \). Note that the boundary condition requires \( g_{0}(\gamma) = 1 \). It can be shown that the function \( g_{0}(\gamma) \) has a maximum, and that the height of the maximum decreases with increasing \( a_0 \). Hence, for \( a_0 \) less than a certain \( a_{0,max} \) the maximum value of \( g_{0}(\gamma) \) is larger than unity and there are two solutions, \( \gamma_1 \) and \( \gamma_2 \), for which \( g_{0} = 1 \). These two values of \( \gamma \) correspond to two different magnetic field geometries with the same flux distribution at the disk. For \( a_0 = a_{0,max} \) there is only one solution, and for \( a_0 > a_{0,max} \) the maximum value of \( g_{0}(\gamma) \) is less than unity so that there are no solutions. Therefore, solutions exist only for \( 0 \leq a_0 \leq a_{0,max} \). The two solutions \( \gamma_1 \) and \( \gamma_2 \) can be represented by a single function \( a_0(\gamma) \), which increases for small \( \gamma \) (\( = \gamma_1 \)) and decreases for large \( \gamma \) (\( = \gamma_2 \)). The quantity \( \Delta \phi \) can be found from Eq. (15) and is found to be a monotonic function of \( \gamma \).

Figure 2 illustrates the evolution of the magnetic field for \( n = 0.5 \). The upper panels show the projections of field lines onto the plane of the disk, and the lower panels show the projections in the meridional plane. The same set of field lines is shown for four values of \( \Delta \phi \) (i.e., four different
times). Note that the field lines expand radially outward as the rotation angle is increased, and that there are large variations in the direction of the magnetic field at the disk surface.

In Figure 3 the magnetic field at the disk surface is plotted as function of rotation angle $\Delta \phi$ for $n = 0.1$ and $n = 0.5$. Figure 3a shows the ratio $B_{\phi,disk}(r,t)/B_{z,disk}(r) = h(\pi/2) = a_0/(n+1)$. Note that $B_{\phi,disk}(r,t)$ reaches a maximum for $\Delta \phi \sim 1.5$ radians and then decreases with increasing rotation angle. $B_{\phi,disk}$ tends to zero when $\Delta \phi$ approaches a certain maximum value, $\Delta \phi_{max}$, which is 2.6 radians for $n = 0.5$ and about 3 radians for $n = 0.1$. No self-similar solution exists for $\Delta \phi > \Delta \phi_{max}$.

Figure 3b shows the ratio $B_{r,disk}(r,t)/B_{z,disk}(r) = f(\pi/2)$. Note that $B_{r,disk}(r,t)$ changes sign as the rotation angle is increased. The time at which $B_{r,disk}$ changes sign depends on $n$. When $\Delta \phi$ approaches $\Delta \phi_{max}$, the magnetic field approaches the open state and $B_{r,disk}(r,t)$ diverges. This is an artifact of the self-similar model and is related to the fact that the total magnetic flux in the model is infinite (singularity at origin).

4. Discussion

The self-similar model indicates that the magnetic links between the star and the outer disk cannot be maintained very long; the enhanced magnetic
Fig. 3. Magnetic field at the disk surface plotted as function of rotation angle: (a) $B_{\phi,disk}$, (b) $B_{r,disk}$.

Pressure due to $B_\phi$ causes the magnetic field to open up when the rotation angle exceeds a few radian. Aly and Kuijpers (1990) proposed that the magnetic field undergoes a series of flare-like events. During these flares the field lines in the magnetosphere are opened, some or all of the azimuthal field is ejected, and reconnection restores the field to something like a potential field configuration. Then the build-up of azimuthal field resumes, until the next flare event. Therefore, the time evolution of the system is punctuated by a quasi-periodic series of flare events. The frequency of flares is determined by that region in the disk where the beat frequency is largest.

Let $\Delta \phi_{rec}$ denote the rotation angle at which the field undergoes a reconnection event. I assume that the field relaxes instantaneously to the potential state. As shown in section 2, the requirement that $B_{z,disk}$ is stationary on the accretion time scale implies $\bar{B}_{r,disk} = 0$, which yields:

$$\int_0^{\Delta \phi_{rec}} B_{r,disk} d(\Delta \phi) = 0. \tag{18}$$

Hence, $B_{r,disk}(r,t)$ must change sign during the energy build-up phase. For the self-similar model this is true if the parameter $n$ lies between 0 and 1 [see Eq. (8)]. This implies that $B_{z,disk}(r)$ falls off with increasing $r$ at a rate which is intermediate between that for a split-monopole field ($r^{-2}$) and a dipole ($r^{-3}$). Moreover, I find that Eq. (18) can be satisfied only if $\Delta \phi_{rec}$ is smaller than the angle $\Delta \phi_{max}$ at which the magnetic field would be fully opened. Equation (18) implies a certain relationship between $\Delta \phi_{rec}$ and the parameter $n$. Therefore, the value of $n$ depends on the precise timing of the reconnection events, which cannot be predicted from the present model.
I conclude that the magnetic flux distribution in the disk depends on the critical angle for energy release in the magnetosphere.

The result that $n$ lies between 0 and 1 suggests that at the inner edge of the disk some of the stellar magnetic field is pulled into the disk and spread out over a wider range of radii. This is a consequence of the non-potential nature of the magnetosphere: at the disk plane the magnetic field lines are curved outward part of the time.

I conclude that the interaction between the disk and stellar field can lead to large variations in magnetic structure on the rotation time scale. Further progress in modeling magnetospheric accretion and comparison with QPO observations requires more realistic time-dependent models of the magnetosphere. In particular the 2-dimensional structure of the field near the inner boundary of the disk must be taken into account.

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References

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