ALFVÉN WAVE HEATING

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Abstract. This review discusses Alfvén wave heating in non-uniform plasmas as a possible means for explaining the heating of the solar corona. It focusses on recent analytical results that enable us to understand the basic physics of Alfvén wave heating and help us with the interpretation of results of numerical simulations. First we consider the singular wave solutions that are found in linear ideal MHD at the resonant magnetic surface where the frequency of the wave equals the local Alfvén frequency. Next, we use linear resistive MHD for describing the waves in the dissipative region and explain how dissipation modifies the singular solutions found in linear ideal MHD.

Key words: MHD – Alfvén waves – Heating

1. Introduction

The theory for heating the solar atmosphere by MHD waves is observationally motivated. The sun does seem to radiate Alfvén waves with significant energy fluxes, but it is not known whether the waves originate from the convective motions, or whether they are the remnants of reconnection events occurring somewhere above the convection zone. In addition MHD waves have the nice property that if they originate at low heights they can in principle heat the temperature minimum region and the chromosphere on their way to the corona and solar wind.

A wave theory for coronal heating must make sure that the energy requirement of the corona can be met by the observed wave amplitudes and must show that the waves are able to dissipate a considerable part of their energy at the coronal level. As far as the energy requirement is concerned, it seems that the observed nonthermal motions can provide sufficient energy to heat the corona (see e.g. Hollweg, 1990). The great difficulty has been considered to be the low efficiency of viscous and ohmic dissipation, which is very weak in a uniform plasma and therefore unable to deposit a substantial amount of energy in the solar corona.

Since the corona is far from being a uniform plasma it is necessary to study the properties of MHD-waves in nonuniform plasmas in order to understand properly the role of MHD-waves in coronal heating.

Alfvén wave heating in nonuniform plasmas was first studied as a means for the supplementary heating of fusion plasmas by Chen and Hasegawa (1974) and Tataronis and Grossmann (1973), and subsequently proposed as

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mechanism for heating magnetic loops in the solar corona by Ionson (1978).

It is only fair to warrant the reader that the scope of the present paper is limited. Indeed, we do not want to embark on the difficult mission of giving a general review of the present status of knowledge of Alfvén wave heating in the solar atmosphere. Instead, we shall concentrate on recent analytic results that help us to understand the basic physics of Alfvén wave heating and enable us to interpret results of large scale numerical simulations. The reader who is interested in a broader overview of the subject is referred to e.g. Kuperus et al. (1981), Heyvaerts (1984), Ionson (1985), Hollweg (1990, 1991), Browning (1991), Goossens (1991).

2. Asymptotic state of Alfvén wave heating

The study of Alfvén wave heating involves forced oscillations in disipative MHD. This means that we have to integrate the time dependent non-linear equations of dissipative MHD in the presence of a time varying force term. This is an ambitious task. Up untill now most studies have used linear theory of wave motions superimposed on an ideal equilibrium state. Even in the context of linear MHD, most investigations have circumvented the time integration by considering the asymptotic or steady state of Alfvén wave heating. In the asymptotic state all the perturbed quantities oscillate with the same frequency as the incoming wave so that the time dependency can be removed out of the mathematical formulation. Detailed results based on large-scale numerical simulations of the asymptotic state of Alfvén wave heating were obtained by Grossmann and Smith (1988) in ideal MHD and by Poedts et al. (1989ab, 1990a) in resistive MHD. Time dependent computations of Alfvén wave heating in linear dissipative MHD have been carried out by Poedts et al. (1990b), and Poedts and Kerner (1993).

In what follows we shall concentrate our attention on the asymptotic state of Alfvén wave heating in linear MHD. All perturbed quantities are put proportional to

\[ \exp(-i\omega t) \]  

with \( \omega \) the frequency of the incoming wave or the external forcing term. The asymptotic state is in principle only valid for \( t \to +\infty \), but in practice it gives an accurate description for \( t \gg t_{\text{crit}} \) where \( t_{\text{crit}} \approx (R_m)^{1/3} \) and \( R_m \) is the magnetic Reynolds number (see Kappraff and Tataronis,1975; Poedts et al. 1990a).

Studies of Alfvén wave heating in the solar corona have almost always used 1D approximations to the equilibrium states in order to reduce the complexity of the mathematical formulation. The equilibrium quantities density \( \rho \), pressure \( p \), and magnetic induction \( \mathbf{B} \) depend on one spatial coordinate only. The two remaining spatial coordinates are ignorable in the differen-
tial equations for the perturbed quantities. The straight cylinder is a classic 1D approximation to the equilibrium state of a coronal loop or a magnetic flux tube. All equilibrium quantities are functions of \( r \) only in a system of cylindrical coordinates \( r, \phi, z \) with the \( z \)-axis coinciding with the axis of symmetry of the cylinder.

Since the equilibrium quantities depend on \( r \) only we can Fourier-analyze the perturbed quantities with respect to \( \phi \) and \( z \) and put them proportional to

\[
\exp[i(m\phi + kz)].
\] (2)

As the time-dependence \( \exp(-i\omega t) \) has already been factored out the perturbed quantities are functions of \( r \) only and the differential equations are reduced to ordinary differential equations.

### 3. Linear ideal MHD

The large values of magnetic Reynolds number \( R_m \) in the corona imply that the dissipative terms in the MHD equations will be unimportant except in narrow layers. In the case of Alfvén wave heating we shall see below that the dissipative terms are only important in a narrow layer around the position where the frequency of the wave equals the local Alfvén frequency. Outside this narrow layer the MHD waves are accurately described by the equations of ideal MHD. Dissipative MHD is only necessary to treat the dissipative layer. Part of the basic physics of Alfvén wave heating can be understood in the context of linear ideal MHD. All but two of the perturbed variables may be eliminated from the linear ideal MHD equations leading to a set of two first-order differential equations for the radial component of the Lagrangian displacement, \( \xi_r \) and the perturbed total pressure \( P' \) \( (P' = p' + B.B'/\mu) \):

\[
\begin{align*}
\frac{D d(r\xi_r)}{dr} &= C_1 r \xi_r - C_2 r P', \\
\frac{D dP'}{dr} &= C_3 \xi_r - C_1 P'.
\end{align*}
\] (3)

Equations (3) govern the linear motions of a compressible cylindrical plasma. They were first obtained in this form by Appert et al. (1974). The coefficient functions \( D, C_1, C_2, \) and \( C_3 \) depend on the equilibrium quantities and on the frequency \( \omega \). Expressions for the coefficient functions can be found in Appert et al (1974) and in Sakurai et al. (1991a). The important coefficient function for the present discussion is \( D \), it takes the form

\[
D = \rho(c^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_C^2).
\] (4)

In equation (4) \( c \) is the adiabatic velocity of sound, \( v_A \) is the Alfvén velocity, \( \omega_A \) is the local Alfvén frequency, and \( \omega_C \) is the local cusp frequency.
squares of these quantities are defined as
\[ c^2 = \gamma p/\rho, \quad v_A^2 = B^2/(\mu \rho), \quad \omega_A^2 = f_B^2/\mu \rho, \quad \omega_C^2 = c^2 \omega_A^2/(c^2 + v_A^2), \quad f_B = k \cdot B, \quad k = (m/r, k). \]

In a nonuniform plasma \( c^2, v_A^2, \omega_A^2, \) and \( \omega_C^2 \) are functions of position. The variation of \( \omega_A \) with position plays a fundamental role in the discussion of Alfvén heating.

The other perturbed quantities (\( p', \rho', \) etc.) can be computed once \( \xi_r \) and \( P' \) are known. For later use we note that the component of the displacement vector in the magnetic surfaces and perpendicular to the magnetic field lines (\( \xi_\perp = (\xi_\phi B_z - \xi_z B_\phi)/B \)) is related to \( \xi_r \) and \( P' \) as

\[ (\omega^2 - \omega_A^2) \xi_\perp = \frac{i}{\rho B} (g_B P' - 2f_B B_\phi B_z \xi_r/\mu r), \]

where

\[ B = (B_\phi^2 + B_z^2)^{1/2}, \quad g_B = \frac{m}{r} - k B_\phi. \]

Equations (3) can be combined into one second order differential equation for \( \xi_r \):

\[ \frac{d}{dr} \left[ f(r; \omega^2) \frac{d(r \xi_r)}{dr} \right] - g(r; \omega^2) r \xi_r = 0, \]

or for \( P' \),

\[ \frac{d}{dr} \left[ \tilde{f}(r; \omega^2) \frac{dP'}{dr} \right] - \tilde{g}(r; \omega^2) P' = 0. \]

Here \( f(r; \omega^2), g(r; \omega^2), \tilde{f}(r; \omega^2), \) and \( \tilde{g}(r, \omega^2) \) are functions of \( r \) and \( \omega^2 \) which can be expressed in terms of the coefficient functions \( D, C_1, C_2, \) and \( C_3 \). Equation (7) was first derived by Hain and Lust (1958).

Equations (3) define an eigenvalue problem with \( \omega^2 \) as eigenvalue parameter when they are supplemented with boundary conditions. In a driven problem \( \omega \) is prescribed. Equations (3) have regular singular points at the zeroes of the coefficient function \( D \). As a consequence we have mobile regular singular points at the positions \( r \) where

\[ \omega^2 = \omega_A^2(r), \quad \omega^2 = \omega_C^2(r). \]

Equation (9a) defines the Alfvén resonance point, while equation (9b) defines the slow resonance point. Since \( \omega_A^2(r) \) and \( \omega_C^2(r) \) are functions of position, these two equations define two continuous ranges in the spectrum which are classically referred to as the Alfvén continuum and the slow continuum. In what follows we shall concentrate on the Alfvén resonance.
We now focus on a frequency in the Alfvén continuum and determine the spatial behaviour of the corresponding perturbation close to the singular point where the condition $\omega^2 = \omega_A^2(r_A)$ is satisfied. It is convenient to introduce the new radial variable $s$ defined as

$$s = r - r_A.$$  \hspace{1cm} (10)

We use series expansions of the coefficient functions around $s = 0$ to obtain simplified versions of the relevant differential equations. These simplified versions are valid in the interval $[-s_A, s_A]$ around the point of resonance where the linear Taylor polynomial is a valid approximation of $\omega^2 - \omega_A^2(r)$; hence $s_A$ has to satisfy

$$s_A < \left| \frac{2(\omega_A^2)'}{(\omega_A^2)''} \right|.$$ \hspace{1cm} (11)

The second order differential equations for $\xi_r$ and $P'$ can be simplified to

$$\alpha \frac{d}{ds} \left( s \frac{d\xi_r}{ds} \right) + \beta \xi_r = 0, \quad \wedge \quad \tilde{\alpha} \frac{d}{ds} \left( s \frac{dP'}{ds} \right) + \tilde{\beta} P' = 0,$$ \hspace{1cm} (12)

where $\alpha$, $\beta$, $\tilde{\alpha}$, and $\tilde{\beta}$ denote constants arising from the series expansions of the coefficient functions. The indicial equations of both of the equations (12) have the double root

$$\nu_{1,2} = 0.$$ \hspace{1cm} (13)

This implies that one of the two independent solutions both for $\xi_r$ and $P'$ has a logarithmic function. It can be shown that the 'large' solutions containing the $\ln |s|$ have to be continuous, whereas as the 'small' (regular) solutions may jump. The solutions for $\xi_r$ and $P'$ diverge logarithmically but the difference in these logarithmic terms remains balanced (see Goedbloed, 1983). The solutions then take the form

$$\begin{cases}
\xi_r(s) = u(s) \ln(|s|) + \left\{ \begin{array}{ll}
\xi_-(s) & s < 0 \\
\xi_+(s) & s > 0,
\end{array} \right. \\
P'(s) = \tilde{u}(s) \ln(|s|) + \left\{ \begin{array}{ll}
P'_-(s) & s < 0 \\
P'_+(s) & s > 0.
\end{array} \right.
\end{cases}$$ \hspace{1cm} (14)

The jump in $\xi_r$ is important, since it is related to the jump in the flux of energy and as a consequence to the absorption of energy when the nonuniform plasma is driven by an external source at a frequency in the Alfvén continuum. This jump in $\xi_r$ is the physical basis for Alfvén wave heating in nonuniform plasmas. The logarithmic singularity is an artefact of ideal MHD and is replaced by a finite value in dissipative MHD. The jump in $\xi_r$.
measures the sharp variation that $\xi_r$ undergoes when one moves across the dissipative layer in dissipative MHD.

For an equilibrium with straight magnetic field lines ($B_\phi = 0$) the differential equation for $P'$ takes a different form:

$$\tilde{\alpha} \frac{d}{ds} \left( \frac{1}{s} \frac{dP'}{ds} \right) + \tilde{\beta} P' = 0, \quad (15)$$

with $\tilde{\alpha}$, and $\tilde{\beta}$ constants. The indicial equation of equation (15) has the two roots

$$\nu_1 = 0, \quad \nu_2 = 2. \quad (16)$$

The solution for $P'$ has no singularities and $P'$ can be considered constant in the vicinity of the Alfvén resonance position. The problem of Alfvén wave heating can then be formulated as a driven oscillator with $P'$ providing the driving (Hollweg, 1987ab). At this point of the analysis the wave under consideration cannot yet be identified as an Alfvén wave confined to a single magnetic surface. We have not yet found the dominant singularities in the solutions and have not yet established the field guided nature of the wave.

Let us now turn to equations (3). The simplified versions of these two equations will enable us to find the fundamental conservation law at the Alfvén resonance point. Equations (3) reduce to

$$\begin{cases} 
    s\Delta \frac{d \xi_r}{ds} = \frac{g_B}{\rho B^2} C_A(s), \\
    s\Delta \frac{dP'}{ds} = \frac{2f_B B_\phi B_z}{\mu r \rho B^2} C_A(s),
\end{cases} \quad (17)$$

where

$$C_A(s) = g_B P' - \frac{2f_B B_\phi B_z \xi_r}{\mu r}. \quad (18)$$

In equations (17) all equilibrium quantities are evaluated at $s = 0$ ($r = r_A$), and

$$\Delta = \frac{d}{dr} (\omega^2 - \omega_A^2). \quad (19)$$

The fact that the right hand members of equation (17) have the common factor $C_A(s)$ which is a linear combination of $\xi_r$ and $P'$ is a key point in the present analysis. It suggests that we should obtain a differential equation for $C_A(s)$. This is readily done by taking the appropriate linear combination of equations (17):

$$s\frac{dC_A(s)}{ds} = 0. \quad (20)$$
Since the large solutions of $\xi_r$ and $P'$ have to be continuous, equation (20) implies that

$$\frac{C_A(s)}{\mu r} = \frac{2 f B B_\phi B_z \xi_r}{\mu r} = \text{constant}. \quad (21)$$

Condition (21) is the fundamental conservation law at the Alfvén resonance point (Sakurai et al., 1991).

The solutions for $\xi_r$ and $P'$ (equation (14)) can now be rewritten as

$$\xi_r(s) = \frac{g_B}{\rho B^2 \Delta} \ln(\lambda s) + \begin{cases} \xi_r(s) & s < 0 \\ \xi_r(s) & s > 0, \end{cases}$$

$$P'(s) = \frac{2 f B B_\phi B_z}{\mu r \rho B^2 \Delta} \ln(\lambda s) + \begin{cases} P'(s) & s < 0 \\ P'(s) & s > 0, \end{cases} \quad (22)$$

The jumps in $\xi_r$ and $P'$ are due to dissipative effects and will be specified later.

Let us now return to the conservation law at the Alfvén resonance point. In an equilibrium with a straight magnetic field ($B_\phi = 0$), the conservation law reduces to

$$[P'] = 0. \quad (23)$$

The solution (22) for $\xi_r$ remains unchanged, $P'$ can be considered to be constant across the point of resonance.

For an equilibrium with a curved magnetic field we can use equation (5) to rewrite the conservation law in terms of $\xi_\perp$:

$$s \xi_\perp = i \frac{C_A}{\rho B \Delta}. \quad (24)$$

Equation (24) expresses that $s \xi_\perp$ is constant across the resonant layer, or that $\xi_\perp$ has a $1/s$-singularity and a $\delta(s)$ contribution which dominate the $\ln |s|$ singularity and the jump found for $\xi_r$ and $P'$. The dominant singularities in the solution reside in the components in the magnetic surfaces and perpendicular to the magnetic field lines (see also Goedbloed, 1983). The dominant dynamics of the wave is contained in $\xi_\perp$ and the wave is polarized in the magnetic surfaces perpendicular to the magnetic field lines. The nonuniform plasma supports an Alfvén wave that is confined to the magnetic surface where the dispersion relation for Alfvén waves in a uniform plasma is locally satisfied. The confinement of the Alfvén wave is not absolute. The Alfvén wave is linked to the outside world as it is coupled to a wave with components normal to the magnetic surfaces. From a dynamic point of view we can consider the wave as an almost pure Alfvén wave confined to its resonant magnetic surface and polarized perpendicular to the magnetic field lines. From the point of view of the energetics $\xi_r$, the component normal to the magnetic surfaces, is essential since it is this quantity which provides the unidirectional transfer of energy to the resonant surface.
4. Linear dissipative MHD

Let us now see how the singular solutions obtained in ideal MHD are modified when we include dissipative effects in the equations. For the present purpose it suffices to consider non-zero electrical resistivity since this removes the singularity present in the ideal equations. The effects of dissipation are generally small and only important in the vicinity of the ideal resonance point. As Sakurai et al. (1991a) we retain in the dissipative terms only the \( r \)-derivatives of the perturbed quantities while we neglect those of the equilibrium quantities. A straightforward calculation then leads to the following set of two differential equations of third order:

\[
\begin{aligned}
D_\eta \frac{d(r \xi_r)}{dr} &= C_1 r \xi_r - C_2 r P', \\
D_\eta \frac{dP'}{dr} &= C_3 r \xi_r - C_1 P',
\end{aligned}
\]  

(25)

where \( D_\eta \) is the differential operator given by

\[
\begin{aligned}
D_\eta &= \rho(c^2 + v_A^2)(\omega_F^2 - \omega_A^2)(\omega^2 - \omega_C^2), \\
\omega_F^2 &= \omega^2(1 - i \frac{\eta}{\omega} \frac{d^2}{dt^2}).
\end{aligned}
\]  

(26)

Equations (25) are formally the same as equations (3), but the coefficient function \( D \) is replaced by the differential operator \( D_\eta \). The singularities are removed from the equations, but the order of the set of differential equations is raised from 2 (in ideal MHD) to 6 (in non-ideal MHD), and in addition the coefficient of the derivative of highest order is proportional to \( \eta \). Since dissipation is only important close to the Alfvén resonance point, we simplify the equations (25) by using Taylor expansions in the same way as we did for obtaining the ideal equations (17):

\[
\begin{aligned}
(s\Delta - i\omega \eta \frac{d^2}{ds^2}) \frac{d\xi_r}{ds} &= \frac{q_B}{\rho B^2} C_A(s), \\
(s\Delta - i\omega \eta \frac{d^2}{ds^2}) \frac{dP'}{ds} &= \frac{2f_B B_s B_z}{\mu r \rho B^2} C_A(s),
\end{aligned}
\]  

(27)

where \( C_A(s) \) is again given by equation (18). As in equations (20) all equilibrium quantities are evaluated at \( s = 0 \) (\( r = r_A \)).

Again we have the common factor \( C_A(s) \) in the right hand sides of equations (27). Goossens et al. (1994) point out that it is straightforward to obtain a differential equation for \( C_A(s) \) in dissipative MHD by taking a linear combination of equations (27):

\[
(s\Delta - i\omega \eta \frac{d^2}{ds^2}) \frac{dC_A(s)}{ds} = 0.
\]  

(28)
Goossens et al. (1994) also find that the dissipative equivalent of equation (24) is

\[
(s \Delta - i \omega \eta \frac{d^2}{ds^2}) \xi_\perp = i \frac{C_A(s)}{\rho B}.
\]  

(29)

The equations (27-29) have no singularities at \( s = 0 \) in contrast to their ideal counterparts.

Dissipation is important when the terms \( s \Delta \) and \( \omega \eta \frac{d^2}{ds^2} \) in the left hand sides of equations (27-29) are comparable. This results in a dissipative layer with a thickness measured by the quantity \( \delta_A \)

\[
\delta_A = \left( \frac{\omega \eta}{| \Delta |} \right)^{1/3}.
\]  

(30)

The thickness of the dissipative layer therefore scales as \( (\eta / | \omega' |)^{1/3} \), a result already obtained by Kappraff and Tataronis (1977) and Hollweg and Yang (1988) and numerically verified by Poedts et al. (1990a). In view of the very large values of the magnetic Reynolds number in the solar corona we have that

\[
\frac{s_A}{\delta_A} \gg 1.
\]  

(31)

Inequality (31) is important for the present discussion. It implies that the interval of validity of the simplified versions of the dissipative MHD equations embraces the dissipative layer and in addition contains two overlap regions to the left and the right of the dissipative layer where ideal MHD is valid. As in Sakurai et al. (1991) we now introduce a new scaled variable

\[
\tau = \frac{s}{\delta_A}
\]  

(32)

which is of order 1 in the dissipative layer, but in view of inequality (31) \( \lim s \to s_A \) corresponds to \( \lim \tau \to \infty \).

With this new variable the equations (27-29) take the form

\[
\begin{align*}
\left\{ \frac{d^2}{d\tau^2} + i \text{sign}(\Delta) \tau \right\} \frac{d\xi}{d\tau} &= i \frac{g B}{\rho B^2 | \Delta |} C_A, \\
\left\{ \frac{d^2}{d\tau^2} + i \text{sign}(\Delta) \tau \right\} \frac{dP}{d\tau} &= i \frac{2fB_B \phi B_z}{\rho B^2 \mu \tau | \Delta |} C_A, \\
\left\{ \frac{d^2}{d\tau^2} + i \text{sign}(\Delta) \tau \right\} \frac{dC_A}{d\tau} &= 0, \\
\left\{ \frac{d^2}{d\tau^2} + i \text{sign}(\Delta) \tau \right\} \xi_\perp &= \frac{-C_A}{\delta_A | \Delta | \rho B}.
\end{align*}
\]  

(33)
Sakurai et al. (1991a) did not obtain the differential equations (28) and (33c) for $C_A$ in dissipative MHD. They focussed on the dissipative equations for $\xi_r$ and $P'$ in an attempt to find the jumps in these quantities across the dissipative layer. They assumed that the ideal conservation law (21) remains valid in dissipative MHD. This assumption implies that the right hand sides of equations (33ab) are constant. Sakurai et al. (1991a) then obtained solutions of the dissipative equations (33ab) in terms of an integral of Hankel functions of order 1/3 of a complex argument. An asymptotic expansion of this solution enabled them to obtain the jump in $\xi_r$.

Goossens et al. (1994) use a more elegant method for obtaining the solutions to equations (33). Inspired by the techniques used by Boris (1968) and Mok and Einaudi (1985) for incompressible perturbations they look for solutions in integral form.

First they show that $C_A$ is also a constant in dissipative MHD for $|s| \leq s_A$. This proves that the ideal conservation law continues to be a conservation law in dissipative MHD and puts the assumption by Sakurai et al. (1991a) on firm grounds. Then they obtain the dissipative solutions for $\xi_r$, $P'$, and $\xi_\perp$ that remain finite for $\tau \to \infty$ as:

\[
\begin{cases}
\xi_r = -\frac{g_B C_A}{\rho B^2 \Delta} G(\tau), \\
P' = -\frac{2f_B B_\phi B_z C_A}{\rho B^2 \mu r \Delta} G(\tau), \\
\xi_\perp = \frac{-C_A}{\delta_A |\Delta| \rho B} F(\tau),
\end{cases}
\]

where

\[
\begin{cases}
F(\tau) = \int_0^\infty \exp(iu\tau \text{sign}(|\Delta|) - u^3/3) du, \\
G(\tau) = \int_0^\infty e^{-u^3/3} \frac{\exp(iu\tau \text{sign}(\Delta) - 1)}{u} du.
\end{cases}
\]

For $|\tau| \leq 1$ Maclaurin expansions of (34-35) give regular solutions in the dissipative layer. Asymptotic expansions of (35) for $\tau \to \pm \infty$ show that

\[
\begin{cases}
\text{Im}(G(\tau)) \to \pm \text{sign}(\Delta) \pi/2 \\
\text{Re}(G(\tau)) \to - \log(|\tau|) \\
F(\tau) \to \frac{i \text{sign}(\Delta)}{\tau}
\end{cases}
\]
These results provide us with the asymptotic behaviour of $\xi_r$, $P'$, and $\xi_\perp$ in the overlap regions where ideal MHD is also valid. In particular they allow us to obtain the jumps in $\xi_r$ and $P'$:

$$
\begin{align*}
\left[\xi_r\right] &= -i\pi \frac{g_B C_A}{\rho B^2 |\Delta|} \\
\left[P'\right] &= -i\pi \frac{2f_B B_\theta B_z C_A}{\rho B^2 \mu r |\Delta|}
\end{align*}
$$

(37)

These jumps and the conservation law were first derived by Sakurai et al. (1991a). An important property of resonant Alfvén wave heating is that the jumps are independent of $\eta$. This implies that the amount of absorbed wave energy is also independent of $\eta$. In addition Goossens et al. (1994) show that

$$
\eta \to 0 \Rightarrow \xi_\perp \to \frac{C_A}{\rho B |\Delta|} \{\delta(s) + i\text{sign}(\Delta) PV(1/s)\}
$$

(38)

in agreement with equation (24). These analytical results provide us with the spatial solutions in the dissipative layer and in the two overlap regions where ideal MHD is valid. As such to enable us to understand the basic physics of resonant Alfvén wave heating. They also help us with the interpretation of the results of large-scale numerical simulations. Finally the jump conditions and the conservation law make it possible to determine the absorption of Alfvén waves without having to solve the dissipative MHD equations. This procedure was used by Sakurai et al. (1991b) for studying the absorption of acoustic oscillations in sunspots. It was generalized to stationary equilibrium states by Goossens et al. (1992). Goossens and Hollweg (1993) used this scheme to obtain conditions for maximal and total absorption and to explain the variation of the spatial solutions with frequency.

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