CORRESPONDENCE

To the Editors of 'The Observatory'

On Scintillation Obfuscation

In two letters to *The Observatory*, Young\(^1\)\(^2\) complains that the formula for scintillation noise:

\[
S = 0.09 d^{-7/8} X^{-1/8} \exp \left( -h/h_0 \right) \tau^{-1/2}
\]  

(1)

has been misattributed to him both by Gilliland & Brown\(^3\) and in the ESA report on the Phase-A Study of *PRISMA*\(^4\). In addition, Young claims\(^2\) that the formula is erroneous, and that it has lent spurious support to an argument for the necessity of carrying out asteroseismic photometry from space. As members of the scientific team investigating the project *STARS*, a successor to *PRISMA*, we write to emphasize that, contrary to the impression one might have been left with after reading Young’s letters\(^1\)\(^2\), equation (1) is approximately correct; it is simply the appellation that, strictly speaking, is erroneous.

Equation (1) is an approximation to the r.m.s. fluctuation \(S\) in the intensity of light from a steady star observed over a continuous time interval \(\tau\), measured in units of the mean intensity. In that equation, \(d\) is the telescope diameter in cm, \(X\) is the airmass (the mass of air per unit area along the line of sight, in units of the mass per unit area measured vertically upwards; it is approximately \(sec\ Z\), where \(Z\) is the angle between the direction of observation and the zenith), \(h\) is the altitude of the telescope above sea level and \(h_0\) is the scale height of the r.m.s. atmospheric refractive-index fluctuations; its value: \(h_0 \approx 8\) km, is approximately the same as the density scale height. The time interval \(\tau\) is measured in seconds. Equation (1) is an explicit example of the generic equation

\[
S = A d^{-7/8} X^{p} \exp \left( -h/h_0 \right) \tau^{-1/2}
\]

(2)

which, for a continuous interval of observation, is equivalent to Young's\(^5\) equation:

\[
S = S_0 d^{-7/8} X^{p} \exp \left( -h/h_0 \right) (\Delta f)^{1/2}
\]

(3)

where \(A\), \(S_0\) and \(p\) are constants, and \(\Delta f\) is the characteristic range of (cyclic) frequency \(f\) accessible from the observations. Lest there be further misunderstanding, we point out immediately that we have not written equations (1) and (3) in exactly the form in which they originally appeared; instead they have been reduced to a common notation. However, they are precisely equivalent to the equations published.

In recognition of Young's extensive contributions to the testing of Reiger's\(^6\) meteorological theory of scintillation and to the application of that theory to astronomical photometric observations, Gilliland & Brown\(^3\) and Appourchaux et al.\(^4\) simply attributed formula (1) to what they believed was the first of Young's papers\(^7\) on the subject. Unfortunately, that attribution was careless, for the formula quoted in Young's paper\(^7\) is essentially equation (3) with values of \(S_0\) and \(p\) that are not quite consistent with equation (1). To be specific, Young explicitly has \(p\) replaced by \(\frac{3}{2}\), whereas the power of \(X\) in equation (1) is closer to the value \(\frac{1}{2}\) suggested by Young in a later paper\(^4\). Moreover, Young points out that \(S_0 = 0.09\) 'appears to be 'a typical value' (when \(p = \frac{3}{2}\)), which, bearing in mind the transformation between \(\Delta f\) and \(\tau\) which we address below,
is also not in precise accord with equation (1). However, since the discrepancy in the resultant values of $S$ is less than the substantial variation of the scintillation arising from day-to-day and site-to-site variations in atmospheric conditions, the distinction between the formulae is hardly significant.

We address separately in the next two paragraphs the values of the factors $A$ and $S_0$ and the value of the exponent $p$. But before doing so, we make our most important point: whereas Young$^2$ simply quotes a ‘typical value’ of $S_0$, Gilliland & Brown$^3$ provide a more detailed calibration of $A$ from observations. Moreover, additional observations$^6,9$, which were available to the PRISMA Science Team prior to publication, add further weight to that calibration. When correctly interpreted, it provides a more reliable estimate of $S_0$, which is somewhat greater than Young’s typical value. As Young$^1$ points out, and as we recommend too, it is that estimate, rather than Young’s, that should be used for estimating actual scintillation noise.

As a prelude to discussing the values of $S_0$ and $A$, we must first address the relationship between them. This depends on the relation between $\Delta f$ and $\tau$. Unfortunately ‘bandwidth’ is not a uniquely defined concept, as one quickly realizes from comparing standard texts on Fourier transforms and power spectra$^9,10,11$. Consequently, since Young$^2$ did not define $\Delta f$ explicitly in his original paper, it is perhaps not very surprising for someone else to have adopted a different definition, particularly when that difference is not material to the scientific inference to be drawn, as is the case here. Indeed, Young himself has been confused by this matter: in the publication$^5$ in which he suggested taking $p = 1 \cdot 75$, he quoted an incorrect formula for $\Delta f$, apparently$^1$ as a result of having misunderstood Robben’s analysis$^{12}$ of noise power spectral density. Nevertheless, it appears that Young$^2$ surmised correctly that Gilliland & Brown$^3$, in relating $S_0$ to $A$ for interpreting Young’s formula, erroneously assumed $\Delta f = \tau^{-1}$ in their conversion of power spectral density to power per inverse observation time$^9$. The definition of $\Delta f$ appropriate to Young’s analysis is the ‘equivalent width’ adopted, for example, by Bracewell$^{10}$. That definition satisfies $\Delta f = \frac{1}{2\tau^{-1}}$ for a continuous observation. Therefore, the correct correspondence between equations (2) and (3) is obtained with the relation: $A = S_0/\sqrt{2}$. The outcome of Gilliland & Brown’s$^3$ calibrations of eqtn. (2), with $p = 1 \cdot 8$, is $A = 0 \cdot 07$, and is independent of that relation. Moreover, it is consistent with the results of subsequent more extensive observations of M67 carried out from a variety of sites$^9$.

We now address the exponent $p$. As a result of advection of the atmospheric turbulence by the wind, the simple power-law dependence of $S$ on the airmass $X$ is inexact. Instead$^{13}$, $S$ is essentially proportional to $W = X^p \left[1 + (X^2 - 1) \sin^2 \theta \right]^{1/2}$, where $\theta$ is the difference between the azimuthal direction of the wind (assumed to be blowing horizontally) and the azimuth of the observation. When $\theta = \pi/2$ or when $X \sin \theta \gg 1$, $W \propto X^p$ with $p = \frac{1}{2}$. This proportionality is obtained also if advection is ignored, which is the origin of Young’s original power law$^7$. But when $\sin \theta \ll (X^2 - 1)^{-1/2}$, $p = 2$. Young offers $p = 1 \cdot 75$ as a compromise$^2$; it provides an approximation to the $\theta$-average of $W$ for ‘moderate’ values of $X$. In the first of his letters$^1$, Young points out that the value of $p$ in equation (1) is $1 \cdot 75$ rounded up, whereas he considers it preferable to round it down. We point out that there is hardly a difference between the two procedures. Not only is there hardly a difference between the values of $X^p$ for moderate values of $X$, but the dependence on $p$ of the calibrated values of $A(p)$ actually cancels much of the variation with $p$ of $X^p$. (No such dependence
on \( p \) of typical values of \( S_0 \) is recognized by Young.\(^1\) For example, the r.m.s. difference between the values given by formula (2) calibrated with \( p = 1.7 \) and \( p = 1.8 \) over airmasses up to 2 is well below \( 1\% \), which is certainly substantially less than the uncertainty in the calibration.

We sympathize with Young\(^2\) regarding the general principle of misattributing formulae, be they wrong or right. In particular, we accept that equation (1) differs somewhat from any of the formulae that Young has published; those of us who are responsible for the careless misattribution apologize, and undertake not to repeat the infelicity. Perhaps Young, who misquoted equation (1) in his letter of complaint,\(^1\) will also take more care. For the future, we commend formula (2) with \( p = 1.8 \) and \( A = 0.07 \) as being preferable not only to Young’s formulae but also to formula (1). As pointed out by Gilliland et al.\(^3\), it implies that scintillation noise power is somewhat greater than Young’s estimate (typically by some \( 50\% \) or more for asteroseismic observations), rather than being less, as has previously been suspected.

Yours faithfully,

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References