Seismic consequence of the Shoemaker–Levy impact

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ABSTRACT
Comet Shoemaker–Levy 9 is about to collide with Jupiter. The impacts will be on the far side. However, seismic disturbances induced by momentum transfer will propagate through the planet to cause waves on the near-side surface, which will begin to concentrate near the antipodes about 1.9 h after each impact. Unfortunately, the amplitudes of the waves are likely to be insufficient to provide an observable diagnostic of the internal structure of the planet.

Key words: waves – comets: individual: Shoemaker–Levy 9 – planets and satellites: individual: Jupiter.

1 INTRODUCTION
Comet Shoemaker–Levy 9 was apparently tidally disrupted in 1992 July, when it passed within $10^5$ km of Jupiter (Chapman 1993a). Now there orbits a string of 21 major fragments, together with lesser debris, which will fall on to the far side of Jupiter between 1994 July 16 and 22 (Beaty & Levy 1994). Most discussion of the likely consequences of the imminent impact has concentrated on radiation that will be emitted by the splash (Sekanina 1993; Chapman 1993b). In addition, however, seismic waves will be generated which, as a result of both their intrinsic propagation and their advection by Jupiter’s rotation, will disturb the visible surface on the near side. It is of obvious interest to consider the seismic response because, if the amplitude of the normal modes of acoustic oscillation excited by the event were to be great enough to permit precise frequency measurement, useful information about the interior structure of Jupiter could be obtained. In particular, it would be possible to detect the abrupt variations in the density stratification (Provost, Mosser & Berthomieu 1993; Lee 1993; Gough & Sekii 1994) resulting from both the variation of chemical composition and the molecular–metallic hydrogen phase transition (Chabrier et al. 1992). Lee & Van Horn (1994) have already computed some of the gravest modes, and have suggested that their amplitudes will indeed be great enough for the frequencies to be measured. The conclusion of this paper is that that will not be the case. Internal gravity (g) waves that are confined to the atmosphere have already been discussed by Harrington et al. (1994).

Sekanina (1993) has discussed the immediate outcome of the impact. A cometary fragment will be ablated as it falls into the planet, and even the largest fragments will have disintegrated completely within, or perhaps immediately beneath, the atmosphere. The characteristic time for dissipating energy and imparting momentum is of order 1 s. This is to be compared with the shortest of the periods of acoustic modes that are contained within the planet, namely $\nu_c^{-1} = 5.2$ min, where $\nu_c$ is the cyclic acoustic cut-off frequency of the atmosphere. Therefore, on the scale of any globally significant seismic mode, each collision may be regarded as being an instantaneous impact at a single point on the Jovian surface.

The Mach number of a fragment as it enters the atmosphere will exceed 70. Consequently a strong shock will be formed, which will propagate away from the point of impact. Much of the upwardly propagating portion of the shock strengthens, especially near the vertical, and ejects part of the atmosphere into outer space (Korobeinikov 1971). The downwardly propagating portion, however, weakens. Indeed, at a depth of about $10^{-3}$ Jovian radii and beyond, linearized acoustics provides an adequate description of the flow. This depth is rather shallower than the position of the highest node of any acoustic oscillation that is contained within the planet. Moreover, the characteristic time of the shock to weaken is only about 1 s. Therefore the global acoustic reaction of Jupiter can adequately be represented as the linearized (adiabatic) seismic response of an instantaneous point impulse. I shall assume that the same is true of the surface gravity (f) modes.

2 INITIAL-VALUE PROBLEM
The equations governing the oscillations of a self-gravitating body (e.g. Unno et al. 1989) are solved as an initial-value problem. This can be conveniently carried out using the Laplace transform. For the purposes of this discussion it is quite adequate to ignore centrifugal distortion, and to
consider the planet as being spherically symmetrical. I also regard the internal angular velocity $\Omega$ to be uniform. Then, since the planet is adiabatically stratified (Chabrier et al. 1992), only $p$- and $f$-modes are excited to a substantial amplitude in the planetary interior. To a first approximation, these are simply advected by the rotation (Cowling & Newing 1949; Ledoux 1951), and the waves can therefore conveniently be described in a frame rotating with the planet, with respect to spherical polar coordinates $(r, \theta, \phi)$ whose pole $\theta = 0$ intersects the surface $r = R$ of the planet at the point of impact. The disturbance velocity $v$ at the surface resulting from the impact of a single cometary fragment at time $t = 0$ can then be represented as a sum of modes of order $n$, degree $l$ and azimuthal order $m$:

$$v(R, \theta, \phi, t) = (m_c/M) v_c \sum_{n,l} q_l E_n^{-1} \left[ W_{nl}^0 \cos \psi \right]$$

$$+ \beta_l u(\omega_n) W_{nl}^1 \sin \psi \right] f_{nl}(t; \theta),$$

where

$$W_{nl}^m = \left[ \begin{array}{c} Y_l^m, \quad u \frac{\partial Y_l^m}{\partial \theta}, \quad \frac{u}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} \end{array} \right]$$

is the surface velocity of a mode, normalized to have unit rms vertical component;

$$Y_l^m(\theta, \phi) = \left[ \frac{2(2l+1)(l-m)!}{(1 + m)!} \right]^{1/2} P_l^m(\cos \theta) \cos m\phi$$

is a normalized spherical harmonic, symmetric in $\phi$ about $\theta = 0$, where $P_l^m$ is an associated Legendre function. Also $q_l = (2l+1)/4$, $E_n$ is the inertia of the mode, defined such that the energy of the mode is $1/2 E_n$ where $M$ is the mass of Jupiter, and $m_c$ and $v_c$ are the mass and the speed of impact of the cometary fragment, whose motion has been taken to subtend an angle $\psi$ with the downward vertical and to lie in the plane $\phi = 0$. The functions $f_{nl}(t; \theta)$ are given by

$$f_{nl}(t; \theta) = \left[ \begin{array}{c} 0 \quad t < \tau(\theta), \quad \exp(-\omega_n t \cos \theta) \quad t > \tau(\theta), \end{array} \right]$$

where $\tau(\theta)$ is the propagation time (at the sound speed $c$) along a direct ray path from the point $I$ of impact to a point $P$ on the surface subtending an angle $\theta$ with $I$ about the origin (see Fig. 1), and $\omega_n$ and $\eta_n$ are the frequency and damping rate of the mode. For modes whose frequencies $\omega_n$ do not exceed the acoustic cut-off frequency $\omega_c = 2\pi v_c$ of the (isothermal) atmosphere, $\eta_n/\omega_n < 1$. Otherwise $\eta_n = \pi \left( c^{-1} \cos \theta \right)^{-1}$, the integral being taken from the centre of the planet to approximately the base of the atmosphere. Finally, the coefficients $\beta_l$ are given by

$$\beta_l = -\frac{2}{3} q_l (l+1)^{-1} \frac{dP_l^j}{d\theta} \bigg|_{\theta=0};$$

they take the values $\beta_1 = 1$, $\beta_2 = 3$, $\beta_3 = (l-1)^{-1} (l+1)/(l-2)$ when $l \geq 3$ is odd, and $\beta_l = (l-2)^{-1} (l^2 - 2l^2 + 2l^2 - 2)$ when $l \geq 4$ is even.

For an approximate evaluation of this solution, one can represent the structure of Jupiter's interior by two superposed adiabatically stratified polytropes of index unity matched at the phase transition. Except in the core, this is a good representation (Chabrier et al. 1992), and it provides an adequate description of even the deeply penetrating low-degree modes, because the core is acoustically quite small. The atmosphere can be approximated by a plane-parallel layer of perfect gas. Then the measure $u(\omega_n)$ of the surface horizontal velocity of a normal $p$-mode in the atmosphere is given, for $\omega < \omega_c$ by

$$u(\omega) = \frac{\omega_0}{\omega_c} \chi^{-2} \left[ 1 - \frac{1}{2} \left( \frac{\omega}{\omega_c} \right)^{-2} \right],$$

where $\omega_0^2 = GM/R^3$, $G$ being the gravitational constant, $\chi = 1.4$ is the adiabatic exponent ($\partial \ln p/\partial \ln \rho$) in the atmosphere, and where $\chi = \omega/\omega_c$. The inertia can be evaluated from the asymptotic eigenfunction, valid for $\omega/\omega_c \gg 1$, which can most conveniently be obtained by matching the asymptotic representation (e.g. Unno et al. 1989; Gough 1993) in the upper layers of the interior acoustic cavity with the eigenfunctions of a plane-parallel polytrope (e.g. Gough 1990). The frequencies then satisfy Duval's (1982) law, which was first discovered empirically for the Sun:

$$\pi(n + \alpha) / \omega = F(w),$$

where $w = \omega/\omega_c$ and $\alpha$ is half the polytropic index; the inertia is then given by

$$E_c = \frac{\omega_0^3}{\omega} \left( 1 - \frac{d \ln F}{d \ln w} \right) \omega_c F.$$

This approximation is valid for all but the low-degree modes of lowest order. For the $f$-modes $(n = 0)$, $u = l(l+1)$ and $E_c = \omega_0^3 l(l+1) = \pi^2 l^2 = \pi^2 (\omega_0/\omega)^2$. Strictly speaking, the sum (1) should also include the inertial modes, which exist by virtue
of the Jovian rotation. These modes, however, have a rather longer response time, comparable with the period of rotation of the planet. They also have somewhat greater inertia. They therefore make only a relatively minor contribution to the seismic response.

3 DESCRIPTION OF THE SOLUTION

The solution represented by equations (1)-(8) can be described as follows. Viewed from a frame rotating with Jupiter, a disturbance propagates across the Jovian surface as rings expanding from the point of impact, in a manner similar to the waves produced by a stone thrown into a pond. The angle $\theta(t)$ subtended by the front of the wave from the point of impact is given by $t = \tau(\theta)$, and is plotted in Fig. 2. Formally, the solution implies that the waves will subsequently converge at the antipode, $\theta = \pi/2$. They will then return to the point of impact, and spread away from it again. The convergences will not mirror the delta-function initial condition at impact, however, because waves with different horizontal wavenumbers propagate across the surface at different speeds, and therefore arrive at the antipodes at different times: regarded as a two-dimensional surface phenomenon, the waves are dispersive. Moreover, the waves do not converge exactly at the same point. One reason for this is that they are not purely advected by the rotation; Coriolis forces cause each component of a given degree $l$ to precess retrogressively about the axis of rotation (Cowling & Newing 1949; Ledoux 1951) with angular velocity $C_0\Omega$, where $C_0\Omega$ varies with $n$ and $l$. Typically, $C_0\Omega \sim 10^{-2}$. Perhaps more significant, however, is the differential advection and distortion of the waves by the non-uniformity in $\Omega$. A substantial radial variation in $\Omega$, if it exists, would cause differential precession in excess of the Coriolis effect (e.g. Gough 1993). Voyager images have revealed (Ingersoll 1990) zonal winds in the atmosphere with speeds up to 100 m s$^{-1}$. If those values are typical of latitudinally varying interior flows, then the zone of convergence could be spread over a solid angle $\sim 10^{-3}$ sr. For waves with $l > l_c = 50$, the diameter of the region of convergence would be comparable with or greater than their horizontal wavelengths. The magnitude of the velocity within the zone might, however, be estimated roughly by evaluating equations (1)-(8) with the coefficients for $l > l_c$ in the series (1) reduced by the factor $(E_0)^{-1/2}$ to take account of the displacement and distortion of the waves. An estimate of the disturbance at the antipodes is illustrated in Fig. 3, for which it was assumed that $\psi = \pi/4$. The time interval plotted is too short for modes with $l \geq l_c$ to contribute significantly, so it was not necessary to take the distortion into account.

4 JOVIAN DIAGNOSTIC CONSEQUENCE

A disappointing conclusion that can be drawn from the analysis is that the event appears to be too weak to improve our knowledge of the internal structure of Jupiter. According to equation (1), the surface velocity amplitude to which a low-degree mode with $\omega < \omega_0$ will be excited is $V = 14 q_2(\omega/\omega_0)^2 (m_1/M) v_c$. To obtain this result I used the low-degree limit of equation (7), namely $\omega \approx (n + l + \frac{1}{2}) \omega_0$ with $\omega_0 \approx 1.67 \omega_a$ implying $F(w) \approx \pi (\omega_0^{-1} - \frac{1}{2} w^{-1})$. To evaluate $E_0$ by equation (8) for the polytropic model. Given that $v_c \approx 59$ km s$^{-1}$, this may be rewritten as $V = 4.5 \times 10^{-2} m_{17} (\omega/\omega_a)^3$ cm s$^{-1}$, where $m_{17}$ is the mass of the cometary fragment in units of $10^{17}$ g. It is estimated that, even for the largest fragments, $m_{17}$ is unlikely to exceed unity (Chapman 1993b; Scotti & Melosh 1993; Sekanina 1993; Weaver et al. 1994). Therefore $V$ is likely to be substantially smaller than the background seismicity, which for some modes appears to exceed 1 m s$^{-1}$ (Mosser et al. 1993). The $f$-modes have somewhat larger amplitudes, but they carry little information about the Jovian interior. The temperature fluctuation per mode in the atmosphere is also very small. A precise prediction cannot be obtained from an adiabatic calculation, but the relative amplitude will certainly be no greater than $(1 - y^{-1})$ of the corresponding Eulerian pressure perturbation: $[\delta T/T] \lesssim (y - 1)gc^{-2} \omega^{-1} V$, where $g$ is the gravitational acceleration in the atmosphere. The value of
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this bound is \(3 \times 10^{-7} q_{l} m_{17}(\omega/\omega_{c})^{2}\) for \(p\)-modes, which is well below the level of detectability.

The rise in the amplitudes with increasing frequency results from a corresponding decline in the modal inertia \(E_{nl}\), which depends in part on the depth of the upper turning point of the mode. For low-frequency modes the comet's momentum is deposited well in the upper evanescent region, and consequently the coupling to the mode is small. The inertia declines also with increasing \(l\), because the depth of penetration diminishes. Consequently, the modes with greatest amplitude are those of high degree with frequencies just below the acoustic cut-off of the atmosphere. A typical low-degree mode gains only \(5.5 \times 10^{-9} m_{17}(\omega/\omega_{c})^{2}\) of the kinetic energy of the cometary fragment at the moment of impact. This result explicitly contradicts the assumption by Lee & Van Horn (1994) that a significant fraction of the total kinetic energy will appear in a grave mode.

5 COMETARY IMPACT ON THE SUN

Finally, it is interesting to consider the seismic response of the Sun to a cometary impact. Isaak (1981) has suggested that such impacts might be responsible for occasional temporary excitation of solar acoustic modes of low degree. For the event to be recognizable, the excess amplitudes would need to be at least comparable with the background seismicity. The response would be most evident at high frequency, where the velocities induced by the impact are relatively large, and the background seismicity relatively small. Taking a typical value of \(E_{nl}=10^{-8}\) from Christensen-Dalsgaard & Berthomieu [equation (8) cannot be used because it is valid only for a polytropic of index unity] for modes with frequencies \(\geq 3\) mHz, and an excess amplitude of \(2\) cm s\(^{-1}\), a cometary mass \(m_{c}=6 \times 10^{17}\) g is required, which is about 6 times the mass of Comet Halley, and which would dissipate in the Sun's outer layers about 10 times the energy of a large flare. Evidently, if any such comet has contributed to the oscillations that have been observed, it could not have been made of such refractory material as Comet Halley, because if it had been its tail would surely have been seen.

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