Progress in Meteor Science

Articles in this section have been formally refereed by at least one professional and one experienced, knowledgeable amateur meteor worker, and deal with global analyses of meteor data, methods for meteor observing and data reduction, observations with professional equipment, or theoretical studies.

Spatial Number Densities and Errors from Photographic Meteor Observations under Very High Activity

Luis Ramón Bellot Rubio

A procedure to compute meteoroid spatial number densities and flux densities from photographs is presented. It follows from the visual method of Koschack and Rendtel with slight changes. Some parameters are recomputed and hints are given on how to produce useful photographic observations. Finally, an analysis of the expected errors is performed. It turns out that the systematic error caused by the uncertainty in the exponent of the radiant zenithal distance correction has important effects on the accuracy of the flux density. Therefore, a detailed investigation of this topic is suggested.

1. Introduction

The recent possibility of enhanced Perseid activity in 1993–1994 and the expected return of the Leonid storm in 1998–1999 have led to a growing interest in the computation of spatial number densities from photographic observations.

Under very high activity, visual observations become more and more inaccurate because of the change in the perception coefficients and the reduction of the available time to note down the data for each meteor. The great advantage of the visual technique (i.e., the possibility of recording meteors down to magnitude +5 or +6) does not hold any longer since the observer cannot keep up with the activity. As a consequence, one has to restrict the count to meteors brighter than a certain magnitude threshold. The “effective” limiting magnitude for visual observations under storm conditions thus becomes comparable to that of photography, which allows us to benefit from the objectivity of the photographic method.

Photography offers a number of advantages. It is possible, for instance, to compute with great precision the area surveyed by the camera field at a given meteor level. Moreover, it may be assumed that the perception coefficients of the camera are constant and even equal to unity if we take into account the meteor limiting magnitude. The photographic technique has also some disadvantages, however. The most important of them are the restricted field of view (which in turn means that less meteors are recorded) and the impossibility of obtaining actual meteor magnitudes. The reason is that the geocentric velocities of some showers (Perseids, Leonids) are near the upper limit of 72 km/s and produce a high percentage of trained meteors [1,2] As the photograph does not separate the light of the meteor itself from the train light, there is a pollution that varies the photographic magnitude of the meteor. However, such an effect may be of secondary importance when calculating the population index, since the meteors just have to be grouped in intervals of one magnitude width and we may assume that a comparable portion of meteors in consecutive magnitude classes show trains [3].

Several parameters are required before spatial number densities, or flux densities, can be computed from photographs. This paper extends the visual method developed by Koschack and

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Rendtel [4] to the photographic case and analyzes in detail the expected error of the calculations.

2. Photographic meteor limiting magnitude and choice of camera field center

Obviously, the larger the number of recorded meteors is, the more accurate the results are. One of the factors that influences the number of recorded meteors is the photographic meteor limiting magnitude, \( plm \). This quantity depends on the camera set-up, the film type and, more importantly, on the meteor angular velocity. Two meteors of the same visual magnitude have different photographic magnitudes if their angular velocities are different. The slower the meteor is, the brighter it appears on the photograph. The variation in apparent magnitude, due to a difference of angular velocity, can be described by

\[
m_1 = m_2 - 2.5p \log \frac{\omega_2}{\omega_1},
\]

where \( m_i \) is the photographic magnitude of the meteor with angular velocity \( \omega_i \) \((i = 1, 2)\), and \( p \) is the Schwarzschild exponent accounting for the failure of the reciprocity law [5,6].

If we want to record as many meteors as possible, we should look for the sky area in which meteors have the slowest angular velocity. The angular velocity of a meteor depends on the altitude \( h \) of its starting point above the horizon and on the angular distance \( \xi \) from the starting point to the radiant [7]:

\[
\omega = \frac{v_\infty}{H} \sin \xi \sin h,
\]

where \( v_\infty \) is the pre-atmospheric velocity of the meteoroid and \( H \) the height of the meteor level.\(^1\) Clearly, the slowest angular velocities are obtained when \( \xi \approx 0 \), i.e., when the meteor appears close to the radiant. In principle, this area is the best choice to point the camera, since it allows the faintest meteors to be recorded and hence improves \( plm \).

One problem of a near-radiant camera field, however, is the identification and photometry of the very short or almost point-like meteor trails occurring there. It also has the great disadvantage of a non-constant photographic limiting magnitude over the field, because the angular speed of the shower meteors does vary strongly near the radiant. Table 1 summarizes the change of \( plm \) across the field of a standard 35 mm, \( f = 50 \) mm camera pointed to the radiant as a function of the radiant altitude \( h_R \). The data have been computed using the angular velocity of meteors at \( \xi = 5^\circ \) and \( \xi = 20^\circ \), and then applying equation \( 1 \) with \( p = 0.8 \). Such enormous differences have to be avoided to determine the number of meteors per magnitude range (which will be used afterwards to obtain the population index of the shower).

<table>
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It should be clear now that the most important criterion is the constancy of \( plm \) over the photograph, so the next step is to find the area of the sky in which the angular velocity of the shower meteors varies as little as possible. We follow the usual procedure and equate the partial derivatives of \( \omega \) in \( 2 \) to zero, which gives the solution \( \xi = 90^\circ \), \( h = 90^\circ \) \([8]\). These requirements cannot be satisfied except when the radiant lies on the horizon and the camera points to the zenith, but it is still possible to minimize the change of angular velocity.

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\(^1\) Notice that, if \( v_\infty \) is given in km/s and \( H \) is given in km, then \( \omega \) is given in rad/s.
First, the values $\xi$ and $h$ should be as large as the geometry allows. Thereto, the camera must be pointed $180^\circ$ away from the radiant in azimuth [8], i.e., $a_f = a_R \pm 180^\circ$. If $a_f = a_R \pm 180^\circ$, then the condition $h \approx 180^\circ - \xi - h_R$ holds, reducing the number of variables in equation (2). This reduction simplifies the calculation of $h_f$, the optimum altitude for the camera center, which turns out to depend on $h_R$ through

$$h_f = 90^\circ - \frac{h_R}{2}. \quad (3)$$

Table 2 lists some numerical values of $h_f$ in terms of the radiant altitude $h_R$. Since the angular velocity of the shower meteors varies little across the camera fields specified by Table 2, the change of the limiting magnitude due to different velocities is also very small (cfr. the third column of Table 2, where $\Delta plm$ has been calculated for a 35 mm, $f = 50$ mm camera). Unfortunately, the angular velocity of the shower meteors reaches its maximum in these fields, which reduces the limiting magnitude to a great extent. Table 3 shows the reduction of $plm$ with respect to a field centered at the radiant. In order to improve $plm$, high-speed films (e.g., ASA 3200) are strongly recommended. It goes without saying that the importance of a meteor storm deserves the best available film.

Table 2 – Optimum camera field elevation $h_f$ as a function of the radiant elevation $h_R$. The third column represents the change of the limiting magnitude $plm$ across the camera field when the longest edge of the photograph is parallel to the horizon. It has been computed by using the variation of the angular velocity of the shower meteors from the center to the edges of the photograph.

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Table 3 – Reduction of the photographic meteor limiting magnitude, $\Delta plm$, for a 35 mm, $f = 50$ mm camera centered at the elevations given in Table 2 with respect to a field pointing to the radiant.

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We do not know how to compute the photographic meteor limiting magnitude of a given exposure on theoretical grounds. Several equations have been tested and showed a good qualitative (but not quantitative) behavior [9]. The major drawback is that they usually ignore a number of relevant effects such as the presence of trains or the darkness of the sky. Meanwhile, simultaneous visual-photographic observations indicate that the typical photographic limiting magnitude for meteors ranges from roughly $+1$ to $+3$, depending on $\omega$ and the film speed [9]. These results suggest setting the reference photographic limiting magnitude as $+3.5$ (remember that the +3 magnitude class includes meteors from $+3.5$ to $+2.5$). Apart from being a reasonable value, this choice has a further advantage: it permits the comparison of spatial number densities obtained from photographs and from visual observations under very high activity.

\footnote{Note that Trigo [9] wrongly sets the optimum azimuth of the camera as $45^\circ$ away from the radiant azimuth. He obtains this result on the assumption that such a field has the highest probability of meteor appearances, but a little thought reveals that this is not true when the negatives are projected onto the meteor layer from the observer’s site.}
3. The projected area at the meteor level

Once the optimum camera field center has been defined, the procedure requires to know the area surveyed by the photograph at the meteor level. From now onwards, we assume a camera field of 2750 x 396, which corresponds to standard film dimensions of 24 mm x 36 mm in a camera with $f = 50$ mm. These values are obtained from the well-known formula $L = 2 \arctan(l/2f)$, where $l$ is the length of the negative in mm, and $L$ the corresponding angular aperture in degrees.

As in [4], the data are reduced to a standard area $A_{red}$ for which there is no extinction $\varepsilon$ and the distance to the observer is assumed to be 100 km. If $A_i$ represents the projected geometrical area of a small portion of the photograph at distance $d_i$ from the camera and extinction $\varepsilon_i$, the reduced area $A_{red}$ may be computed from

$$A_{red} = \sum_i A_i r_i \log_{10} \frac{100}{d_i} - \varepsilon_i,$$

where the index $i$ is such that all zones of the photograph enter the summation.

The details for the calculation of $A_{red}$ are explained in the Appendix. The numerical procedure assumes that the longest edge of the photograph is parallel to the horizon. Average extinction values from [10] have been used. We must carefully define the height $H$ of the meteor level. Any characteristic height of the meteor trajectory depends on velocity, mass and zenith angle (in decreasing order) [11, 12]. If we select the height $H_0$ of the beginning of the photographic trajectory as the height of the meteor layer, $H$ will also depend on $pim$. However, there are two reasons in favor of this choice. First, deriving an accurate relationship between the above-mentioned parameters is still impossible due to the lack of data on individual showers. Secondly, the dependence of $H_0$ on mass and zenith angle is much less important than its dependence on velocity (which is not true if we use, for instance, the height of maximum intensity). This way, only one height is required for each shower. The small variations of $H$ that might be present due to the slight dependence of $H_0$ on mass and zenith angle will be reflected in an increase of the standard deviation of $H$.

The best photographic compilation of heights for different showers is the survey of Jacchia et al. [11]. The limiting magnitude in that survey was about +2.5, close to the photographic limiting magnitude achievable nowadays by standard cameras equipped with high sensitivity films. Since the threshold of detection is similar in both cases, we may use the mean values of $H_0$ reported in [11] as the height $H$ of the meteor layer. It turns out that $H = (100 \pm 9)$ km for all meteors (including sporadics), whilst for Perseids $H$ increases up to $(114 \pm 3)$ km. The paper by Jacchia et al. does not contain enough data for Leonids, and thus we use the graphically reduced Super-Schmidt meteors of McCrosky and Posen [13]. Despite the poorer quality of these meteors, the heights given in [13] still constitute a valid approximation, yielding $H = (118 \pm 6)$ km for Leonids.

Figure 1 shows $A_{red}$ when $H = 100$ km. The qualitative behavior of $A_{red}$ as a function of the population index differs from the visual case (cfr. Figure 5 in [4]). The smaller the altitude of the camera field is, the larger the photographic reduced area becomes. The population index does not change this trend, except for very high $r$-values that never occur in practice. Consequently, we shall obtain more reliable results when the radiant is near the zenith, since the surveyed area will be larger (cameras pointing to smaller elevations).

Tables 4–6 give $A_{red}$ for $45^\circ \leq h_f \leq 90^\circ$ and $H = 100$ km, 114 km, and 128 km, respectively. The error $\Delta A_i$ when computing the geometrical areas $A_i$ can be ignored since its contribution to $\Delta A_{red}$ never exceeds 2 km$^2$. The uncertainty of the extinction values has not been taken into account as it depends on the sky conditions; however, it should be negligible for fields near the zenith. Moreover, small $r$-values (which are likely to occur during storms) reduce the influence of $\Delta \varepsilon$. The most important error is thus the error $\sigma_H$. Accordingly, we expect $\Delta A_{red} \approx 0.08 A_{red}$ for $H = (100 \pm 9)$ km, $\Delta A_{red} \approx 0.025 A_{red}$ for Perseids ($H = (114 \pm 3)$ km), and $\Delta A_{red} \approx 0.06 A_{red}$ for Leonids ($H = (118 \pm 6)$ km).

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Figure 1 – Reduced areas $A_{red}$ as a function of the population index $r$ and the camera field elevation $h_f$. The height of meteor appearances is assumed to be $H = 100$ km.

Table 4 – Reduced areas $A_{red}$ (km$^2$) as a function of $r$ and the camera field center elevation $h_f$ for $H = 100 \pm 9$ km. The maximum error $\Delta A_{red}$ is 8% of $A_{red}$.

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Table 5 - Reduced areas $A_{\text{red}}$ (km$^2$) as a function of $r$ and $h_f$ for $H = 114 \pm 3$ km (the Perseid case). The maximum error $\Delta A_{\text{red}}$ is 2.5% of $A_{\text{red}}$.

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Table 6 - Reduced areas $A_{\text{red}}$ (km$^2$) as a function of $r$ and $h_f$ for $H = 118 \pm 6$ km (the Leonid case). The maximum error $\Delta A_{\text{red}}$ is 6% of $A_{\text{red}}$.

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4. Calculation of the spatial number density

In complete analogy with the method developed by Koschack and Rendtel [4], the spatial number density of particles causing meteors of photographic absolute magnitude at least +3.5 may be obtained from

$$\rho(m \leq +3.5) = \frac{\text{ZHR}_o \times c(r)}{3600 \times A_{\text{red}}(r, h_f, H) \times v_\infty},$$

(5)

where \(\text{ZHR}_o\) is the observed photographic zenithal hourly rate and \(v_\infty\) the geocentric velocity of the shower meteors in km/s. The correction factor \(c\) was introduced to account for the loss of meteors due to perception, and is given by

$$c(r) = \frac{\sum_{m = -\infty}^{+3} r^m}{\sum_{m = -\infty}^{+3} r^m \times p_{ph}(3.5 - m)},$$

where \(p_{ph}(\Delta m)\) represents the probability of perception of a meteor with \(\Delta m = plm - m\). When the favorable camera field center is selected, the photograph records any meteor brighter than \(plm\), irrespective of its position on the field. The fact that \(\Delta plm\) is very small across the photograph ensures the validity of the above statement. As a consequence, \(p_{ph}(plm - m) \equiv 1\) while \(m \leq plm\), and thus \(c(r) = 1\) for any value of the population index.

The photographic zenithal hourly rate can be computed from

$$\text{ZHR}_o = \frac{N}{T} \times \sin^{-1} h_R \times r^{3.5 - plm},$$

(6)

where \(N\) represents the number of recorded meteors whose beginning points are inside the camera field boundaries and \(T\) the exposure time of the photograph. Note that the reference photographic meteor limiting magnitude has already been used in \(c(r)\) and equation (6).

We further define the flux density \(Q\), related to the spatial number density through

$$Q(m \leq 3.5) = 3600 \times v_\infty \times \rho(m \leq 3.5).$$

(7)

The expressions for \(Q(M \geq M_0)\) and \(\rho(M \geq M_0)\), the flux density and the spatial number density of particles with masses larger than a certain value \(M_0\), respectively, can be taken from [4]. Bearing in mind equations (5) and (6), it is possible to rewrite equation (7) as

$$Q(m \leq 3.5) = \frac{N}{T} \times \sin^{-1} h_R \times \frac{r^{3.5 - plm}}{A_{\text{red}}(r, h_f, H)}.$$

(8)

There are two different ways to estimate \(Q\) from a given set of individual measurements \(\text{ZHR}_i\). The first one is averaging the values \(\text{ZHR}_i\) and obtaining a single value \(\text{ZHR}_{av}\) which allows the calculation of \(Q\). The second method is computing one individual value \(Q_i\) from each individual measurement \(\text{ZHR}_i\) and then averaging these estimates \(\bar{Q}\). Since the flux density depends on \(h_f\) through the reduced area \(A_{\text{red}}\), the first procedure cannot be applied. Therefore, the best approximation to \(Q\) is the average, \(\bar{Q}\), of the individual values \(Q_i\). Now we deal with the expected error of \(\bar{Q}\), namely \(\sigma_{\bar{Q}}\).

Let \(\sigma_i\) be the uncertainty when determining \(Q_i\). The error \(\sigma_i\) comes from the uncertainties associated to each of the variables entering equation (8). The errors \(\sigma_N, \sigma_T, \sigma_{h_R}, \sigma_r, \sigma_{plm}, \sigma_{h_f}, \) and \(\sigma_H\) are random, independent errors. Accordingly, the global uncertainty they produce in \(Q_i\) is found using the method of the quadratic sums. We are not yet finished, however, since the radiant zenithal-distance correction \(C_R \equiv \sin^{-a} h_R\) adds a further error to \(\sigma_i\). If only a pure geometrical correction is performed, the exponent \(a\) turns out to be +1.00. It has been proved however that the decrease of the meteor brightness for large values of \(h_R\) modifies \(a\) (Roggemaus, [14]). From observational data, Zvoláňková [15] found \(a = +1.47 \pm 0.11\), which agrees well with
the work of Roggenmann. Thus, the use of $a = +1.00$ leads to a systematic error in $Q_i$. The presence of a systematic error strongly affects the computation of $\sigma_Q$, but, once its value has been estimated, it can be handled with the techniques of the covariance matrix and the theory of error propagation [16]. Notice that there are no other systematic trends as the only possible source for them would be the limiting magnitude correction $C_{plm} = r^{3.5-\text{plm}}$ and the recorded range of magnitudes is small enough.

Now suppose that $Q_i$ is affected by a systematic error $S_i$ and a random error $\sigma_i'$. Since both errors are independent of each other, it follows from the central limit theorem that the total error $\sigma_i$ can be obtained by adding the two in quadrature, i.e.

$$\sigma_i = \sqrt{\sigma_i'^2 + S_i^2}.$$  

Hence, as far as the error of one individual estimate $Q_i$ is concerned, there are no changes from the method of quadratic sums when systematic errors have to be included. It is possible to approximate $S_i$ by

$$S_i \approx \left| \frac{\partial Q_i}{\partial a} \Delta a \right|,$$

where $\Delta a$ represents the probable uncertainty of the radiant zenithal distance correction exponent $a$. We may use $\Delta a \approx 0.47$ according [15]. From equation (8) one thus obtains

$$S_i \approx Q_i \Delta a \ln(\sin^{-1} h_{Ri}).$$  

(10)

The numerical values of $T_i$, $h_{Ri}$, and $h_{f}$ can be known with high accuracy, and so the errors $\sigma_T$, $\sigma_{h_{Ri}}$, and $\sigma_{h_{f}}$ produce negligible contributions to $\sigma_i'$. The other sources of error are $\sigma_N$, $\sigma_r$, $\sigma_{plm}$, and $\sigma_H$. Thus we have

$$\sigma_i' = \sqrt{\left( \frac{\partial Q_i}{\partial N_i} \sigma_{N_i} \right)^2 + \left( \frac{\partial Q_i}{\partial r_i} \sigma_{r_i} \right)^2 + \left( \frac{\partial Q_i}{\partial plm_i} \sigma_{plm_i} \right)^2 + \left( \frac{\partial Q_i}{\partial H_i} \sigma_{H_i} \right)^2}.$$  

(11)

From equation (8), one finds

$$\frac{\partial Q_i}{\partial N_i} = \frac{Q_i}{N_i};$$

$$\frac{\partial Q_i}{\partial r_i} = \frac{\partial Q_i}{\partial r_i} \left( \frac{3.5-\text{plm}}{A_{\text{red}i}} \right) = (3.5 - \text{plm}) \frac{Q_i}{r_i} + \frac{Q_i}{A_{\text{red}i}} \left| \frac{\partial A_{\text{red}i}}{\partial r_i} \right|;$$

$$\frac{\partial Q_i}{\partial plm_i} = Q_i \ln r_i;$$

$$\frac{\partial Q_i}{\partial H_i} \sigma_{H_i} = \frac{\partial Q_i}{\partial H_i} \frac{\partial A_{\text{red}i}}{\partial H_i} \sigma_{H_i} = -\frac{2Q_i}{A_{\text{red}i}} \varepsilon A_{\text{red}i} = -2Q_i \varepsilon,$$

where $\varepsilon = 0.08$ for $H = (100 \pm 9)$ km, $\varepsilon = 0.025$ for Perseids ($H = (114 \pm 3)$ km), and $\varepsilon = 0.06$ for Leonids ($H = (118 \pm 6)$ km). While the random fluctuations in the number of meteors $N$ can be accounted for by using a Poisson distribution, i.e., $\sigma_N = \sqrt{N}$, the errors $\sigma_r$ and $\sigma_{plm}$ must be obtained from global analyses and experiments, respectively. With regard to $\partial A_{\text{red}i}/\partial r_i$, the data given in Tables 4-6 should be sufficient. Combining the previous results and equation (9), the standard deviation $\sigma_i$ of the estimate $Q_i$ becomes

$$\sigma_i = Q_i \sqrt{\frac{1}{N_i} + \left( \frac{3.5-\text{plm}}{r_i} \right)^2 + \left( \frac{\partial A_{\text{red}i}}{\partial r_i} \right)^2} \left[ \sigma_{r_i}^2 + \sigma_{plm_i}^2 \ln^2 r_i + 4\varepsilon^2 + \left( \frac{S_i}{Q_i} \right)^2 \right].$$  

(12)
The flux density $Q(m \leq 3.5)$ must be computed as the weighted mean of the individual measurements $Q_i$, i.e.,
\[
\overline{Q} = \frac{\sum Q_i}{\sum \frac{1}{\sigma_i^2}}
\]
The values $Q_i$ are not mutually independent since they share a systematic error coming from the uncertainty of the zenithal radiant distance correction. As a consequence, the standard deviation $\sigma_{\overline{Q}}$ of $\overline{Q}$ is given by
\[
\sigma_{\overline{Q}}^2 = \sum_j \left( \frac{\partial \overline{Q}}{\partial Q_j} \sigma_j \right)^2 + 2 \sum_j \sum_{k \neq j} \frac{\partial \overline{Q}}{\partial Q_j} \frac{\partial \overline{Q}}{\partial Q_k} \text{cov}(Q_j, Q_k),
\]
where $\text{cov}(Q_j, Q_k)$ represents the covariance of $Q_j$ and $Q_k$, i.e., the degree of correlation between $Q_j$ and $Q_k$ introduced by $\Delta a$. The values $Q_j$ and $Q_k$ have systematic errors $S_j$ and $S_k$ and also random errors $\sigma_j$ and $\sigma_k$. This feature can be treated by considering $Q_j$ as having two parts, $Q_j^R$ with random error $\sigma_j^R$ and $Q_j^S$ with systematic error $S_j$. The same applies to $Q_k$. By this definition, $Q_j^R$ and $Q_k^R$ are independent of each other and of $Q_j^S$ and $Q_k^S$, whereas $Q_j^S$ and $Q_k^S$ are completely correlated. It can be shown [16] that $\text{cov}(Q_j, Q_k) = \text{cov}(Q_j^S, Q_k^S) = S_j^2S_k$ or, by virtue of equation (10),
\[
\text{cov}(Q_j, Q_k) = Q_jQ_k(\Delta a)^2 \ln(\sin^{-1} h_{Rj}) \ln(\sin^{-1} h_{Rk}).
\]
The partial derivatives entering equation (13) read
\[
\frac{\partial \overline{Q}}{\partial Q_j} = \frac{1}{\sum_i \frac{1}{\sigma_i^2}} \frac{1}{\sigma_j^2},
\]
whence
\[
\sum_j \left( \frac{\partial \overline{Q}}{\partial Q_j} \sigma_j \right)^2 = \frac{1}{\sum_i \frac{1}{\sigma_i^2}},
\]
and thus
\[
\sigma_{\overline{Q}}^2 = \frac{1}{\sum_i \frac{1}{\sigma_i^2}} \left( 1 + \frac{2(\Delta a)^2}{\sum_i \frac{1}{\sigma_i^2}} \sum_j \sum_{k \neq j} Q_jQ_k \frac{\ln(\sin^{-1} h_{Rj}) \ln(\sin^{-1} h_{Rk})}{\sigma_j^2\sigma_k^2} \right).
\]
If $\Delta a = 0$, i.e., if the error due to the uncertainty of the zenithal radiant distance is neglected, equation (14) provides the usual standard deviation of a weighted mean in which all the errors are independent. But in general $\Delta a \neq 0$. While the net effect of random errors decreases in size when the number of measurements increases, the effect of systematic errors does not fall off at all.

Equation (14) implies that, for a sufficiently large set of individual measurements $Q_i$, the error $\sigma_{\overline{Q}}$ is dominated by the systematic error that stems from $\Delta a$. The only way to improve $\sigma_{\overline{Q}}$ is reducing the uncertainty $\Delta a$. This will require more research work and a firm determination by the IMO to change the standard procedure if necessary. Meanwhile, the only possibility to keep $\sigma_{\overline{Q}}$ as low as possible is to observe with $h_R \approx 90^\circ$, since in that case the covariance of any pair of individual estimates $Q_j$ and $Q_k$ becomes nearly zero (cfr. equation (10)).

If spatial number densities are sought instead of flux densities, the above procedure still applies. The uncertainty $\sigma_{f}$ is then given by $\sigma_{f} = \sigma_{\overline{Q}}/3600v_\infty$, which comes directly from equation (7).
5. Conclusions

The method given above permits the determination of both spatial number densities and flux densities from photographic records. Only slight changes have had to be made to the visual procedure of Koschack and Renidel [4] to make it applicable to photography. The presence of a systematic error coming from the radiant zenithal distance correction must be investigated more thoroughly, since it also affects the analysis of visual data. The current IMO method does not treat systematic errors and, although the whole computations may look nice, the results can be a complete disaster if they are not properly accounted for.

Appendix

We devote this Appendix to the computation of $A_{\text{red}}(h_f)$. A rectangular coordinate system with origin at the camera lens is selected for further use. The $z$-axis points towards the zenith, while the $y$-axis is directed along the projection of the azimuth of the camera field center onto the perpendicular plane to the $z$-axis. In this reference frame, the meteor layer may be described by an spherical surface with equation

$$x^2 + y^2 + (z + 6371 \text{ km})^2 = (6371 \text{ km} + H)^2,$$  \hspace{1cm} (15)

where $H$ represents the height of meteor appearances.

Figure 2 – Diagram of a camera. The $y'$-axis represents the projection of the camera field center. Point $P$ on the negative is specified by angles $\alpha'$ and $\beta'$ or, equivalently, by vector $\overline{OB}$.

The aim is to calculate the geometrical projected area $A_1$ of a small zone of the photograph. Figure 2 shows a diagram of the camera. Any point $P$ on the negative is specified by two angles $\alpha'$ and $\beta'$ whose maximum values are 13°5 and 19°8 respectively. Both angles define the positional vector $\overline{OB}$ as

$$x' = R \sin \beta';$$
$$y' = R \cos \beta' \cos \alpha';$$
$$z' = R \cos \beta' \sin \alpha',$$  \hspace{1cm} (16)

where $R = |\overline{OB}|$.

Now suppose that the camera field center has an altitude $h_f$ above the horizon ($xy$-plane). This is equivalent to the rotation of the $x'$- and $y'$- axes around the $x'$-axis by an angle of $-h_f$ (see Figure 3, left). The coordinates of vector $\overline{OB}$ in the $xyz$-system are thus obtained via the matrix of the rotation, i.e.,
Figure 3 – Left: Position of the camera's coordinate system presented in Figure 2 with respect to the $xyz$-system when the camera field center is aimed at angle $h_f$ above the horizon. Right: Definition of angles $h$ and $\beta$ for vector $\overrightarrow{OB}$. Angle $h$ represents the elevation of point $P$ above the horizon ($xy$-plane).

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos h_f & -\sin h_f \\
  0 & \sin h_f & \cos h_f
\end{pmatrix} \begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix},
\]

whence

\[
x = R \sin \beta'; \\
y = R \cos \beta' \cos (\alpha' + h_f); \\
z = R \cos \beta' \sin (\alpha' + h_f).
\]

Writing $h' \equiv \alpha' + h_f$, the parametrical form ($p$ being the parameter) of the straight line defined by vector $\overrightarrow{OB}$ becomes

\[
\begin{align*}
x &= p \tan \beta' / \cos h' \\
y &= p \\
z &= p \tan h'.
\end{align*}
\]

It is now easy to solve equations (15) and (18), which gives the projection of point $P$ onto the meteor level. The whole photograph is sampled by letting $\alpha'$ run from $-13^\circ 5$ to $+13^\circ 5$ and $\beta'$ from $-19^\circ 8$ to $+19^\circ 8$. (Note the symmetry, which means that we only have to consider $0^\circ \leq \beta' \leq 19^\circ 8$.)

The last important parameter is the altitude $h$ of vector $\overrightarrow{OB}$ above the horizon, since it provides the extinction $\varepsilon$ associated to $P$. From Figure 3, right, and formulae (18), one obtains

\[
\begin{align*}
h &= \arctan(\tan h' / \cos \beta); \\
\beta &= \arctan \left( \frac{\tan \beta'}{\cos h'} \right).
\end{align*}
\]

The method to compute $A_{red}$ for a given camera center altitude $h_f$ works then as follows:

1. The negative is discretized on a mesh of points separated by $0^\circ 02$. Four of these points define a small square that will be projected onto the meteor level to obtain the corresponding geometrical area $A_i$.

2. The projection of the vertices of each square using equations (15) and (18) produces a trapezoid at the meteor level. The coordinates of the trapezoid vertices are $(x_i, y_i, z_i)$ with $i = 1, \ldots, 4$. Since the grid step is small enough, the trapezoid can be approximated by a
rectangle. The area $A_i$ is thus the product of the length of two of its adjacent edges. In order to minimize the error, $A_i$ is taken as the arithmetic mean of the two possible products $l_1 \times l_2$ and $l_3 \times l_4$. Accordingly, $\Delta A_i$ is the maximum difference between $A_i$ and $l_1 \times l_2$ or $l_3 \times l_4$.

3. The distance $d_i$ from the camera to the trapezoid is computed. By applying formula (19) we get the altitude above the horizon and select the relevant extinction value.

4. The geometrical area $A_i$ is corrected for distance and extinction.

5. The whole procedure runs until all the squares on the negative have been projected. Finally, the results are summed up to obtain $A_{\text{red}}$.

As an example, Figure 5 shows the camera field boundaries at the meteor level when $h_f = 50^\circ$ and $H = 100$ km (only $xy$-coordinates are shown).

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**Figure 4** - Projection of the camera field boundaries onto the meteor level for $h_f = 50^\circ$ and $H = 100$ km. We assume a 35 mm, $f = 50$ mm camera with the longest edge of the photograph parallel to the horizon.

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**Acknowledgments**

Many people contributed to the development of the photographic technique. Among them, we must cite Josep M. Trigo, Ralf Koschack and Jürgen Rendtel. Some of the topics discussed in Sections 2 and 3 were first put forward by these meteor workers and were never published. I am indebted to Robert Hawkes and Jürgen Rendtel for their useful comments to the paper. I also appreciate the help of Mark Kidger, who corrected the English of the manuscript and discussed several points concerning the whole procedure. David Asher is warmly acknowledged for making available some important references.
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Received April 1994; revised June 1994; accepted July 1994.