THE ROTATION OF PHOTOPHERIC MAGNETIC FIELDS: A RANDOM WALK TRANSPORT MODEL

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ABSTRACT

In an earlier study of solar differential rotation, we showed that the transport of magnetic flux across latitudes acts to establish quasi-stationary patterns, thereby accounting for the observed rigid rotation of the large-scale photospheric field. In that paper, the effect of supergranular convection was represented by a continuum diffusion, limiting the applicability of the calculations to large spatial scales. Here we extend the model to scales comparable to that of the supergranulation itself by replacing the diffusive transport with a discrete random walk process. Rotation curves are derived by cross-correlating the simulated photospheric field maps for a variety of time lags and spatial resolutions. When the lag between maps is relatively short (≤15 days), the midlatitude correlation functions show two distinct components: a broad feature associated with the large-scale unipolar patterns and a narrow feature originating from smaller magnetic structures encompassing from one to several supergranular cells. By fitting the broad component, we obtain the rigid rotation profile of the patterns, whereas by fitting the narrow component, we recover the differential rate of the photospheric plasma itself. For time lags of 1 month or greater, only the broad feature associated with the long-lived patterns remains clearly identifiable in the simulations.

Subject headings: Sun: magnetic fields — Sun: rotation

1. INTRODUCTION

The solar rotation rate and its variation with latitude are often measured by tracking magnetic features on the photosphere. Somewhat paradoxically, the results are found to depend on the type of magnetic tracer used (see the reviews of Howard 1984; Schröter 1985). Small, concentrated, and presumably short-lived features exhibit strong differential rotation similar to that derived by Newton & Nunn (1951) for recurrent sunspots. On the other hand, large, weak, long-lived features, in particular the unipolar magnetic regions studied by Bumba & Howard (1965, 1969), show a far more rigid rotation. Both types of rotation may coexist at the same latitude.

Snodgrass (1983) obtained a rotation curve characteristic of smaller scale features by cross-correlating Mount Wilson Observatory (MWO) magnetic maps taken 1–4 days apart. His result may be expressed as

\[ \omega(\theta) = 13.38 - 2.30 \cos^2 \theta - 1.62 \cos^4 \theta \text{ deg day}^{-1}, \]

where \( \omega \) is the synodic angular velocity and \( \theta \) denotes colatitude. Equation (1) is in good agreement with the classical Newton-Nunn formula; moreover, while the rates are about 2% faster at all latitudes, the Snodgrass curve is essentially parallel to that derived from Doppler measurements (Howard et al. 1983; Snodgrass, Howard, & Webster 1984), suggesting that it approximates the rotation of the photospheric plasma itself. More recently, Komm, Howard, & Harvey (1993a) have cross-correlated high-resolution daily magnetograms taken at National Solar Observatory/Kitt Peak and obtained a rotation profile similar to equation (1), but with a faster equatorial rate. By numerically simulating the evolution of the photospheric field, Sheeley, Wang, & Nash (1992) inferred a synodic angular velocity of 13°46 day\(^{-1}\) for the equatorial plasma, in good agreement with the measurements of Komm et al. (1993a) but significantly faster than the 13°38 day\(^{-1}\) of Snodgrass (1983) or the 13°39 day\(^{-1}\) of Newton & Nunn (1951). Nevertheless, we may continue to regard equation (1) as representa-tive of the steep differential rotation shown by individual, small-scale magnetic features and by the photospheric plasma itself.

Very different results are obtained when long, continuous time series of magnetograph data taken near central meridian are autocorrelated (Wilcox & Howard 1970; Wilcox et al. 1970; Stenflo 1974, 1989). This method, and the similar one of cross-correlating successive Carrington maps of the photospheric field (Sheeley, Nash, & Wang 1987, hereafter SNW), tends to pick up long-lived features that recur from one rotation to the next. The derived rotation rates generally agree with equation (1) in the low-latitude sunspot belts but are much faster (by an amount that varies over the sunspot cycle) at latitudes above \( \sim 30^\circ \), where the photospheric field is dominated by the large unipolar regions.

A physical explanation for the observed quasi-rigid rotation was proposed by SNW. The mechanism is based on the flux transport model (Leighton 1964; DeVore, Sheeley, & Boris 1984; Wang, Nash, & Sheeley 1989), which attributes the evolution of the large-scale photospheric field to the eruption of magnetic flux in the sunspot belts and to the subsequent dispersal of this flux by photospheric differential rotation, supergranular convection, and a poleward bulk flow. SNW showed that the meridional component of the flux transport acts to offset the rotational shearing of the photospheric flux distribution, giving rise to quasi-rigid magnetic patterns. A simple analogy might be drawn to a line of ducks stretching across a stream, with the current faster on one side of the stream than the other. If the ducks just drifted with the current, the line would become increasingly sheared with time; but if each duck swims toward the opposite bank and the line is continually replenished by new ducks entering the stream from the near bank, a stationary pattern will be formed. The stream, of course, corresponds to the differentially rotating photosphere, while the ducks represent the individual magnetic flux elements that originate in the sunspot belts and migrate poleward under...
the influence of supergranular convection and meridional flow. It is apparent that the patterns themselves will tend to corotate with their active region sources, whereas the flux elements of which the patterns are constituted will rotate at the local rate of the photospheric plasma.

In the analytical and numerical calculations of SNW, the Snodgrass formula (1) was used to represent the rotation of the photospheric plasma. Also, following Leighton (1964), the effect of the nonstationary supergranular convection was approximated as a continuum diffusion process. However, as is evident from inspection of magnetograms taken with relatively high spatial resolution, supergranular flows act not only to spread magnetic flux over the solar surface, but also to concentrate it at the cell boundaries to form the observed network. Because such “small-scale” processes are not described by the diffusion approximation, the transport equation used by SNW is not applicable on the spatial and temporal scales of the supergranulation itself.

Stenflo (1989, 1990, 1992) has criticized the mechanism proposed by SNW on the following grounds. (1) The rotational phase velocity derived from their transport equation does not reduce to equation (1) in the limit of small time lags. Thus, when the simulated photospheric field maps are cross-correlated, the rigid pattern rate is obtained even when the maps are separated by only a few days, contrary to the result of Snodgrass (1983). (2) Snodgrass used coarse-binned MWO data with a longitude resolution of ~1° and a latitude resolution ranging from ~1° at the equator to ~20° near the poles; the areal size of each bin corresponds to about two to three supergranular cells near central meridian. According to Stenflo, the diffusion approximation should be valid on such spatial scales, so that the inability of the model to recover the Snodgrass rotation law implies that the SNW mechanism is untenable.

Despite long-standing observational evidence (summarized by Howard 1992) for the active region origins of the large-scale patterns, Stenflo (1989, 1990) has proposed instead that they are formed in situ, through the continual emergence of magnetic flux on timescales longer than a few days but shorter than 1 month. In his view, the photospheric field is dominated at all latitudes by newly erupted flux rotating at the local plasma rate, as described by the Snodgrass law. On the other hand, to account for the results of the autocorrelation analyses, the eruptions are assumed to be highly organized on global scales, giving rise to patterns that rotate at the rigid rate of their subsurface sources. Stenflo then uses the autocorrelations to deduce the rotation rate at the bottom of the convection zone.

Recently, Snodgrass (1992) has cross-correlated MWO daily maps using lags of 30 days and more, rather than 1–4 days, as in his 1983 study. Surprisingly, he obtains a rotation curve similar to that for the short time lags, instead of the rigid profile found from autocorrelation measurements or by cross-correlating successive Carrington maps. He notes that his daily magnetic maps are dominated by features with longitudinal widths ranging from 3° to 15°, that this “fine structure” is what is being locked onto even for long time lags, and that such features must therefore have half-lives on the order of a solar rotation (not just a few days). More generally, Snodgrass concludes that the rotational behavior shown by the photospheric field depends not so much on the lifetimes, but rather on the sizes of the features being tracked, with smaller (but not necessarily short-lived) structures rotating more differentially.

The purpose of this study is to extend the range of the SNW model to smaller spatial scales by replacing the diffusion approximation by a discrete random walk process. With this more realistic treatment of the flux transport, we shall examine the rotational behavior of the simulated photospheric field and its dependence on time lag and spatial resolution. The improved model will allow us to address some of the questions raised by Snodgrass and Stenflo.

2. A RANDOM WALK TRANSPORT MODEL

We employ spherical coordinates (r, θ, φ), with the photosphere located at r = R⊙, colatitude θ measured from the north rotational pole, and longitude φ increasing westward in the direction of the solar rotation; the time coordinate will be denoted by t. In describing the transport of magnetic flux over the solar surface, SNW represented the nonstationary supergranular convection by a continuum diffusion process with a turbulent rate coefficient κ. The time evolution of the photospheric field B(θ, φ, t), assumed to be radially oriented (see Svalgaard, Duvall, & Scherrer 1978; Howard 1991), was then determined from the equation

$$\frac{\partial B}{\partial t} = -\alpha(\theta) \frac{\partial B}{\partial \phi} + \kappa \nabla^2 B - \frac{1}{R⊙ \sin \theta} \frac{\partial}{\partial \theta} [v(\theta) B \sin \theta] + S.$$  

(2)

Here ∇^2 denotes the θ and φ components of the Laplacian, ω(θ) is the angular velocity of the plasma (given by eq. (1)), v(θ) is the meridional flow speed, and S(θ, φ, t) is a source term representing the eruption of new flux in the form of bipolar magnetic regions (BMRs).

As noted in § 1, the diffusion approximation produces smooth flux distributions and cannot describe transport effects on the spatial and temporal scales of the supergranulation itself. The diverging flow pattern within each supergranular cell concentrates magnetic flux at its boundaries, and in particular, at the vertices formed by the cell with its neighbors. To represent such “fine structure,” we replace the diffusive transport by a discrete random walk process in which the step length is related to the supergranule size, and the time between steps to the supergranule lifetime. Observations indicate an average diameter of 31,200 ± 2,300 km for supergranules (Wang & Zirin 1989). Their mean lifetime is somewhat uncertain: estimates range from ~20 hr based on cross-correlations of Dopplergrams to ~50 hr based on visual tracking of individual supergranules (see Wang & Zirin 1989 and references therein).

Our numerical procedure for simulating the effect of the nonstationary supergranular convection is as follows. The solar surface is represented by a rectangular grid with 128 equal longitude intervals and 64 equal sine-latitude intervals. The area of each grid cell, given by 4πR⊙^2/(128 × 64) = π(15,440 km)^2, is thus close to that of an actual supergranule. At each computational time step Δt ~ 1 day, a constant fraction f of the 128 × 64 grid cells are chosen at random. All of the magnetic flux within each selected cell is then moved to a new position (θ_{new}, φ_{new}), located at a fixed linear distance l but a random direction z relative to the center of the original cell. (The angle z lies in the range 0° to 360° and is measured with respect to the local longitude axis, φ = φ_{local}) The transported flux is distributed among the four nearest cell centers using bilinear interpolation. All of the flux rearrangements during a given time step are performed simultaneously (rather than sequentially).
The above procedure represents a discrete random walk of flux elements on a fixed supergranular "lattice"; the random walk is characterized by a mean step length \( l \) and an average time between steps \( \tau = f^{-1} \Delta t \). (In reality, of course, it is the nonstationary nature of the supergranular network itself that gives rise to the random walk of magnetic flux on the Sun.) As a simple test case, we apply this algorithm to a unipolar flux distribution having the initial form

\[
B(\theta, \phi, 0) = 100 \, \text{G} \exp \left\{ -\frac{\left[ (\theta - 90^\circ) / 10^\circ \right]^2}{2} \right\},
\]

which represents a Gaussian with a latitudinal half-width of 10° centered around the equator. For the parameter choice (\( \tau = 4 \) days, \( l = 15,400 \) km), Figure 1a (solid line) shows the longitudinally averaged flux distribution as it appears after the lapse of \( t = 2 \) yr. Also plotted in the same panel (dotted line) is the corresponding result obtained by setting \( \kappa = 300 \, \text{km}^2 \, \text{s}^{-1} \) in the diffusion equation

\[
\frac{\partial B}{\partial t} = \kappa \nabla^2 B.
\]

In Figure 1b, the same calculations are repeated with the parameter choices (\( \tau = 2 \) days, \( l = 15,400 \) km) and \( \kappa = 600 \, \text{km}^2 \, \text{s}^{-1} \). From the agreement between the solid and dotted curves in both Figures 1a and 1b, we conclude that the rate at which flux spreads in latitude is approximately the same in the random walk and diffusion models, provided that we take

\[
\kappa \approx 0.45 \frac{l^2}{\tau}.
\]

In the above examples, the step size \( l \) for the random walk was chosen equal to the effective radius of a grid (supergranule) cell. When \( l \) is made progressively larger compared to the cell radius, the numerical coefficient relating \( \kappa \) to \( l^2/\tau \) is found to decrease rapidly toward the "classical" value of 0.25. The relationship between this scaling factor and the grid spacing may be understood as follows. In general, for a two-dimensional random walk, the directionally averaged spreading rate is given by

\[
\kappa = \frac{1}{4} \frac{\langle s^2 \rangle}{\tau},
\]

where \( \langle s^2 \rangle \) is the mean square displacement undergone by a flux element during the time \( \tau \). If the displaced flux is interpolated linearly among the nearest grid points, the flux-weighted value of \( \langle s^2 \rangle \) exceeds \( l^2 / \tau \) because of the disproportionately large contribution provided by the "outermost" of the neighboring points. Let us denote the linear dimensions of a grid cell by \( \delta_\theta \) and \( \delta_\phi \). When the mean step length \( l \) is much greater than \( \delta_\theta \) and \( \delta_\phi \), \( \langle s^2 \rangle / l^2 \to 1 \), and we recover the continuum result

\[
\kappa = \frac{l^2}{4 \tau}.
\]

On the other hand, if both \( l < \delta_\theta \) and \( l < \delta_\phi \), it is easy to show that

\[
\kappa = \frac{1}{2\pi} \frac{\delta_\theta + \delta_\phi}{l} \frac{l^2}{\tau}.
\]

Thus, the smaller the ratio of step length to cell size, the greater the "superdiffusion.”

The effective value of \( \kappa \) given by equation (8) represents an average of the random walk transport over all directions. Because of the nonuniform nature of the computational grid, the rate at which flux spreads in the simulations in fact depends on both direction and latitude. If we consider only the latitudinal component of the transport, then

\[
\kappa = \frac{1}{2} \frac{\langle s^2 \rangle}{\tau} = \frac{1}{\pi} \frac{(\delta_\phi / l)^2}{\tau},
\]

where \( \langle s^2 \rangle \) denotes the mean square displacement in the \( \theta \)-direction, and the second equality holds provided \( l < \delta_\phi \). Since the grid points are spaced at intervals of 0.031 in sine latitude, the value of \( \delta_\phi \) varies from 21,800 km at the equator to 44,000 km at latitude 60°. For a mean step length \( l = 15,400 \) km, equation (9) reduces to the numerically deduced result (5) at the equator. On the other hand, because of the increasing latitudinal elongation of the grid cells toward the poles, an even stronger "superdiffusion" should occur at high latitudes. Indeed, from an inspection of Figure 1b, it can be seen that the random walk algorithm transports more flux to the poles than the "equivalent" diffusion model. This artifact of the computa-
tional grid, which will affect quantitative modeling of polar field evolution but has no substantive effect on the simulations of magnetic rotation presented in this paper, might be eliminated in the future by employing a mesh where all cells have similar shapes as well as equal areas.

Although we have not attempted a detailed optimization of the random walk parameters, we shall hereafter adopt a step length \( l = 15,400 ~\text{km} \) and a mean time between steps \( \tau = 2 \) days \( (f = 0.5) \), so that the flux contained in each cell is displaced on the average once every 2 days by an amount equal to the cell "radius." The resulting rate of latitudinal flux transport corresponds, near the equator, to the diffusion constant \( \kappa = 600 \, \text{km}^2 \, \text{s}^{-1} \) that Wang et al. (1989) inferred by modeling the evolution of selected active regions and of the Sun's polar fields using equation (2). Moreover, the adopted value of \( \tau \) is consistent with the mean supergranule lifetime of \( \sim 50 \) hr measured by Wang & Zirin (1989). We have verified, however, that the rotational properties of the simulated photospheric field are relatively insensitive to the choice of the random walk parameters.

We now proceed to simulate the evolution of the photospheric field during sunspot cycle 21. We employ a transport equation similar to equation (2), except for the replacement of the diffusion term by the random walk algorithm described above: symbolically,

\[
\kappa \nabla^2 B \rightarrow R_{\tau}, \{B\}.
\]

(10)

As in SNW, the Snodgrass formula (1) will be taken to represent the rotation rate of the plasma. We assume a poleward surface flow of the form

\[
s(\theta) = \mp 11.2 \, \text{m} \, \text{s}^{-1} \sin^3 \theta \cos \theta ,
\]

(11)

which attains its peak speed of \( 10 \, \text{m} \, \text{s}^{-1} \) at latitude \( 30^\circ \) in each hemisphere. The adopted flow amplitude is similar to that inferred by Wang et al. (1989) from the long-term evolution of the polar fields; it is also consistent with the measurements of Komm et al. (1993b) based on correlation tracking of small photospheric magnetic features. Although the results of this paper do not depend on the exact latitude dependence of the flow, the profile was arbitrarily adjusted to yield concentrated polar fields resembling those observed near sunspot minimum.

The starting date for the simulations is 1976 August 17, coinciding with the commencement of Carrington rotation (CR) 1645. In accordance with measurements by Svalgaard et al. (1978), the initial background field is assumed to have the poleward-peaked distribution

\[
B(\theta, \phi, 0) = \pm 11.5 \, \text{G} \, \cos^8 \theta ,
\]

(12)

where the minus sign is taken in the southern hemisphere. Subsequently, magnetic doublet sources are deposited onto the photospheric grid according to their observed eruption times, locations, strengths, and axial orientations. The properties of the 2700 doublets, representing BMRs that appeared on Kitt Peak daily magnetograms between 1976 August and 1986 April, are described in Wang & Sheeley (1989).

As an illustrative example, Figure 2a shows the simulated photospheric field on 1983 January 5. For comparison, we also display the observed flux distribution in the form of the Kitt Peak synoptic map for CR 1730 (Fig. 2b), as well as the corresponding photospheric field map computed using the diffusion approximation with \( \kappa = 600 \, \text{km}^2 \, \text{s}^{-1} \) (Fig. 2c). Allowing for the greater noise level of the observations, the random walk model reproduces both the small-scale, grainy structure and the large-scale unipolar patterns seen in the Kitt Peak map. By contrast, the field distribution derived with the diffusive transport equation (2) is perfectly smooth outside the sunspot belts, quite unlike the observed map; the midlatitude regions are completely dominated by the broad, backward-swept unipolar patterns, with no trace of fine structure.

In deriving his rotation curve (eq. [1]), Snodgrass (1983) employed coarse-binned MWO data with a longitudinal resolution ranging (at central meridian) from \( 3^\circ 4 \) at the equator to \( 5^\circ \) near the poles. Although our computations are performed on a somewhat finer grid (with longitude intervals of \( 2^\circ 8 \)), the simulated 128 \times 64 maps can be smoothed to a spatial resolution comparable to that used by Snodgrass by averaging over a \( 3 \times 3 \) moving "window" centered on each grid cell. The result, again for 1983 January 5, is displayed in Figure 3a along with the MWO synoptic map for CR 1730 (Fig. 3b). The smoothed random walk simulation is very similar in appearance to the MWO field distribution; both show nonuniform structure above the sunspot belts on scales larger than the supergranulation itself but smaller than the unipolar patterns. Because the MWO Carrington maps are constructed by averaging daily measurements, the fine structure seen in Figure 3b would be even more prominent in the original daily maps used by Snodgrass (1983.) This "patchiness" is completely absent from the diffusion approximation simulation, which has been replotted in Figure 3c.

In order to demonstrate the stochastic nature of the fine structure, we have repeated the random walk computation using different initial seeds for the random number generator (which selects grid cells and determines the directions in which their flux is to be moved). Figure 4 shows three variants of the simulated photospheric field during 1983 January 5, where only the initial seed has been changed; a \( 3 \times 3 \) smoothing has again been applied to the maps. It is apparent that, while the global patterns remain essentially the same, the patterns consist of clumps of flux whose sizes, shapes, and locations vary greatly from one map to the other. Consider, for example, the large, positive-polarity "plume" issuing from the northern-hemisphere active region near the right edge of the maps. As outlined by its strong-field contours, this plume appears as a short, wide structure in Figure 4a but as a long, relatively narrow one in Figure 4b; in Figure 4c, it is broken up into two distinct components, each with a longitudinal extent of \( 15^\circ - 20^\circ \). It is also interesting to note how the polar regions, where the nonaxisymmetric large-scale patterns are barely discernible, are dominated by random clumps of flux whose longitudinal sizes range from \( \sim 3^\circ \) to \( \sim 20^\circ \). Such clumps are also present in the MWO synoptic map shown in Figure 3b.

The remainder of this paper will focus on the rotational properties of the simulated photospheric field. The analysis will employ 27 photospheric maps calculated at 3 day intervals between 1982 December 22 and 1983 March 14 (CR 1730–1732). Rotation curves will be derived by cross-correlating pairs of maps separated by fixed time intervals ("lags") ranging from 3 to 27 days. Before performing the correlations, we subtract the longitudinally averaged component of the field from each of the maps; the removal of the axisymmetric field greatly increases the amplitude of the correlation signal relative to the background at middle to high latitudes. For each pair of such maps, the correlation is computed latitude-by-latitude using the fast Fourier transform; the correlations for
all pairs are then averaged and curve-fitted to determine the mean longitudinal displacement at each latitude.

3. CORRELATION FUNCTIONS

It is instructive to examine the correlation curves obtained by applying the above procedure to the simulated photospheric field maps. We consider the different latitude zones separately.

3.1. Middle Latitudes

From both the observed and simulated photospheric maps displayed in Figures 2–4, it is apparent that the large-scale unipolar patterns dominate the latitude range between \( \sim 30^\circ \) and \( \sim 60^\circ \). The characteristic appearance of the midlatitude correlations is illustrated in Figure 5 for a variety of time lags and spatial resolutions. In each panel, the normalized correlation amplitude obtained by averaging the correlations...
Fig. 3.—(a) Photospheric flux distribution on 1983 January 5, as simulated using the random walk model and smoothed by taking a $3 \times 3$ running pixel average over the original $128 \times 64$ map (displayed in Fig. 2a). (b) MWO synoptic map for CR 1730. The measured field strengths were divided by $\sin \theta$ to correct for line-of-sight projection, multiplied by a factor of 1.8 to compensate for line profile saturation (see Svalgaard et al. 1978), and interpolated from the original $91 \times 34$ map into a $128 \times 64$ grid. (c) Diffusion approximation simulation of the photospheric flux distribution on 1983 January 5 (same as Fig. 2c). The gray-scale calibration for these maps is the same as that used in Fig. 2, with white (black) denoting $B > 5 \text{G}$ ($B < -5 \text{G}$).

between individual pairs of maps) is plotted as a function of the longitudinal lag. All of these examples have been computed at 47° N using the random walk model.

Figure 5a shows the result of cross-correlating pairs of unsmoothed $128 \times 64$ photospheric maps spaced 3 days apart. The correlation function (thick line) has two distinct components: a broad "hill," surmounted by a narrow spike. The hill, with a full width at half-maximum (FWHM, measured with respect to zero correlation amplitude) of order 90°, originates from the large-scale unipolar patterns. The spike, with a width of order 5°, is produced by the small clumps of flux of which the patterns are comprised (see Fig. 2a). Also plotted in Figure 5a are a Gaussian fit to the hill (thin solid line) and a three-point parabolic fit to the spike (dotted line). The center of the Gaussian is shifted 2° to the right of the midpoint of the parabola (in the direction of longer lags), implying that the patterns rotate about 6% faster than the individual clumps of flux at 47°N.
In Figure 5b, a 3 day lag has again been used, but a running 3 x 3 pixel average was first applied to the simulated maps. Despite the spatial smoothing, the two-component structure seen in the unsmoothed case of Figure 5a clearly survives. However, the amplitude of the spike decreases relative to that of the hill-like feature; also, while the smoothing does not change the FWHM of the hill, which remains of order 90°, the width of the spike increases from ~5° to ~10°. This behavior is readily understood by comparing the unsmoothed and smoothed photospheric field maps displayed in Figures 2a and 3a, respectively; while leaving the global patterns essentially unchanged, smoothing tends to increase the size of the individual clumps but to decrease their average intensities (and thus the relative strength of their correlation signal). As indicated by the Gaussian and parabolic fits in Figure 5b, the hill is shifted 2:2 to the right of the spike just as in the unsmoothed simulation, so that the patterns again rotate 6% faster than the clumps.

As shown by Figures 5c (derived from the unsmoothed maps) and 5d (derived from the smoothed maps), the two-component structure survives even when the time interval between pairs of maps is increased from 3 to 12 days. However, the Gaussian-like hill is now overwhelmingly dominant, while the spike is beginning to approach the noise level. Neverthe-
less, by fitting the two components as before, we find that the hill is centered ~9° to the right of the spike, implying once again that the patterns rotate ~6% faster than the clumps.

When the time interval between pairs of maps is increased to 27 days, as shown by Figures 5e (unsmoothed case) and 5f (smoothed case), the spike is no longer clearly identifiable, and the correlation curve resembles a Gaussian with noise superposed. Thus, for this longer time lag, only the rotation rate of the patterns can be unambiguously determined.

It is interesting to compare the correlation curves of Figure 5, based on the random walk model, with the corresponding ones obtained when the diffusion approximation is applied. For this purpose, we cross-correlate pairs of 128 × 64 maps computed using equation (2) with k = 600 km² s⁻¹ (as before, the simulated maps span the interval from 1982 December 22 to 1983 March 14). The resulting correlation functions at 47°N are displayed in Figures 6a (3 day lag) and 6b (27 day lag). Both the short and long time lags yield the same smooth, broad curve with FWHM ~9°; there is no trace of the narrow features seen in the random walk correlations of Figure 5. In view of the perfectly smooth patterns that characterize the midlatitude field distribution in the diffusion model (see Fig. 2c), this result is hardly surprising. It is also evident that the Gaussian-like curves of Figure 6 are essentially identical to the broad “hills” in the random walk correlations of Figure 5: both are produced by the large-scale patterns.

3.2. High Latitudes

At progressively higher latitudes, the large-scale photospheric flux distribution becomes increasingly symmetrized by rotational shearing (see SNW); thus the contribution of the unipolar patterns to the rotational signal is expected to weaken relative to that of the small-scale clumps. Near the poles, where
the large-scale field is nearly axisymmetric, the clumps completely dominate the correlations in the random walk model. Figure 7 illustrates the behavior of the correlations at 72°N. When the smoothed random walk simulations are cross-correlated using a 3 day lag (Fig. 7a), the correlation function consists of a single narrow spike; the broad "hill" present at midlatitudes is now entirely suppressed. The spike is the signature of the differentially rotating clumps seen near the pole in Figure 3a. However, because these clumps are relatively short-lived, the result obtained when the lag is increased to 27 days is essentially noise (Fig. 7b). A very different behavior is found when the diffusion model is used. Because no clumps are formed in this case, the weakly nonaxisymmetric patterns provide the only correlation signal; a smooth, broad, Gaussian-like curve is thus obtained for lags of both 3 days (Fig. 7c) and 27 days (Fig. 7d).

3.3. Low Latitudes

In the sunspot belts, the correlation signal is completely dominated by active regions and a relatively simple behavior results. Figure 8 illustrates the correlation functions at 15°N, where the following cases (all computed using the unsmoothed 128 × 64 photospheric maps) are shown: (a) random walk model, 3 day lag; (b) random walk model, 27 day lag; (c) diffusion model, 3 day lag; (d) diffusion model, 27 day lag.

Fig. 7.—Correlation functions at latitude 72°N. (a) Random walk model, 3 day lag, 3 × 3 smoothing; (b) random walk model, 27 day lag, 3 × 3 smoothing; (c) diffusion model, 3 day lag; (d) diffusion model, 27 day lag.
Fig. 8.—Correlation functions at latitude 15°N. (a) Random walk model, 3 day lag, unsmoothed maps; (b) random walk model, 27 day lag, unsmoothed maps; (c) diffusion model, 3 day lag; (d) diffusion model, 27 day lag. In all four cases, we see the same tall, narrow peak, with little evidence for any other systematic component. (Because the correlation signal weakens as the time interval between maps increases, the correlations show greater noise and asymmetry for the 27 day lag than for the 3 day lag. The same effect is evident in the high-latitude examples of Fig. 7.) The FWHM of the peak is of order 10°, with the longer time lag giving rise to a small but perceptible broadening. It is interesting to note that the random walk and diffusion models produce practically identical results at these latitudes: the correlations are tracking the active regions, whose sizes, strengths, and lifetimes are largely model independent.

With increasing latitude, the tall, sharp peak progressively widens; Figure 9 illustrates the correlation functions at 29°N. The broadening signals the growing dominance of the large-scale unipolar patterns, formed as the remnant flux from decaying active regions spreads poleward. (In the correlations of Fig. 9, the secondary maximum located 180° from the main peak reflects the presence of an overall four-sector polarity structure at 29°N, which may be seen in the photospheric field maps of Fig. 2.) Although not shown here, the correlation peak also undergoes a perceptible broadening close to the equator, which may again be attributed to the presence of decaying flux originating from the sunspot belts (see the discussion of the “equatorial bulge” in § 3 of SNW).

4. ROTATION CURVES

It is now evident that the rotation profiles derived from the random walk simulations will depend not just on the time lag between maps, but also on the manner in which the correlations are fitted. Thus, if the location of the maximum correlation amplitude is used to determine the longitudinal phase shift at each latitude (so that we fit only the narrow spike representing the highest points in the correlation curve), the resulting profile will be strongly differential for short time lags. In this case, we are tracking relatively small-scale magnetic features at latitudes above the sunspot belts. If instead we choose to fit the broad Gaussian-like component of the mid-latitude correlations, the profile will be more rigid, reflecting the rotation of the large-scale patterns.

These principles are illustrated by Figures 10a and 10b, where two entirely different rotation profiles have been derived with the random walk model using the same 3 day lag. In both cases, we have cross-correlated unsmoothed 128 × 64 maps of the photospheric field spanning the interval from 1982 December 22 to 1983 March 14. In Figure 10a, the phase velocities were determined by applying a three-point parabolic fit to the highest peak of the correlation function at each latitude. The resulting profile (thick line) agrees very well with the Snodgrass law (thin line), which represents the plasma rotation rate in the simulation. In Figure 10b, the envelope of the correlation func-
rotation at each latitude was fitted with a Gaussian, supplemented by a second-degree polynomial to represent the background. The profile derived in this manner is far more rigid than the Snodgrass curve, showing much faster rotation above the sunspot belts. In this case, the rotation rates become indeterminate at high latitudes, where the broad component of the correlation function degenerates into noise. As noted in § 3.1.2, the large-scale field is nearly axisymmetric near the poles and the rotational signal is dominated by small-scale structure.

We next consider how the rotation curves derived from the random walk model depend on spatial resolution. For this purpose, we smooth the original $128 \times 64$ maps by taking running pixel averages over "windows" of fixed area size; we then perform the cross-correlations as before, again using a 3 day lag. Figures 11a and 11b illustrate the effects of $3 \times 3$ and $5 \times 5$ smoothing, respectively. The thick line in each panel shows the rotation rates obtained by fitting a parabola to the highest peak in the correlation at each latitude (as in the unsmoothed case displayed in Fig. 10a). Neither of these profiles differs substantially from the Snodgrass curve (thin line), although as the resolution is degraded there is a perceptible "pulling off" at midlatitudes. We conclude that, for short time lags, the plasma rate can effectively be recovered even when the photospheric field maps are smoothed over an area encompassing several supergranules. This result was anticipated in the earlier sections, where it was found that the random walk model produced clumpy structure on scales intermediate between the supergranulation and the global patterns. The spikelike feature contributed by this differentially rotating structure survives in the short-term correlations despite the low spatial resolution. For comparison, the dotted lines in Figures 11a and 11b indicate the rotation rates obtained by fitting the broad component of the correlation functions; as in the corresponding unsmoothed case of Figure 10b, these quasi-rigid profiles characterize the rotation of the large-scale unpolar regions.

We now examine the effect of taking longer time lags. The rotation curves plotted in Figure 12 were derived from maps spaced 12 days apart; in both cases, a $3 \times 3$ smoothing was applied to the simulations. The more rigid profile (dotted line) was obtained by fitting the broad envelopes present in the midlatitude correlations, whereas the more differential profile (thick solid line) resulted from fitting the narrow peaks. The latter curve closely follows the Snodgrass law except at midlatitudes, where there is a significant "pulling off" toward faster (more pattern-like) rotation rates. The deviation from the local plasma rate indicates that the narrow correlation feature is now beginning to blend with the envelope.

When the time lag is increased to 27 days, the spikelike signal generated by the relatively short-lived clumps is no longer distinguishable from the noise level, so that only the broad "hill" associated with the long-lived patterns can be unambiguously fitted at midlatitudes. The resulting rigid profile, as derived from both the unsmoothed and smoothed simulations, is shown in Figure 13; it closely resembles the Gaussian-fit cases for the 3 day and 12 day lags.

Finally, for comparison purposes, we display in Figure 14 the rotation profiles calculated with the diffusion model for
time lags of 3 days (thick solid line) and 27 days (dotted line). Since no narrow features were present in the correlations above the sunspot belts in either case, Gaussian-type fitting was performed. Both time lags yield the same rigid profile characterizing the large-scale patterns. Unlike in the corresponding random walk cases shown in the previous figures, the absence of noise in the high-latitude correlations allows the pattern rates to be determined poleward of 60°.

5. DISCUSSION AND CONCLUSIONS

In earlier studies of magnetic flux transport on the Sun, observed properties of the large-scale photospheric field were modeled successfully by representing the effect of supergranular convection by a turbulent diffusion with rate coefficient \( \kappa \) (see, e.g., Wang et al. 1989). However, the diffusion approximation was unable to reproduce the “grainy” structure characterizing the observed flux distribution on scales comparable to (and even exceeding) that of the supergranulation itself. This limitation has now been removed by replacing the diffusive transport by a discrete random walk process. The latter gives rise to two main physical effects: first, like diffusion, it transports flux across latitudes and establishes large-scale unpolar patterns; second, unlike diffusion, it produces nonuniform structure of clumping of flux on scales ranging from one to many grid (supergranule) cells. These “systematic” and “stochastic” effects of the random walk have enabled us to study the rotation of both large- and small-scale magnetic features, rather than just the global patterns considered in SNW. Our findings may be summarized as follows.
Fig. 12.—Rotation curves derived using a 12 day lag (random walk model). The photospheric field maps were smoothed by taking a $3 \times 3$ running mean. *(Thick solid line)* Profile obtained by parabolic fitting to the highest peak in the correlation function; *(dotted line)* profile obtained by Gaussian fitting to the broad envelope of the correlation function; *(thin solid line)* Snodgrass law.

1. When the simulated photospheric field maps are cross-correlated using short time lags ($\lesssim 15$ days), the midlatitude correlation functions display two distinct components: a sharp, narrow spike originating from magnetic structures with longitudinal widths ranging from $\sim 3^\circ$ to $\sim 20^\circ$, and a broad “hill” with a characteristic width of $\sim 90^\circ$ originating from the large-scale unipolar patterns. By fitting the narrow component, we recover the differential rotation of the plasma, whereas by fitting the broad component, we obtain the quasi-rotation of the patterns.

2. For long time lags ($\gtrsim 27$ days), only the broad “hill” remains identifiable in the midlatitude correlations. Fitting this Gaussian-like feature again yields the rigid rate of the large-scale patterns; the rotation profile is essentially identical to that obtained by fitting the broad component in the short-term correlations.

3. When the simulated maps are smoothed to the spatial resolution of the MWO coarse data, clumpy structure encompassing up to several supergranules is seen at all latitudes. By cross-correlating the smoothed maps at short time lags and fitting the highest peak in the correlations, we find that these relatively short-lived clumps rotate at the differential rate of the plasma.

4. Near the poles, the large-scale patterns are nearly axisymmetric, and the rotational signal is dominated by smaller-scale structure.

5. In the sunspot belts, both the short- and long-term correlations display a single, sharp peak originating from the differentially rotating active regions.

We can now understand why Snodgrass (1983), cross-correlating MWO magnetic maps spaced 1–4 days apart, obtained a strongly differential profile similar to the Doppler one and characteristic of the photospheric plasma. In determining the longitudinal displacements, Snodgrass fitted a parabola to the highest peak of his correlation functions, thus singling out the smallest resolvable structures rather than the broad patterns. According to Snodgrass (1992), these narrow structures had longitudinal widths of $3^\circ$–$15^\circ$ and dominated the appearance of his daily maps. The present simulations suggest that they represent a statistical clumping resulting from the random walk of flux elements on the supergranular “lattice.” Because the clumping is a purely stochastic rather than a systematic latitudinal transport effect, the structures rotate at the local photospheric rate. Our simulations also suggest that, had Snodgrass attempted to fit the broad envelope rather than the highest peak of his correlation functions at latitudes $\sim 45^\circ$, he would have obtained the rigid pattern rate. (The correlation amplitudes of Snodgrass are, in fact, based on the linear or “Pearson” correlation coefficient. However, the resulting correlation curves are similar in shape to those derived from the ordinary cross-correlation, provided that, as done here, the longitudinally averaged com-
ponent of the field is subtracted before the cross-correlation is computed.)

As mentioned in § 1, Stenflo (1989, 1990, 1992) has proposed that the large-scale unipolar patterns are formed in situ and that their rigid rotation reflects the properties of their subsurface "sources," rather than the effect of latitudinal flux transport. The main basis of Stenflo's argument is that Snodgrass employed relatively low-resolution data in his 1983 analysis; thus, the steep differential rotation that he found referred to the "large-scale" field, which should rotate rigidly for any time lag if the latitudinal transport mechanism were at work. However, as we have now seen, the random walk process gives rise to ubiquitous, differentially rotating structures that range up to several supergranules in size, but are still smaller than the unipolar patterns themselves. Thus Snodgrass was measuring not the pattern rate, but that of the intermediate-scale clumps of which the patterns are comprised.

The random walk model presented here cannot account for the existence of differentially rotating, intermediate-scale magnetic structures with lifetimes of 27 days and longer, as inferred recently by Snodgrass (1992). If we exclude the possibility of an error in Snodgrass's observational analysis, the failure of the present model to produce sufficiently long-lived clumps may indicate the need for a random walk algorithm that leads to greater bunching of flux. The shape and structure of the observed correlation curves may help resolve this question.

In conclusion, this study provides further support for the idea that the large-scale unipolar regions are formed by the transport of magnetic flux from the sunspot belts. The latitudinal transport processes—supergranular convection and meridional flow—are also responsible for the rigid behavior of these patterns. Our random walk simulations have demonstrated explicitly how the rotation of the photospheric field may be quasi-rigid on global scales and yet strongly differential on smaller spatial scales.

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REFERENCES


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