DIRECTIVITY OF THE RADIO EMISSION FROM THE K1 DWARF STAR AB DORADUS

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ABSTRACT

We present measurements of the spectrum and polarization of the flaring radio emission from the K1 dwarf star AB Doradus, together with previously reported single frequency measurements (with no polarization information) on 3 other days. On all 4 days spanning a 6 month period, the emission was strong and, when folded with the stellar rotation period, showed similar time variations with two prominent peaks at phase 0.35 and 0.75. These peaks coincide in longitude with two large starspots identified from the stellar optical light curve and have half-power widths as small as 0.1 rotations and no larger than 0.2 rotations. The modulated emission shows no measurable circular polarization, and its two peaks have different turnover frequencies.

We discuss four contrasting models that can reproduce the observed properties of the modulated emission. Each model places a stringent upper limit on the source size and therefore a lower limit on its brightness temperature. In the first model the modulation is produced purely by the geometrical effects of occultation and requires sources with brightness temperature up to $10^{13}$ K. In the second model we include the effects of limb darkening but find that neither the required source dimension nor its brightness temperature is significantly changed. In the third model the emission has a high directivity imposed extrinsically by coronal structures that absorb the radiation along all but the radial direction (to the stellar surface) and requires sources with brightness temperatures up to $10^{11}$ K. In the fourth model the directivity is intrinsic to the emission process itself combined with a magnetic structure of a particular shape and requires sources with brightness temperatures up to $6 \times 10^{10}$ K.

The high brightness temperatures, the broad-band and unpolarized nature of the radiation, and the coincidence of the radio peaks with large starspots suggest that the modulated emission of AB Dor is produced by gyroresonance from ultrarelativistic electrons, that is synchrotron emission. This is in contrast to the Sun, which even in flares produces relatively few such energetic electrons. If as Readhead (1994) suggests the maximum (intrinsic) brightness temperature attainable by synchrotron emission is $\sim 10^{11}$ K, then only models incorporating emission with high directivity—imposed extrinsically or, more likely, intrinsic to the source—can explain all the observed properties of the modulated emission. This then is the first indirect evidence that the incoherent radio emission of active stars can be highly directive.

Subject headings: polarization — radiation mechanisms: nonthermal — radio continuum: stars — stars: activity — stars: individual (AB Doradus) — stars: late-type

1. INTRODUCTION

Active late-type dwarf stars of spectral classes G–M can have radio luminosities many orders of magnitude higher than that of the Sun. Nevertheless, because optical measurements indicate that strong photospheric magnetic fields on these stars are organized in complex bipolar regions, one view is that their radio emission—like that of the Sun—originates from coronal magnetic field structures which are relatively compact (e.g., see discussion in White, Kundu, & Jackson 1989). This view received strong support recently from the detection of rotational modulation of radio emission from a K1 dwarf star, AB Doradus, believed to be near or at zero-age main sequence (Lim et al. 1992, hereafter Paper I). Thus, the relatively high radio luminosity of active late-type dwarf stars requires a large surface area coverage of compact radio-emitting magnetic structures or a high brightness temperature, or both. In this paper, we present indirect evidence that the incoherent radio emission of active late-type dwarf stars may achieve a much higher brightness temperature than previously thought.

As discussed in Paper I, certain conditions must be fulfilled for the radio emission of a star to vary periodically over many rotations, as observed on AB Dor. First, the magnetic field structure(s) containing each radio source must persist at the same stellar longitude for long intervals, in the case of AB Dor for up to hundreds of stellar rotations. In Paper I, the authors provided tentative evidence that the magnetic structures involved are located at two longitudes where large and long-lived (months to years) starspots or starspot groups are inferred—from optical light-curve analysis—to occur. Second, because the lifetime of nonthermal electrons is likely to be much less than one stellar rotation, the radiating electrons must be replenished almost continuously, presumably by a
near-continuous dissipation of magnetic energy. Third, in order for the radio sources to be eclipsed, naturally the magnetic structures involved must be smaller than the stellar disk. Finally, to explain the narrow peaks (with half-power widths of 0.1–0.2 rotations) in the radio light curve of AB Dor, the radio emission may be required to be highly directive. This property, not explored in detail in Paper I because of the lack of radio spectral and polarization measurements, is important because little is known about the directivity of radio emission processes on stars other than the Sun. As we shall show, it challenges our present understanding of the generation of radio emission on AB Dor. Despite this, the observed directivity can be used to place stringent constraints on the size, and therefore brightness temperature, of radio sources on this star.

In this paper, we report the first measurements of the spectrum and polarization of the rotationally modulated radio emission from AB Dor. We use these results, together with those from Paper I, to investigate models to explain the properties of the modulated emission. This paper is organized as follows. In § 2 we describe the observations and data analysis. In § 3 we present the measured radio light curves and discuss the profile of the modulated component. In § 4 we examine the relationship between the modulated emission and the locations of starspots. In § 5 we present four contrasting models that illustrate the physical concepts involved in explaining the observed properties of the modulated emission. In § 6 we discuss the implications of these results for the radio emission process and the directivity of the emission. Finally, in § 7 we summarize our conclusions.

2. OBSERVATIONS AND DATA ANALYSIS

We observed AB Dor with the Australia Telescope Compact Array (ATCA) on 10 separate days beginning on 1990 December 17 and ending on 1991 January 17. (These observations coincided with the period the X-ray satellite observatory ROSAT was scanning past AB Dor regularly during its all-sky survey.) On all but 1 or perhaps 2 of these days, apart from the odd highly impulsive and usually highly circularly polarized flare with duration less than 1 hr, the radio emission of AB Dor was relatively weak (a few mJy) and displayed only low-level if any detectable time variability; this may correspond to the quiescent state of the star. We shall report these observations in full elsewhere. In this paper we concentrate on the observation of 1991 January 7, when the stellar radio emission was both strong and had a temporal profile similar to that reported in Paper I. The onset of high radio activity occurred between January 4 and 7, and the star was still somewhat elevated in flux relative to its quiescent state in the subsequent observation on January 10.

In the observation we used five antennas of the ATCA located on the 3 km railtrack, and switched between the 20, 13, 6, and 3 cm bands (center frequencies of 1472, 2368, 4740, and 8640 MHz respectively) every 15 minutes throughout the entire observation of duration ~14 hr (1.1 stellar rotations). At each wavelength measurements were made in 32 separate frequency channels spanning a bandwidth of 128 MHz and comprised a ~10 minute integration on AB Dor followed by a short scan of a secondary calibrator, PKS 0454–810. For absolute amplitude calibration we used the primary flux calibrator PKS 1934–638, which we assumed to have a flux density of 14.816 Jy at 20 cm, 11.784 Jy at 13 cm, 6.130 Jy at 6 cm, and 2.983 Jy at 3 cm. These are recently revised values (which may be subject to further small revisions in the future), such that the 6 cm flux densities presented in Paper I are 5.0% higher than the revised values.

We edited the data using AIPS, discarding mainly narrow-band interference at 20 and 13 cm, and calibrated the data using MIRIAD. The latter has to be used to calibrate polarization data taken with the ATCA, which unlike the VLA employs linearly polarized feeds. We then imaged the data at each wavelength separately using AIPS. As in previous observations (Paper I; Lim 1993) the dMe star Rossiter 137B, separated by ~9" from AB Dor, was detected as a steady (quiescent) source during the observation. After subtracting model visibilities for Rst 137B from the data, we plotted the flux density of AB Dor at each wavelength as a function of rotation phase using a period of 0.51479 days, with the epoch of zero rotation phase at HJD = 2,444,296.575. The resulting radio light curves can therefore be compared directly with those in Paper I (see, however, translation error described in § 3.1), and with the optical light curves (indicating the position of starspots) in Innis et al. (1988) and Anders, Coates, & Thompson (1992).

3. RESULTS

3.1. Radio Light Curves

In Figure 1 we plot the radio light curve of AB Dor at 6 and 3 cm measured on 1991 January 7, together with the light curves (presented in Paper I) at 6 cm measured on 1990 July 24, September 27, and October 1. Each curve has been plotted on the same amplitude scale, but offset in amplitude from the curve below (all the light curves have approximately the same base level). The 1990 radio light curves have been corrected for a translation error made in Paper I, requiring a rotation phase of 0.15 to be subtracted from the data plotted in Figure 2 of that paper, and rescaled in flux density for the reason described in § 2. As can be seen, on 1991 January 7 the radio light curve of AB Dor displays two peaks in emission centered at or near rotation phases 0.35 and 0.75, very similar in temporal morphology to the 1990 radio light curves. Furthermore, the two peaks in emission have flux densities comparable to their 1990 counterparts, with—as on 1990 September 27 and October 1—the peak at phase 0.75 being stronger than that at phase 0.35.

In Figure 2 we plot the radio light curve of AB Dor measured in Stokes I and V at all four wavelengths on 1991 January 7. Each data point represents an individual scan of duration ~10 minutes and has an error bar of length ±1 σ. Some of the data points at 13 cm have relatively large error bars because at those times interference was particularly bad, and correspondingly more visibilities had to be discarded. Note that the enhanced emission preceding phase ~0.5 seen at 20, 13, and 6 cm, but apparently not at 3 cm, has a different spectrum to the peak at phase 0.75, and has decayed significantly in intensity by the subsequent rotation. This enhanced emission is probably the wing of the peak seen at phase 1.35 in the subsequent rotation; both have spectra rising between 20 and 6 cm, but falling between 6 and 3 cm. A short-lived, apparently narrow-band, and highly circularly polarized (~100%) flare can be seen at phase 0.45 at 20 cm, superposed on the modulated emission. Apart from this impulsive event, the temporal morphologies of the emission at all four wavelengths are remarkably similar. At its peak (phase 0.75) this emission has a degree of circular polarization of 10% ± 5% at 20 cm, 6% ± 4% at 13 cm, 3% ± 2% at 6 cm, and 0% ± 4% at 3 cm.
(the quoted uncertainties are $1 \sigma$), all statistically compatible with null polarization.

In Figure 3 we plot the spectral index of the emission on 1991 January 7 as a function of rotation phase. Because the measurements at the four wavelengths were not made contemporaneously, we have interpolated the data points at 20, 13, and 6 cm shown in Figure 2 to the time of the data points at 3 cm. We then calculated, for each data point, the spectral index of the emission between 20 and 13 cm (Fig. 3a), 13 and 6 cm (Fig. 3b), and 6 and 3 cm (Fig. 3c). As can be seen, the peak at phase 0.75 has a spectrum rising between 20 and 3 cm with a spectral index as steep as $0.54 \pm 0.08$ (between 13 and 6 cm), indicating optically thick emission at all four wavelengths. The peak at phase 1.35 has a spectrum rising between 20 and 6 cm with spectral index as steep as $0.56 \pm 0.25$ (between 13 and 6 cm), again indicating optically thick emission up to 6 cm. This peak, however, has a falling spectrum between 6 and 3 cm with a spectral index of $-0.76 \pm 0.25$, compatible with optically thin emission at 3 cm. Note that the enhanced emission preceding phase 0.5 has a similar spectrum to the peak at phase 1.35, further strengthening our earlier suggestion that this emission is the wing of the peak seen in the subsequent rotation at phase 1.35. The spectral behavior of the emission does not appear to change significantly during the rise and decay phase of either peaks, but this behavior cannot be studied in detail because of insufficient time resolution and signal-to-noise ratio.

3.2. The Modulation Profile

The radio light curve of AB Dor comprises two components, a sharply modulated component with peaks at or near rotation phases 0.35 and 0.75, together with an underlying component.
not apparently modulated by rotation but which could in principle consist of several weak modulated sources. In Figure 4 we fit (by eyeball) three different profiles to each of four representative peaks that have good phase coverage and good signal-to-noise ratio; they are the peaks at phase 0.75 on 1990 September 27 (Fig. 4a) and October 1 (Fig. 4b), and the peaks at phase 0.35 at 6 cm (Fig. 4c) and phase 0.75 at 3 cm (Fig. 4d) on 1991 January 7. As can be seen, above the e-width of the fitted profiles the modulated components can be adequately fitted by Gaussian profiles with half-power widths occasionally as small as 0.1 rotations, and never larger than 0.2 rotations. At the other extreme, they also could be fitted by profiles with straight sloping sides and a sharp or flat top, but with approximately the same half-power widths. The relevance of the fitted profiles will become clear in § 5.

4. RELATIONSHIP BETWEEN RADIO PEAKS AND STARSPOrTS

The photometric light curve of AB Dor encompassing the period of our observations has been published by Anders et al. (1992). Although the light curve changes markedly on timescales of several months or less, over the period from late 1989 to early 1991 at least one, and occasionally two, large starspots or starspot groups are evident centered at rotation phases 0.25–0.35 and 0.70–0.75. For example, in 1990 January two starspots are detectable at the above-mentioned rotation phases, whereas in 1990 November and 1991 January only the starspot at phase 0.35 is detectable. By 1991 February, however, two starspots again are detectable, centered at rotation phases 0.30 and 0.75. Interestingly, a year later in 1992 February the same two starspots appear to be still present. This long time sequence of optical observations suggests that between 1990 and 1992 two starspots or starspot groups evolved independently at or near rotation phases 0.35 and 0.75,
and greatly strengthens the claim made in Paper I that the radio peaks are coincident in longitude with the location of large starspots.

5. MODELS FOR THE MODULATION

Here we consider a number of models for the modulated radio emission, each of which has to explain the observed modulation profile as well as the intensity, spectrum, and polarization of the radiation. The models we consider are not an exhaustive list of the possibilities, but are meant to demonstrate the physical concepts likely to be involved. The first model is the simplest in that it involves emission from unobscured (bare) sources where the modulation is produced purely by the geometrical effects of occultation. In the second model we include the effect of limb darkening due to absorption by an overlying stratified atmosphere, or to a mechanism recently found to cause limb darkening of 20 cm active region emission on the Sun. In the third model the modulation is produced by coronal structures that absorb the radiation along all but a few lines of sight, or duct the radiation along a particular direction (i.e., the directivity is extrinsic to the source). In the fourth model the modulation is produced by the intrinsic directivity of the radiation mechanism itself, combined with a confining magnetic structure of a particular shape.

5.1. Bare Sources with No Intrinsic Directivity

The radiation from an optically thick slab (i.e., a source with a vertical thickness much smaller than its lateral dimension, as seen from the stellar surface) carried by rotation across the stellar disk will appear to have a cosine dependence on the rotation phase. Suppose that the rotation axis is tilted along the line of sight with one pole toward us (the inclination of the axis in the plane of the sky will not affect the modulation profile). If the inclination of the star’s rotation axis to the line of sight is i, the latitude of the source is θ, and the longitude (or rotational phase) is ϕ in spherical coordinates referred to the rotation axis, with θ = 0 at the visible pole and ϕ = 0 at meridian transit, then the projected area of a source varies as ζ H(ξ), where H(x) is the Heaviside step function and

\[
ζ = \sin \theta \sin i \cos \phi + \cos \theta \cos i ;
\]

the source is visible as long as ζ > 0, and the fraction of the actual source area seen in projection is ξ. This formula will be modified if the source is significantly above the stellar surface, but the modulated part of the flux will still have essentially a cos θ dependence on rotational phase.

AB Dor is thought to be at an inclination of i ≈ 60° (Robinson & Collier Cameron 1986). If the radio source is actually in the hemisphere whose pole is away from us, then the modulation profile can be quite narrow in phase; e.g., if θ = 135°, \( ζ = [1/2(2^{1/2})][3^{1/2} \cos \phi - 1] \) and the source will be visible for only 109° (0.30) of rotational phase, with a maximum projected area of 26% of its true area. If the source is at θ = 148°, then it will be visible for only 45° (0.12) of rotational phase and have a maximum projected area of only 3.5% of its true area. The source is never visible, in the limit of zero height, if θ > 150°. In Figure 5 we fit the modulation profile predicted by equation (1) for a source at θ = 148° to the peak at phase 0.75 on 1990 September 27. As can be seen, the fit is adequate above the e-width of the profile; in the wings, the predicted profile depends on the way in which the source is revealed by the edge of the disk, and hence depends on the source height and shape.

By contrast to an optically thick source, an optically thin source of any shape produces a modulation that is essentially a step function. While the source is visible the flux is constant, and the sharpness of the step during the transition from "off" to "on" reflects the way in which the source is revealed by the edge of the stellar disk and therefore depends on the source size; the duration of the "on" state depends greatly on the height and latitude of the source. A source at height \( h \) above the surface is visible as long as

\[
\left(1 + \frac{h}{R_\star}\right)\sqrt{1 - \xi^2} > 1 \quad \text{or} \quad \xi > 0 ,
\]

where \( R_\star \) is the stellar radius. The first condition corresponds to the source being seen in projection above the limb, whereas the second corresponds to it being in front of the visible face of the star. This model can produce a source visible only for a narrow range of rotational phase provided that the source is very low (\( h \ll R_\star \)), and that it occupies the same narrow range of latitude as deduced above for an optically thick slab.

The different temporal profiles predicted for the modulated emission depending on whether the source is optically thick or optically thin permits a test of this model. On 1991 January 7 the peak at phase 0.35 appears to be optically thick at 6 cm, but optically thin at 3 cm. In Figure 6 we fit the profiles predicted by equation (1) and equation (2), respectively, for a source at \( \theta = 135° \) to the peaks at 6 cm (Fig. 6a) and 3 cm (Fig. 6b). As can be seen, above their e-widths the predicted profiles can adequately fit the data points. Because of the relatively poor signal-to-noise ratio of the data at 3 cm, however, this apparent agreement cannot be regarded as a particularly stringent test of the model.

The above model therefore appears to be able to reproduce the observed modulation profile on AB Dor, so long as the source is suitably located on the stellar disk. As illustrated in Figure 7, the constraints imposed by the observations are very severe. If the inclination of the star is truly 60°, then for the observed radiation to have a half-power width of 0.2 rotations the sources must lie between the latitudes θ = 135° (indicated by the dotted latitude in Fig. 7) and θ = 150° (the southern
range of latitudes over many months. Because they must both be in the same hemisphere, the two sources do not represent the poles of an oblique rotator. If we relax the unrealistic assumption that the sources are everywhere at the same height, then the apparent degree of modulation will be reduced, with the consequence that the range of possible latitudes is even smaller than that inferred above.

Given the very small projected area of the sources, to produce radiation with the observed intensity they must have extremely high brightness temperatures. At a distance of $\sim 25$ pc and with a radius $R_\odot \approx 1.3 R_\odot$ (see Paper I), the brightness temperature, $T_b$, of radio sources on AB Dor is given by

$$T_b = 7.5 \times 10^6 S_{\text{mJy}} v_{\text{GHz}}^2 (r_d/R_\ast)^{-2},$$

where $S_{\text{mJy}}$ is the flux density in mJy, $v_{\text{GHz}}$ is the observing frequency in GHz, and $r_d$ is the projected source radius. The latter is given by the relation

$$r_d \approx \sin (\Delta \phi/2) \cos (\theta - \bar{\theta}) R_\ast,$$

where $\Delta \phi$ is the longitude range spanned by the source; in this case, $\Delta \phi$ is approximately equal to the possible latitude range spanned by the source. Thus, to produce a profile with a width of 0.2 rotations, the source can have a projected radius of only $r_d \approx 0.13 R_\ast$, with the consequence that to produce the peak at phase 0.75 on 1991 January 7 it must have $T_b \approx 2 \times 10^{12}$ K at 20 cm and $T_b \approx 1 \times 10^{11}$ K at 3 cm. To produce a profile with a width of only 0.1 rotations, the source can have a projected radius of just $r_d \approx 0.02 R_\ast$, and hence to produce the peak at phase 0.75 on 1990 September 27 it must have $T_b \approx 1 \times 10^{13}$ K (at 6 cm). In § 6, we show that this model is probably made invalid by the fact that the inferred source brightness temperature can exceed the upper limit attainable by synchrotron emission by up to two orders of magnitude.

### 5.2. Limb Darkening

A hot source embedded in a cooler stratified atmosphere can show some directivity because absorption by this atmosphere in the line of sight between the source and the observer increases as the source approaches the limb. This is a form of limb darkening. The opacity, $\tau$, of a plane parallel atmosphere in hydrostatic equilibrium, and which has a density scale height $h$, and a vertical height $h_0$, is given by

$$\tau \approx 0.1n_0^2 T^{-3/2}v^{-2}h_0 \cos \phi)^{-1}[1 - \exp (-2h_0/h)].$$

where the electron density $n$ changes with (vertical) height $h$ as $n = n_0 \exp [-h/h_0]$, $T$ is the temperature of the atmosphere, and $v$ is the observing frequency. For typical parameters in the atmosphere of AB Dor, that is $T \approx 3 \times 10^6$ K and therefore $h_0 \approx 0.26 R_\ast$ (see Lim 1991), and $n_0 \approx 1.6 R_\ast$ (the height of the Keplerian corotation radius of AB Dor, beyond which matter cannot be gravitationally bound), the resultant limb darkening can have a significant effect on the observed modulation profile provided that $n_0 \geq 10^9$ cm$^{-3}$ at 20 cm and $n_0 \geq 10^{10}$ cm$^{-3}$ at 3 cm, values perhaps not impossible at the base of the stellar corona.

One might hope that the additional directivity imposed by limb darkening can make the occultation model described in § 5.1 less restrictive by increasing the range of possible latitudes that the source can occupy. We find that the addition of limb darkening can indeed permit the source to occupy a wider range of latitudes and still produce a modulation profile of the observed width, but only if the atmosphere is very optically
thick even in the radial direction. For example, a source at \( \theta = 130^\circ \) can produce a modulation profile with a width of 0.2 rotations only if \( \tau \approx 1.2 \) in the radial direction, with the consequence that the inferred source brightness temperature is even higher than in the simpler occultation model of § 5.1.

Recently Aschwarden & Bastian (1994) found that at 20 cm the brightness temperature of active region emission on the Sun appears to be lower at the limb than when the region is on the disk. The reason for this apparent limb darkening is unknown; absorption by cool low-lying coronal loops surrounding the active region was suggested as a possibility. Statistically, the observed limb darkening can be described by the relation

\[
T_d(\phi) / T_d(0) = 0.4 + 0.6 \cos^2 \phi,
\]

where \( T_d(0) \) is the brightness temperature of the source at central meridian. The addition of this form of limb darkening to the occultation model of § 5.1 increases the possible range of latitudes that a source can occupy by at most only \( \sim 5^\circ \) and does not significantly change the inferred source dimension nor its brightness temperature.

5.3. Absorption or Ducting by Coronal Material

On the Sun both soft X-ray (e.g., Pallavicini et al. 1979; Webb & Zirin 1981) and radio (White, Kundu, & Gopal-Krishna 1991) observations indicate that the density in the corona immediately over the umbra of a sunspot is depressed relative to the surrounding active corona. One can envisage that the corona over the umbral of starspots on active stars like AB Dor also represent density "wells," and that the surrounding active corona is relatively much thicker than on the Sun. The density and temperature of this surrounding material are free parameters, and can plausibly be in a regime where free-free absorption of centimeter-wavelength radio emission is optically thick. This will have the effect of putting high absorbing "walls" around the umbra; any radio source low over the umbra will only be seen when the line of sight to it does not intersect a wall. This effect will render the source visible for only a narrow range of rotational phase, depending on the height of the walls and the height and dimension of the radio source.

With this model, we can reproduce the observed light curve with a suitable—but ad hoc—choice of model parameters. For example, if the walls have a temperature of \( \sim 3 \times 10^6 \) K and a density structure described by the plane parallel atmosphere of the previous model, then for the walls to be optically thick (integrated through its entire vertical height) to free-free absorption at 3 cm the electron density at the base of the well must be \( 1-2 \times 10^{10} \) cm\(^{-3} \). The source size predicted by this model depends on the directivity imposed on the radiation; the maximum source size corresponds to the case where the escaping radiation is everywhere directed radially outward from the stellar surface, whereby the projected source radius is given by equation (4) with \( \Delta \phi \) approximately equal to the angular half-power width of the peaks. Realistically, to achieve such a high directivity the electron density at the top (and hence also the base) of the wall is required to be implausibly high, and so the inferred source size should be regarded as an extreme upper limit. In this limit, and also in the most favorable situation where the source is located at \( \theta = i \), to produce the peak at phase 0.75 on 1991 January 7 the source must have \( r_s \approx 0.6 R_\ast \), and therefore \( T_\ast \approx 9 \times 10^8 \) K at 20 cm and \( T_\ast \approx 5 \times 10^8 \) K at 3 cm; to produce the peak at phase 0.75 on 1990 September 27 the source must have \( r_s \approx 0.3 R_\ast \), and therefore \( T_\ast \approx 6 \times 10^{10} \) K (at 6 cm). Such high brightness temperatures imply that the emission must be produced by synchrotron emission (see § 6).

Synchrotron theory predicts that the spectrum of a radio source in the strong fields near an umbra will have a high turnover frequency. For example, with a field strength of only 100 G, the turnover frequency for electrons with a power-law energy spectral index of \( -4 \) would be of order 50 GHz (a density of \( 10^8 \) cm\(^{-3} \) and source size \( 10^2 \) cm are assumed, but the result depends only weakly on these quantities whereas it is roughly proportional to the field strength; Dulik 1985). Although the peak at phase 0.75 on 1991 January 7 has a turnover frequency above 8 GHz, the peak at phase 0.35 has a turnover frequency below 8 GHz corresponding to a weak magnetic field more appropriate to a height of order \( R_\ast \). The requirement of relatively weak magnetic fields at low heights over the umbra of a starspot may invalidate this model. Although we cannot rule out similar density wells over lower field regions of the surface, in that case we cannot appeal to the solar analogy.

A variant on the above model is to invoke refraction in coronal structures to provide directivity. In this case, the radio emission reflects off the walls of a duct and propagates upwards until the walls become sufficiently transparent (Duncan 1979). This is the usual explanation for the observed directivity of metric solar radio bursts (e.g., see reviews in McLean & Labrum 1985). Because the plasma frequency in the walls of the duct must exceed the observing frequency, to duct radiation at 8 GHz the walls must have an electron density \( n > 10^{12} \) cm\(^{-3} \). As shown above, free-free opacity will absorb all the radiation at densities about 2 orders of magnitude below that at which refraction becomes important; for ducting to be important at these high frequencies without also incurring severe free-free absorption, the density gradients between the low-density and high-density regions of the corona must be very sharp (density scale lengths of \( \lesssim 10^{-2} \) cm for typical stellar coronal temperatures). On this basis, we regard directivity due to absorption to be more likely than ducting.

5.4. Radiation with Intrinsic Directivity

The radiation from a single ultrarelativistic electron (with Lorentz factor \( \gamma \)) gyrating in a magnetic field is emitted into a narrow cone with axis in a direction along the particle’s instantaneous motion, and half angle \( \psi \approx 1/\gamma \). For electrons with, for example, \( \gamma \approx 10 \) (corresponding to an effective temperature \( T_{\text{eff}} \approx 6 \times 10^{10} \) K), the radiation cone is only 12° wide. This property can lead to beamed radiation, e.g., from an electron beam moving towards the observer, as in the case of superluminal radio sources. Beams traveling in closed magnetic loops in the corona of active late-type stars will have a very short lifetime against precipitation into the chromosphere, and thus maintaining the radio emission by such beams would require enormous energy. Instead, it is energetically more efficient to produce the radio emission with a population of trapped electrons, which have a longer lifetime. Such a population is a natural consequence of magnetic loop containment, whereby electrons with moderate to small pitch angles precipitate into the dense chromosphere, leaving only trapped electrons with large pitch angles; specifically, only electrons with pitch angles \( \angle \geq \arcsin \left( B_{\text{loop}} / B_{\text{tot}} \right)^{1/2} \) where \( B_{\text{loop}} \) is the magnetic field strength at the loop top and \( B_{\text{tot}} \) that at the coronal footpoint, will be trapped. The lifetime of these ultrarelativistic electrons will be determined by scattering into smaller pitch
angles due to interactions with plasma waves (e.g., see the discussion of scattering and energetics by Kundu et al. 1987).

A trapped electron population can still produce directed synchrotron radiation under the right conditions. First, the electrons must have only a narrow range of large pitch angles. This requires the confining magnetic field structure to diverge only slowly with height; e.g., to trap electrons with pitch angles in the range 70°–90°, the structures must have $B_{\text{max}}/B_{\text{foot}} \approx 0.88$. Because of its slow divergence, the magnetic field of the loop need not be unrealistically strong to produce the observed turnover frequency in the modulated emission; indeed, to produce a turnover frequency below 8 GHz as observed for the peak at phase 0.35 on 1991 January 7, the loop magnetic field must be quite weak. At a given location the radiation from such a pitch angle distribution is directed only along a narrow range of angles perpendicular to the field line, and so the overall radiation pattern depends on the shape of the magnetic loop. Two examples of magnetic structures suitable for producing the observed modulated profile will be mentioned below.

First, consider a magnetic loop in which the legs are vertical and much longer than the near-horizontal portion encompassing the loop top. This structure will produce strong emission when the loop is at the eastern or western limb and thus naturally explains the two peaks separated by $\sim 180°$ of rotational phase. (A variant of this model is to invoke emission from only the footpoints of a loop of whatever size and shape; in this case, however, the emission at all four wavelengths must originate from the same locations, an unlikely situation given the wavelength dependence of synchrotron radiation on the magnetic field strength.) This model implies that the radio source is not coincident with the large starspots deduced from the stellar optical light curves. It has the serious weakness that as the modulated emission originates from just one magnetic structure, the two peaks in emission should have essentially identical properties (e.g., the same intensity on 1990 September 27, and the same spectrum on 1991 January 7), contrary to observations.

Instead, consider a magnetic loop in which the loop top is much longer than the legs, and furthermore is very flat. Such a loop would be required for each modulated component, and will produce a peak in emission whenever the loop crosses the stellar central meridian. The emission will have high directivity provided that the horizontal axis of the loop lies along or near to an east-west direction; if the horizontal axis of the loop lies along a north-south direction, the observed radiation will not have any directivity in the azimuthal direction, and, depending on the inclination, may not be illuminating us. With this model, the spatial coincidence of the radio sources with the longitudes of large starspots is preserved. Furthermore, the two peaks in emission can have different properties—as observed—reflecting the different physical conditions in their respective regions.

This model requires sources with the same maximum lateral dimension as in the previous model (§ 5.3). It has the advantage, however, that to produce radiation with the same observed peak intensity, the source can have a lower brightness temperature; because all the radiation is emitted into a narrow opening angle $\Delta \theta$ in the azimuthal direction, the inferred brightness temperature is a factor of $\Delta \theta/2$ lower than in the case where the radiation has no intrinsic directivity. Thus, in the most favorable situation where the source is located at $\theta = i$, to produce the peak at phase 0.75 on 1991 January 7 the source must have $T_B \approx 6 \times 10^{10}$ K at 20 cm and $T_B \approx 3 \times 10^{10}$ K at 3 cm; to produce the peak at phase 0.75 on 1990 September 27, the source must have $T_B \approx 2 \times 10^{11}$ K (at 6 cm). Compared to the model where the directivity is imposed extrinsically by absorption, the source can occupy a wider range of stellar latitudes and still not exceed the maximum brightness temperature attainable by synchrotron emission.

6. IMPLICATIONS FOR RADIO EMISSION PROCESS

In Paper I a coherent emission process, which intrinsically can be highly directive (as observed on the Sun), was suggested as a possible mechanism for the modulated emission. The spectral and polarization measurements presented in this paper, however, suggest otherwise. Both solar and stellar observations have shown that the radiation from coherent emission processes is usually highly impulsive, almost always narrowband or highly structured in frequency, and usually (although not always) moderately to highly circularly polarized. All these properties were displayed by the short-lived flare on AB Dor at 20 cm on 1991 January 7. The modulated emission, by contrast, had a relatively smooth temporal profile, was broadband, and essentially unpolarized ($\leq 10\%$ circularly polarized), properties suggestive of an incoherent emission process.

The models we presented demonstrate that the sources responsible for the modulated emission must have a lateral dimension substantially smaller than the stellar diameter. Here we cite another piece of evidence, following a line of argument advanced by White et al. (1989). The highly circularly polarized flare at 20 cm on 1991 January 7 is produced by a coherent emission process, such as an electron-cyclotron maser (Melrose & Dulk 1982), presumably radiating at low harmonics of the gyrofrequency. The source responsible for modulated emission at 20 cm, on the other hand, is produced at much higher harmonics of the gyrofrequency (i.e., in a region of weaker magnetic fields), and therefore is located higher in the stellar corona. If this source had covered the entire star, then it (being optically thick) would have obscured the underlying radiation from the coherent flare; clearly this is not the case. We can think of only one reason why one might wish to invoke the participation of large magnetic loops in the radio emission process, and that is the curious but so far consistent occurrence of two peaks in emission whenever the radio emission appears to be strong and modulated in phase with the stellar rotation. This may indicate that the two active regions on approximately opposite sides of the star have a common magnetic origin (located presumably deep inside the star), or are connected by loops of stellar dimension. Whatever the reason, our argument that the radio sources responsible for the modulated emission must be substantially smaller than the stellar disk seems incontrovertible.

The majority of late-type dwarf stars (apart from the Sun) have very small apparent angular diameters, making it difficult to effectively constrain radio source sizes using Very Long Baseline Interferometry (VLBI). So far only two successful VLBI measurements of late-type dwarf stars have been reported, both of the dMe flare star YZ Canis Minoris; Hewitt et al. (1990) reported a source size of $D_\theta \leq 5 D_\star$ (where $D_\star$ is the stellar diameter) for weakly polarized emission, whereas Benz & Alef (1991) reported a source size of $2 \pm 1 D_\star$ for weak but highly circularly polarized emission (here we have assumed, like Hewitt et al. 1990, a stellar diameter of 0.5 mas). In the vast majority of observations where there is no direct nor (until now) indirect way to constrain the size of incoherent radio
sources, to be prudent authors usually assume a source the size of the star when estimating brightness temperatures. (In the case of coherent flares, occasionally light travel time arguments can be used to constrain source sizes; the brightness temperature of coherent emission, however, is not directly related to the energy of the radiating electrons.) With this perhaps generous estimate of the source size, the brightness temperature inferred for the incoherent radio emission of active late-type dwarf stars in usually $\sim 10^8$ K (see White et al. 1989), rarely exceeds $10^9$ K (Kundu et al. 1988; Paper I), and only once has been inferred to attain $\sim 10^{10}$ K (Bastian & Bookbinder 1987). Such brightness temperatures imply gyroemission from mildly relativistic electrons, which in the convention of solar radio terminology is known as nonthermal gyrosynchrotron emission. This is the mechanism responsible for impulsive microwave flares—by far the most common types of microwave flare—on the Sun. In this view, all detectable incoherent flares from active late-type dwarf stars are similar to solar flares, but simply scaled up in spatial size.

The very high brightness temperatures inferred for the modulated emission of AB Dor (up to $T_B \geq 6 \times 10^{10}$ K), however, suggest that the solar analogy may not be always apt. Such high brightness temperatures, the broad-band and unpolarized nature of the radiation, and the apparent coincidence of the radio peaks with starspots suggest that the modulated emission is produced by gyroemission from ultrarelativistic electrons (up to $\gamma \geq 10$, or energy $E \geq 5$ MeV), that is synchrotron emission. The observed spectral slope of $\sim 0.5$ in the optically thick regime is broadly consistent with this interpretation; in the case of a homogeneous source synchrotron emission has a predicted spectral slope of 2.5 in the optically thick regime, but in the more realistic situation where the source size increases with decreasing frequency (because of the divergence of the source magnetic field), the spectral slope can be considerably smaller. The radiation from synchrotron emission is at best very weakly circularly polarized, and so perhaps unlike other nonthermal emission processes we do not have to appeal to almost perfect cancellation of oppositely circularly polarized radiation to explain the observed unpolarized radiation. It is noteworthy that synchrotron emission (of relativistic electrons) is not a significant contributor to any class of solar radio emission, with the possible exception of moving type IV bursts in the metric wavelength range (for which other models are presently favored). This implies that active stars such as AB Dor are relatively much more efficient at accelerating electrons to ultrarelativistic energies on a steady basis than are solar flares. Our observations support the idea that the site of such energetic electron acceleration is associated with large starspots or large starspot groups.

Recently Readhead (1994) showed that, under a wide range of physical conditions, the brightness temperature of synchrotron emission appears to be limited by a hitherto unknown mechanism that maintains approximate equipartition in energy between the emitting particles and the magnetic field in the emitting region. Accordingly, the maximum brightness temperature attainable by synchrotron emission is, in the rest frame of the source, $\sim 10^{11}$ K; radiation with significantly higher measured brightness temperatures requires Doppler beaming by the relativistic motion of the source toward the observer, possible for jets in extragalactic sources but hardly likely for magnetically confined radio sources on nearby stars. If this argument (and our argument that the modulated emission is produced by synchrotron emission) proves to be correct, then the condition that the inferred source brightness temperature cannot exceed $\sim 10^{11}$ K immediately rules out simple geometrical models (§ 5.1), even when the effects of limb darkening are included (§ 5.2). This condition also makes models where the directivity is imposed extrinsically by absorption highly restrictive, that is possible only under the extreme conditions of radially directed radiation and a very narrow range of source locations near latitudes $\theta \approx 0^\circ$; the absorption model of § 5.3 suffers further from the unlikely requirement of relatively weak coronal magnetic fields at low heights over starspot umbrae. The model that is most able to reproduce all the observed properties of the modulated emission and at the same time not exceed the maximum brightness temperature attainable by synchrotron emission for a small range of source locations near $\theta \approx 0^\circ$, is one in which the emission has intrinsically high directivity (§ 5.4). Whether the directivity is imposed extrinsically or is intrinsic to the source, the modulated emission of AB Dor provides the first indirect evidence that the incoherent radio emission of late-type dwarf stars can be highly directive.

7. CONCLUSIONS

We have presented measurements of the spectrum and polarization of the rotationally modulated radio emission on AB Dor. This emission displays two prominent peaks with different turnover frequencies, and no measurable circular polarization. The two peaks are centered at or near rotation phases 0.35 and 0.75, coincident in longitude with large starspots or starspot groups identified from the optical light curve. They have half-power widths as small as 0.1 rotations and no larger than 0.2 rotations.

We have investigated four different models to explain the modulated emission, each of which has to reproduce the observed modulation profile as well as the intensity, spectrum, and polarization of the emission. We found the following:

1. Modulation produced purely by the geometrical effects of occultation requires sources confined to a narrow range of latitudes near the limb whose pole is tilted away from us. The lateral dimension of the source cannot be larger than 0.13 $D_\star$, and can be as small as 0.02 $D_\star$, with the consequence that the inferred brightness temperature can be as high as $10^{13}$ K.

2. The addition of limb darkening, whether by absorption in an overlying plane parallel atmosphere or to a mechanism recently found to cause limb darkening of 20 cm active region emission on the Sun, to the geometrical effects of occultation does not significantly alter the possible latitude range that the source can occupy. Neither the inferred source dimension nor brightness temperature is significantly changed, and if anything, the latter may be required to be higher.

3. Modulation produced by directivity extrinsic to the source, such as vertical coronal structures situated around a starspot umbra that absorb the radiation along all but a few lines of sight, requires sources with a lateral dimension no larger than 0.6 $D_\star$ and as small as 0.3 $D_\star$. In the most favorable situation where the radiation is everywhere directed radially, and where the source is located at latitudes $\theta \approx 0^\circ$, the inferred brightness temperature can be as high as $10^{11}$ K. This model suffers from the requirement that, to produce a turnover frequency below 8 GHz as observed for one of the peaks in the modulated emission, the coronal magnetic field at low heights over the starspot umbra must be very weak, only $\sim 10$ G.

4. Modulation produced by the intrinsically high directivity
of synchrotron emission also requires sources with the same dimensions as in the previous model. Because the radiation is beamed into a narrow opening angle $\Delta \phi$ in the azimuthal direction the brightness temperature of the source can be a factor of $\sim \Delta \phi/2$ lower but nonetheless can be up to $6 \times 10^{10}$ K. This model suffers from the requirement of magnetic structures with peculiar shapes.

The inferred high brightness temperatures, the broad-band and unpolarized nature of the radiation, and the coincidence of the radio peaks with starspots suggest that the modulated emission of AB Dor is produced by gyroemission from ultrarelativistic electron (up to $\gamma \geq 10$, or $E \geq 5$ MeV), that is synchrotron emission. This is the first true indirect evidence for radio emission from ultrarelativistic electrons on a late-type dwarf star and implies that active stars such as AB Dor are relatively much more efficient at accelerating electrons to ultrarelativistic energies on a steady basis than are solar flares. If as suggested by Readhead (1994) the maximum (intrinsic) brightness temperature attainable by synchrotron emission is $\sim 10^{11}$ K, then the modulated emission of AB Dor also is required to have high directivity. This then is the first indirect evidence that the incoherent radio emission of active stars can be highly directive. In our opinion, as the models presented well demonstrate, the actual physical process that gives rise to this high directivity remains to be satisfactorily explained.

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