VLA\textsuperscript{1} STEREOSCOPY OF SOLAR ACTIVE REGIONS. I.
METHOD AND TESTS
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ABSTRACT

We develop a new technique for extracting three-dimensional information from multiday solar VLA observations. While standard stereoscopic methods provide a three-dimensional view of an object by combining simultaneous observations from two different aspect angles, we relax the condition of simultaneity and exploit solar rotation to vary the aspect angle. The solar radio images are decomposed into Gaussian source components, which are then cross-correlated in maps from preceding and following days. This provides measurements of the three-dimensional position of correlated source centroids.

In this first paper, we describe the stereoscopic method and perform tests with simulated and real radio maps (from the VLA at 20 cm), in order to study the accuracy of altitude measurements, and the limitations introduced by (i) source confusion, (ii) source motion, and (iii) the assumed differential rotation rate. The tests demonstrate that (i) the information content of a VLA map relevant for stereoscopic correlation can be conveniently represented in terms of a small number of Gaussian components; (ii) the fitting of the three-dimensional source position is stable within a numerical accuracy of $\lesssim 0.02$ map pixels; (iii) the relative accuracy of the altitude determination is uniform over the solar disk, and (iv) source confusion does not affect the accuracy of stereoscopic position measurements for sources with a signal-to-noise ratio of $\gtrsim 36$.

Subject headings: methods: numerical — Sun: activity — Sun: radio radiation — techniques: image processing

1. INTRODUCTION

The principles of stereoscopy were first described in 1832 by Charles Wheatstone, a contemporary of Michael Faraday. Stereoscopy, a method of creating the illusion of depth perception by recording two images of the same three-dimensional (3D) object from two slightly different aspect angles, came into use after public demonstrations by Sir David Brewster in 1851. Today practical applications of stereoscopy range from medical X-ray images to airborne photographic mapping of the Earth's surface. In astronomy, the discovery of planetoids or comets is commonly accomplished by use of stereocomparators, where the parallax effect of nearby moving objects is distinguishable against the background stars. Stereoscopic observations of solar flares by different spacecrafts (ISEE 3, ICE, PVO, Helios B, HEAO A, Venera, Prognoz) have been used to study the directivity of hard X-ray emission (Kane et al. 1980, 1988), to place altitude limits on partially occulted flares behind the limb (Kane et al. 1979, 1982; Kane 1983), and to achieve differential imaging of the hard X-ray sources (Kane 1981). At radio wavelengths, the beaming pattern of solar type I radiation has been determined through stereoscopic observations (the Stereo-1 experiment; Caroubalos & Steinberg 1974; Steinberg, Caroubalos, & Bougeret 1974).

Here we develop a stereoscopic approach to study the radio emission from the Sun's slowly varying component. Instead of combining two solar images recorded simultaneously from two different vantage points, we use the same telescope at two different times—solar rotation yields a change in the aspect angle during the interval between the two observations. However, the advantage of using a single instrument is traded off against complications introduced by temporal variability of the source between observations. Therefore, we are constrained to study quasi-stationary phenomena which persist for many days—that is, radio emission from solar active regions, or the so-called slowly varying component.

For pointlike sources, the stereoscopic method reduces to one of simple triangulation, while for extended sources, a full 3D perspective can potentially be reconstructed via tomographic methods. The stereoscopic effect in human perception is accomplished through a geometric interpretation of the parallax of correlated features between the left and right eye—uncorrelated features do not lead to a stereoscopic effect. We employ the same idea here. By cross-correlating radio maps obtained from two different aspect angles, the source height can be extracted. We employ an image decomposition algorithm which reduces a two-dimensional (2D) map to a relatively small number of source components. The source components are then used for stereoscopic correlation. Tests show that VLA 20 cm maps can be represented by $\approx 200$ elementary components, reproducing details of the map down to the nominal dynamic range of $\approx 20:1$. The technique is far more efficient than is cross-correlation of two maps, each of which is of order $512 \times 512$ pixels in size.

The stereoscopic method is described in \S 2. We performed a number of tests (\S 3) to demonstrate (i) the fidelity of image representation using a "multiresolution" image decomposi-
tion technique; (ii) the accuracy of centroid position measurements; (iii) the accuracy of altitude measurements for simple, stationary sources; (iv) the effect of source confusion on the accuracy of altitude measurements; (v) the errors induced by the difference between the assumed and actual solar differential rotation rates. The tests involve the use of simulated and real (VLA) data. The accuracy of 3D position measurements and strategies to disentangle the effects of horizontal source motion, differential rotation rate, and time evolution are summarized in § 4. This first paper on VLA stereoscopy describes the method—we will describe the first results of stereoscopic VLA observations in a companion paper (Aschwanden & Bastian 1994; Paper II).

2. THE METHOD OF VLA STEREOSCOPY

Aperture synthesis by the VLA provides a 2D map of the Sun projected onto the celestial sphere. The observed brightness distribution can be mapped onto a 3D brightness distribution by various means (Aschwanden, Bastian, & White 1992), stereoscopy being one such method. The basic idea of this study is to use the Sun's rotation to, in effect, view its radio brightness distribution from different aspect angles. VLA maps produced on different days can then be cross-correlated to constrain the 3D structure of stationary radio sources. The method described has some elements in common with medical X-ray tomography, but is necessarily inferior because of the substantially lower scanning rate and poorer aspect angle coverage. However, the stereoscopic correlation of radio sources from different aspect angles can nevertheless constrain 3D models of the coronal density distribution.

2.1. Preliminary Considerations

For the stereoscopic approach described here, the time range of a useful observation sequence may vary from a few hours up to half a solar rotation. The variability of the radio sources is the main factor limiting the accuracy of stereoscopic position measurements because it causes ambiguities in the identification of correlated features. Quasi-stationary radio sources are therefore required for the time interval of stereoscopic correlation. What frequency provides quasi-stationary sources with measurable altitudes? The VLA currently supports observations in seven frequency bands ranging from 74 MHz to 23 GHz. Solar radio emission at the lowest frequencies (74 and 333 MHz) is often dominated by the presence of type I sources. Type I sources often produce highly variable radio emission. Moreover, observation at these frequencies is also handicapped by significant wave refraction and possibly, by wave ducting effects, which produce a considerable shift between apparent and true source positions. While low-frequency emissions are unsuitable for the stereoscopic analysis described here, stereoscopic observations using ground- and space-based radiometers by Caroubalos & Steinberg (1974) and Steinberg et al. (1974) have established the beaming pattern of type I sources.

On the other hand, solar radio emission at high frequencies (> 5 GHz) is often dominated by gyroresonance emission (e.g., review of Dulk 1985), originating in high magnetic fields associated with sunspots in active regions. Interpretation of radio sources due to gyroresonance emission is complicated by its dependence on the magnetic field strength and orientation. High-frequency components due to free-free emission tend to lie low in the solar atmosphere. The parallax effect is therefore small and difficult to measure stereoscopically.

Radio emission at intermediate frequencies (decimeter wavelengths) is primarily due to free-free emission from the quiet Sun and from active regions. For example, while the quiet Sun is optically thin in the corona at 20 cm, with comparable contributions to the brightness from the corona and the transition layer (e.g., Kundu 1965; Dulk & Gary 1983), the quiescent emission from active regions is likely optically thick, with the bulk of emission originating from coronal heights. On the basis of these considerations, we favor a frequency of 1.4 GHz (λ ≈ 20 cm) for radio stereoscopy.

The interpretation of the 3D structure of a radio source depends on the optical properties of the emitting plasma. For an optically thick source, most of the observed emission originates from a surface where the optical depth is near unity. For the optically thin case, the observed brightness represents a line-of-sight integral through the source. Thus, the 3D position as determined by stereoscopic correlation from different aspect angles has different meanings for optically thick or optically thin emission. We illustrate this for the particular case of a coronal loop in Figure 1.

For the case of an optically thin plasma loop (with homogeneous density and temperature) viewed from the vertical direction (α = 0° in Fig. 1, left), the observed brightness distribution shows two local maxima (corresponding to the footpoints of the loop) because the line-of-sight emission measure is largest in those directions. Mapping the same loop from a different angle, say from an α = 45° in the vertical plane of the loop, the radio map will show a maximum in the direction where the line of sight is tangential to the inner radius of the circular loop.

The intersection of the two lines of sight (at P₁ in Fig. 1) represents the stereoscopically determined position for these two particular aspect angles. Varying the two aspect angles by a small amount, the stereoscopic intersection point will wander along the inner radius of the circular loop (e.g., P₂ for the aspect angle pair α = 45° and 90° in Fig. 1). For widely separated aspect angles, the intersection point can lie outside the outer loop radius. In the case of a faulty correlation (of two unrelated features), the inferred position can lie far from the structure (e.g., position P₃ in Fig. 1). We have developed a strategy designed to exclude such faulty positions by restricting the valid altitude range, as we describe further below.

In the case of optically thick emission, the stereoscopically determined positions from different aspect angles usually are closer to the centroid of the structure. In the case of a semi-circular loop (Fig. 1, right), the intersection points (P₃ and P₄ in Fig. 1) of two lines-of-sight pairs are located inside the inner radius of the loop. Note that the spatial separation between the position P₁ (optically thin loop) and P₃ (optically thick loop) can differ by an amount as large as the loop radius.

It is important to understand the meaning of stereoscopically obtained position information, because there is no unambiguous definition of the position of an extended 3D structure. It is clear from the example in Figure 1 that the same structure can lead to different “positions” depending whether the plasma is optically thick or thin, or on which pair of aspect angles is used in determining the position. In addition, the position depends on the topology of the object. For spherically symmetric objects only, the stereoscopic position remains invariant for different aspect angles. For the case of a coronal loop, which is believed to be one of the most fundamental structures in the solar corona, the centroid of the brightness distribution is likely to be found near the tangent of the loop for the optically thin case. Therefore, a statistical center-limb...
effect can result in both the optically thin or optically thick case. We therefore emphasize that our first stereoscopy study is mainly of statistical nature and has been carried out with an aim toward understanding aspect angle-dependent opacity effects.

2.2. Image Decomposition and Reconstruction

The first step in the stereoscopic analysis is to organize the information of a 2D image into a form suitable for a cross-correlation analysis. A VLA map is typically 512 × 512 pixels in size. However, the features of interest for stereoscopic analysis typically fill a small fraction of the pixels with emission. What is needed is a method of image decomposition which (i) reproduces the 2D data with high fidelity, (ii) conserves the positional information of source centroids with high accuracy, (iii) uses a minimal number of elementary components, and (iv) identifies the position and width of structures suitable for stereoscopic correlation. The most economical decomposition method was found by using Gaussians of various widths and amplitudes as elementary components. We decompose a 2D image simply by iterative subtraction of 2D Gaussian components, whose widths and amplitudes are fitted to the highest residual peak, until the noise limit is reached. The fact that the radial brightness distribution of 20 cm sources in VLA maps tends to be intrinsically Gaussian-like makes this natural decomposition method efficient. This property is illustrated in Figure 2, where 40 radial scans through the brightest sources of a 20 cm VLA map are shown, normalized and overlaid on a Gaussian with unity amplitude and width.

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The decomposition of the source into elementary Gaussian components should not be confused with the CLEAN algorithm commonly used for image deconvolution in radio astronomy. The CLEAN algorithm and its many variants deconvolves the instrument response function (i.e., sidelobes) from synthesis maps through an iterative procedure wherein the source is modeled as an ensemble of δ-functions with various amplitudes and locations. The purpose of the source decomposition described here serves only to represent the observed 2D brightness distribution in a more economical manner in order to improve the efficiency of the data transformations and the cross-correlation analysis subsequently performed.

For a mathematical description of the decomposition algorithm, we denote the brightness distribution of a 2D map with a discrete function \( f(x_i, y_j) \), where the indices \((i, j)\) refer to map pixels, with map size \( N_x \times N_y \). The map can be represented by a sum of \( N \) elementary components with Gaussian radial brightness distribution, such as

\[
f(x_i, y_j) = \sum_{k=1}^{N} P_k \exp \left( -\frac{(x_i - X_k)^2 + (y_j - Y_k)^2}{W_k^2} \right),
\]

where each Gaussian component is characterized by four parameters: the center position \((X_k, Y_k)\), the amplitude \(P_k\), and the width \(W_k\). While each of the \( N \) components is characterized by four parameters in the 2D map, the transformation into 3D space can be performed by generalizing to five parameters \((X_k, Y_k, R_k, P_k, W_k)\), where \(R_k\) represents the distance of the radio source to the Sun center. Obviously, assumptions must be made concerning \(R_k\) a priori.

The decomposition into Gaussian components can be performed by the following simple algorithm: we subtract iteratively the brightest component, identifying the position of the center \((X_k, Y_k)\) by the absolute flux maximum \(P_k\), and evaluate the Gaussian width \(W_k\) by scanning the flux profile in \(x\) and \(y\)-direction around the center \((X_k, Y_k)\). From the four scanned radial flux profiles \(f(r)\) (in \(x\), \(-x\), \(y\), \(-y\) direction), we select the least confused direction (by secondary sources) and interpolate a Gaussian curve through three appropriate points near the peak. An example of this algorithm is illustrated in Figure 3, where the iterative decomposition and corresponding reconstruction is shown in detail. The application to a full disk solar VLA map is shown in Figure 4, where most of the features of the original map are reproduced by only 200 elementary components. The reconstructed map can reproduce all fine structures of the original map by increasing the number of elementary components. However, 200 components seem to produce sufficient quality considering the dynamic range of the cleaned VLA maps, which is rarely better than 100:1 for the data set employed for this purpose.

### 2.3. Stereoscopic Correlation

The image decomposition method described in the previous section produces a series of \( N \) Gaussian components, each specified by the four parameters \((X_k, Y_k, P_k, W_k)\), \(k = 1, \ldots, N\), sorted according to decreasing brightness. The brightness components usually represent the peaks (local maxima) in the 2D brightness distribution, while fainter components reproduce fine structures or extended diffuse emission. Local maxima in the brightness distribution represent features suitable for stereoscopic correlation. Stereoscopic correlation becomes less feasible for fainter sources, because of higher time variability (shorter lifetimes), increasing source confusion, and a poorer signal-to-noise ratio.

In the following we describe the coordinate transformation required for stereoscopic correlation. We describe the position

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**Fig. 3.** Decomposition of the 2D brightness distribution into elementary components by iterative subtraction of Gaussian components (first and third columns). The reconstruction of the 2D image is shown by superposing iteratively the Gaussian components (second and fourth columns), for the iterations 1–5, 10, 20, 50. The reconstructed image after 50 iterations (bottom right) is indistinguishable from the original image (top left).
of a source component by the spatial 3D coordinates \((X_k, Y_k, R)\) in map 1, and by \((X_k', Y_k', R)\) in map 2, where \(R\) is a variable
distance from the Sun center, and \(k\) is a component identification
number in both maps. The two VLA maps are denoted by
\(f_1 = f(x, y, t_1)\) and \(f_2 = f(x', y', t_2)\), where the times \(t_1\) and \(t_2\)
refer to two different observation days. The heliographic
coorinate system in both maps is described by the position angle
\(P\) with \(P_1 = P(t_1)\) and \(P_2 = P(t_2)\), and by the heliographic
latitude \(B_0\), with \(B_1 = B_0(t_1)\) and \(B_2 = B_0(t_2)\), and longitude
\(L_0\) of the central point of the disk, with \(L_1 = L_0(t_1)\) and \(L_2 = L_0(t_2)\).

To allow for the likelihood that different active regions
possess different effective heights, individual features within the
maps are cross-correlated. Two different sets of criteria were
used to select subimages for cross-correlation. The first set
selected entire active regions for cross-correlation, so the
boundary of the subimage was located in areas of the quiet Sun
where the flux level was very low. Because the cross-correlation
is weighted by emission from the entire active region, the
inferred height is relatively insensitive to short-term variability
of discrete parts of the active region. The second selection
of features for cross-correlation simply selected all local maxima.
This second approach was, of course, subject to greater ambiguity
due to short-term variability in the relevant active regions.
In either case, a given feature was typically composed of a
number of elementary components. All coordinate transformations
performed on a given source for the purpose of
cross-correlation assumed that the source was characterized by
a single value of \(R\); hence, the ensemble of elementary
components whose sum characterized the source was assumed to
have the same value of \(R\).

First, we perform image transformations which are not
dependent on the third space coordinate and do not affect the
stereoscopic measurement of the unknown source altitude \(R\).
We rotate both maps (with \(x\)-axis in right ascension and \(y\)-axis in
decimation) by the position angles \(P_1\) and \(P_2\), in order to
orient the solar rotation axis parallel to the \(y\)-axis. The map
coordinates \((x, y)\) are then related to the heliographic coordinates
\((L, B)\) by the transformation:

\[
\begin{align*}
  (y - y_0) &= R \sin (B - B_0), \\
  (x - x_0) &= R \cos (B - B_0) \sin (L - L_0),
\end{align*}
\]

(2)

where \((x_0, y_0)\) is the pixel position of the disk center in the 2D
image. For a given value of \(R\) assumed for a component \(k\), we
then can obtain the heliographic coordinates \((L_k, B_k)\) from the
back-transformation of equation (2),

\[
\begin{align*}
  B_k &= B_0 + \arcsin \left( \frac{Y_k - y_0}{R} \right), \\
  L_k &= L_0 + \arcsin \left[ \frac{X_k - x_0}{\sqrt{R^2 - (Y_k - y_0)^2}} \right].
\end{align*}
\]

(3)

To perform a stereoscopic correlation we convert the
components \((X_k', Y_k', R)\) of map 2 (specified by \(L_2, B_2\)) into the
heliographic coordinates \((L', B')\) with equation (3), using an
assumed value of \(R\). We then transform the heliographic coordinates
\((L', B')\) from time \(t_2\) to \(t_1\), correcting for differential
rotation (Allen 1973) via

\[
\begin{align*}
  L &= L' + (L_2 - L_1) \left( 1 + \frac{3^\circ}{13'45'} \sin^2 B' + \frac{\Delta L_{\text{rot}}}{13'45'} \right), \\
  B &= B' + (L_2 - L_1) \frac{\Delta B_{\text{rot}}}{13'45'},
\end{align*}
\]

(4)
where $\Delta L_{\text{rel}}$ and $\Delta B_{\text{rel}}$ allow for the possibility of relative or "systematic" motion of the source with respect to the general differential rotation (see § 3.3). Finally, equation (2) is used with the constants $(L_1, B_1)$ and $R$ to convert the heliographic coordinates into the coordinate system of map 1, i.e., $(x, y)$.

The 2D cross-correlation between the untransformed subimage $f(x, y)$ and the transformed subimage $f_2(x', y')$, $y'$, is then performed, where the normalized cross-correlation coefficient $C(r) = C[f_1 f_2(r)]$ is defined by

$$C(f_1, f_2) = \frac{\text{Cov}(f_1, f_2)}{[\text{Var}(f_1) \text{Var}(f_2)]^{1/2}} = \frac{\int f_1 f_2 - f_1 \cdot f_2}{[(f_1^2 - f_1 \cdot f_2)(f_2^2 - f_2 \cdot f_2)]^{1/2}},$$

yielding a normalized coefficient $C(f_1, f_2)$ in the range of $[-1, +1]$.

The entire algorithm is then repeated for a given pair of subimages with a different value assumed for $R$. The process continues until the cross-correlation function $C(r)$ is adequately sampled as a function of $R$. It was found, in practice, that assumed heights sampling the range $R/R_{\text{sun}} = [1.0, 1.1]$ were adequate in most cases. From the maximum value of $C(r)$, an effective source height is inferred.

3. TEST RESULTS

We have performed some elementary tests (i) to ensure the correct encoding of the stereoscopic algorithm, (ii) to quantify the accuracy of the measured altitudes and positions, and (iii) to study the effects of source confusion. The influence of some other effects such as time evolution, systematic source motion, and an erroneous differential rotation rate is studied in Paper II.

3.1. Accuracy of Source Position Measurement

We performed a test of the accuracy of stereoscopic position measurements under ideal conditions, assuming (1) pointlike sources (Gaussians with finite width), which are (2) well-separated, to avoid source confusion, and (3) stationary in time (constant flux) and space (no relative motion). For this purpose we simulated a map by distributing 48 Gaussian sources onto a heliographic grid between $-45^\circ$ and $+45^\circ$ latitude, each source separated by $30^\circ$ in latitude and longitude. From an initial map with arbitrary position angle $P$ and Sun center coordinates $L_0, B_0$ we generate 10 maps by applying the solar differential rotation rate for a fixed radius of $R = 1.05 R_{\text{sun}}$, and by transforming the heliographic coordinates into right ascension and declination coordinates. Each map is $512 \times 512$ pixels, each pixel $6^\circ$ in size. This provides 10 simulated maps in the plane of the sky, where typically 24 sources are visible at one time, which are then used as input for the stereoscopic correlation. The accuracy of the 3D position of the simulated sources is, of course, only limited by the numerical accuracy since no noise was included.

We perform stereoscopic correlations for each pair of maps at two subsequent days, yielding about $2(10 - 1)24 = 432$ source correlations. Each stereoscopic scan generates the altitude range of $1.0 < R/R_{\text{sun}} < 1.1$, where the step is chosen in such a way, that the projected interval in altitude corresponds to the map resolution. The exact altitude is found by parabolic interpolation of the correlation coefficient. After the measurement of the altitude $R_h$, the heliographic coordinates $(L_h, B_h)$ can be evaluated from the map position $(x_h, y_h)$ by equation (3).

Figure 5 shows the difference between the model value and the fitted position of the center of the Gaussian point sources, in units of arcseconds (measured horizontally on the solar surface). For comparison, the map pixel resolution is shown, which varies as $6^\circ \cos (\pi/2, R/R_{\text{sun}})$ versus the angular center-limb distance $R/R_{\text{sun}}$. This test demonstrates that the model accuracy of the centroid fit (of a Gaussian with finite width) is stable within an interval of $\leq 0.01$ map pixel size, and somewhat less accurate near the limb following the $\cos(\theta)$ dependence.

3.2. Accuracy of Source Altitude Measurement

With the same test we checked the numerical accuracy of altitude measurements, and whether the accuracy depends on the position, e.g., on the center-limb distance. Altitude and heliographic position are related through equation (3) and cannot be measured independently in the stereoscopic method. If the heliographic position were known a priori, the altitude measurement would be most exact near the limb, while the position measurement is most inaccurate near the center of the disk because of the sin $(\theta)$ dependence of the projection angle. For stereoscopic correlations, however, the altitude is a function of the heliographic position, and the $\cos (\theta)$ dependence of the projected heliographic position and the sin $(\theta)$ dependence of the altitude complement each other, leading to a constant accuracy of altitude measurements across the solar disk.

The difference between the exact altitudes (as specified in the generation of rotated maps) and the stereoscopically evaluated altitudes are shown for the $(\approx 400)$ correlations in Figure 6. The accuracy of altitude measurements is found to be always better than 0.4, except for the three singular cases out of 400. This corresponds to a value of about 0.02 pixel size resolution at
3.3. Differential Rotation Rate and Systematic Motion

The accuracy of altitude measurements by stereoscopic correlation depends critically on the assumed solar differential rotation rate in equation (4). Errors in the assumed differential rotation rate may be evaluated a posteriori as statistical effects from many stereoscopic correlations. In order to get an idea about the magnitude of the induced error, we consider several standard differential rotation laws:

\[ \Omega_1 = 13^\circ 45 - 3^\circ \sin^2 B \text{ deg day}^{-1} \] (Allen 1973),
\[ \Omega_2 = 13^\circ 380 - 2^\circ 297 \sin^2 B - 1.624 \sin^4 B \text{ deg day}^{-1} \] (Snodgrass 1983),
\[ \Omega_3 = 13^\circ 340 - 2^\circ 119 \sin^2 B - 1.832 \sin^4 B \text{ deg day}^{-1} \] (Howard et al. 1990),

where \( B \) is the heliographic latitude. For the stereoscopic correlation analysis, we used \( \Omega_i \) (Allen 1973). In comparison, for a latitude of \( B = 20^\circ \), the two other differential rotation laws yield slightly lower values than Allen: \( \Omega_2 - \Omega_1 = 0^\circ 010 \) and \( \Omega_3 - \Omega_1 = -0^\circ 032 \) deg/day. In Figure 7, we show the expected error on the stereoscopically measured altitude of radio sources, for deviations of \( d\Omega \) = 0.01, 0.02, 0.05, and 0.10 deg day\(^{-1}\). The effect is opposite for east and west sides of the solar disk, and causes the largest error near the center of the disk. Provided there is not too much scatter in the measurement of altitudes by other effects, we are able to determine deviations from the assumed rotation rate from the statistical difference between the eastern and western altitude values measured near the central meridian.

While the assumption of an incorrect photospheric differential rotation law leads to errors in the stereoscopically deduced source altitudes, the analysis presupposes that the radio sources correlate with the photosphere. This may not be the case—radio sources may have a relative or "systematic" motion relative to the general differential rotation. Indeed, relative motions have been measured for small magnetic photospheric features by Howard, Harvey, & Forcagh (1990). There are different ways to measure this horizontal source motion of radio sources. The relative motion \( \Delta B_{rel} \) in heliographic latitude can directly be evaluated from the latitude difference \( (B - B') \) in transformation (4). The relative motion \( \Delta L_{rel} \) in heliographic longitude can be measured from the lon-

Fig. 7.—The effect of an error in the assumed solar differential rotation rate on the accuracy of stereoscopic altitude measurements is shown. The induced error in altitude (relative to an average height) is displayed for deviations of 0.01 to 0.1 deg day\(^{-1}\) from the effective solar differential rotation rate. The error is largest close to the central meridian, because of the singular behavior in the projection of altitude differences near the Sun center.


gitude difference \((L - L')\) in transformation (4) only if the differential rotation rate is well-known and if the source altitude is constant. Because the latter two conditions are not well-known for radio sources, we can evaluate these effects only on a statistical basis, or by comparison with photospheric features in coregistrated magnetograms.

The relation between horizontal source motion in longitude \(\Delta L_{rel}\) in deg day \(^{-1}\) and the induced error \(\Delta R\) in the stereoscopically measured altitude \(R\) follows from geometric considerations:

\[
\Delta R = -R \left(1 + \frac{L - L_0}{n_\Phi \Delta L_{rel}}\right)^{-1},
\]

where \((L - L_0)\) is the longitude difference to the central meridian and \(n_\Phi = (L_1 - L_2)/\Omega_t\) is the number of days between the two measurements used for stereoscopic correlation. The effect of the error \(\Delta R\) in the stereoscopic measurement of the altitude \(R\) by neglecting the relative motion \(\Delta L_{rel}\) is exactly the same as caused by an error in the assumed solar differential rotation rate \(\Omega_t\), as shown in Figure 7.

3.4. Effect of Source Confusion

In practice, we expect the accuracy of source positions to be limited by factors other than those described above. The dynamic range of the 20 cm maps used in this study is typically several times 10:1. The main factor which limits the dynamic range of the map is "confusion noise" (e.g., Bastian 1989). When imaging the Sun with the VLA, the primary beam of the instrument is filled with emission on a large range of spatial scales. Furthermore, the emission varies in time and space due to intrinsic variability of physical conditions within the source, and because of the Sun's rotation. Under these conditions the ability to identify and remove the sidelobes of discrete sources of emission via deconvolution is limited by the sidelobe clutter itself. In addition, source variability causes the convolution relation between the source brightness distribution and the instrument response function to break down, limiting the fidelity with which the true brightness distribution can be recovered.

Another source of confusion results from the stereoscopic procedure. The brightness distribution is decomposed into elementary components as described in § 2. An asymmetric brightness distribution can introduce some error in the Gaussian fit of the primary source centroid, as can two or more closely spaced local maxima. In the worst cases, one source may overlap a second one, e.g., near the limb, and a disentangling is impossible, thus the second source gets lost in the image decomposition. However, this case can be retrieved by permutating the sequence of maps in the stereoscopic correlation.

In order to estimate the degree of error introduced by the finite dynamic range of the maps used in practice, and by the errors in the source identification and fitting procedure, we performed a test with the map from 1989 August 13, shown in Figure 4. Decomposing this map into 200 components, and rotating the source components backward by 3 days with a fixed height of \(r/R_{sun} = 1.05\), we simulated a second artificial map. Before cross-correlating individual sources, random noise was added to each map separately to simulate a dynamic range of 20:1. We believe this is a conservative test—true confusion noise contains a correlated component which was not modeled by our test. The results of the test are summarized in Figure 8.

![Figure 8](image)

**Fig. 8.**—Stereoscopic correlation between the VLA map from 1989 August 13 (see Fig. 4) and the 3 days backward rotated map. The accuracy of source altitude measurements is degraded for those components which possess a signal-to-noise ratio of less than 3 \(\sigma\).

The error in the altitude measurement is plotted as a function of the brightness temperature. We find that the error in altitude determinations is small for those cases where the signal-to-noise ratio exceeds 3 \(\sigma\) (or a brightness above 0.33 MK). The deviation of the normalized correlation coefficient CC from unity serves as a useful indicator of source confusion. We find that essentially all successful correlations possess a correlation coefficient \(C(r) > 0.8\), a threshold which is used for subsequent analysis.

3.5. Effect of Time Variability

The time variability of individual sources can potentially lead to faulty altitude determination because the stereoscopic algorithm employed here finds correlated features by maximizing the correlation coefficient within a limited range of longitude positions that correspond to a projected height range of \(\pm 0.1\) solar radii. In the unfortunate case when a source visible on day 1 fades out on day 2, the stereoscopic algorithm may find a second source of competing brightness at a nearby heliographic longitude. However, we can eliminate most such faulty correlations, (1) by requiring a high correlation coefficient \((CC \geq 0.8)\), (2) by performing correlations in both forward and reverse time direction, and (3) by performing double correlations (if triple observations from 3 different days are available).

We have found, in practice, that source variability does not present a severe problem. For the data set described in paper II, for example, the observed distribution of lifetimes of active region structures at 20 cm wavelength was found to have an exponential distribution with an e-folding timescale of 18 days (see Fig. 4 in Paper II). Of 66 sources analyzed (in Paper II), 80% were found to have an observed lifetime of longer than 3 days, and were observed on more than 2 different days. Most of the correlated sources showed a correlation coefficient of \(\geq 0.8\) (see Fig. 6 in Paper II), which indicates a good temporal stability for most of the observed features. We might also keep in mind that the radio structures observed here represent 4 hr averages, which largely smooth out transient brightenings, e.g., as seen in soft X-ray time-lapse movies by YOHKOH (Shimizu et al. 1992).

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4. SUMMARY

We have demonstrated that a combination of multiple solar radio maps, such as maps obtained by the VLA at 20 cm on different days, can be used to measure the 3D position of quasi-stationary radio sources. The method is based on the stereoscopic correlation of deconvolved source components. The procedure consists of the following steps:

1. Two or more high-quality, cleaned VLA maps from different observation times are decomposed into (1) source components which correspond to local peaks of the 2D brightness distribution or (2) components which represent the residual fine structures, allowing us to extract subimages that encompass selected local structures.

2. Subimages around the local peaks of the brightness distribution with similar heliographic positions in a pair of maps are extracted for stereoscopic correlation, where the coordinates of the second map have to be transformed in time (applying the solar differential rotation with variable distance r to the Sun center) to the coordinate system of the first map.

3. The cross-correlation coefficient \( C(r) \) is then computed from the two subimages (rotated by the radius r into the same coordinate system). The radius \( r \) is varied in the coordinate transformation within a range of \( 1.0 < r/R_{\odot} < 1.1 \), \( i = 1, \ldots, N \), and the cross-correlation \( C(r) \) is computed for each transformation \( r \).

4. The altitude of the radio source is found from the maximum of the cross-correlation coefficient \( C(r) \), i.e., \( \max(C) = C(r_{0}) \), and by quadratic interpolation between the neighboring values around the peak at \( r_{0} \).

5. A complete set of 3D positions of all source centroid components can be used as fiducial points for a 3D model of the coronal density distribution, or to generate approximate projections from different aspect angles, or into heliographic coordinates (synoptic map).

The stereoscopic correlation is subject to various dynamical effects, such as the differential solar rotation, systematic horizontal and vertical source motion relative to photospheric features, or temporal evolution of the source. Furthermore, the correlation of corresponding features may be hampered by source confusion and errors in the assumed dynamical parameters (such as the solar differential rotation rate). From tests with simulated data and rotated VLA maps we find the following specifications for the different error sources.

1. The numerical accuracy of stereoscopic 3D position measurements of pointlike, unconfused, stationary sources is limited only by the accuracy of the centroid fit method. For Gaussian sources we obtain a stability of \( \pm 0.01 \) map pixel resolution in the determination of the centroid position in our code, after applying the stereoscopic coordinate transformation.

2. The accuracy of altitude measurements of source centroids is found to be constant as function of the center-to-limb distance, with a numeric accuracy of \( \pm 0.02 \) map pixels. However, the absolute accuracy of altitude measurements is probably dominated by other effects (e.g., dynamic range, rotational smearing, optical depth effects, time variability) rather than by the numeric accuracy of the centroid fit.

3. The effects of source confusion are severe for sources with a brightness \( T_{\text{B}} \leq 0.3 \) MK in our Gaussian decomposition method. However, the normalized correlation coefficient serves as a useful discriminator against faulty altitude measurements due to source confusion. Such cases are excluded from analysis.

4. The rotation rate of radio sources can be measured against the photospheric differential rotation rate. A difference of \( 0.02 \) deg day \(^{-1} \), in the rotation rate, (corresponding to the scatter of photospheric differential rotation rates derived with different tracer methods), introduces a error of \( \approx 6^\circ \) in the determination of the altitude for sources close \((<10^\circ)\) to the central meridian.

5. Temporal evolution of radio sources is probably the most severe constraint in stereoscopic correlations. The effect can only be studied statistically in a large number of stereoscopic correlations. The correlation coefficient is believed to be a good indicator of temporal evolution and persistence of active regions.

The objective of this study is to apply the stereoscopy method in a systematic way to solar VLA data and to demonstrate the feasibility and accuracy of this new approach. We believe that VLA stereoscopy provides a useful tool for the study of quasi-stationary solar radio sources. However, we emphasize that the positional information of stereoscopy is dependent on the opacity model. Conversely, the stereoscopic information can be used on a statistical basis to test the assumed opacity models. We present a stereoscopic analysis of a sequence of VLA maps recorded in 1989 July/August in a subsequent paper (Aschwanden & Bastian 1994). After the first exploration of VLA stereoscopy we hope to be able to constrain a model on the optical properties of radio-emitting plasmas in active regions. Future applications of the stereoscopic method may provide fiducial points for tomographic 3D reconstruction of active region structures (see also discussion in Aschwanden et al. 1992).

REFERENCES


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