SOLAR TOMOGRAPHY

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ABSTRACT

Tomographic imaging has provided the medical profession with unprecedented three-dimensional views of the internal structure of the human body. Similar techniques can provide solar physicists with an equally spectacular view of the three-dimensional structure of the solar corona, providing a new tool for addressing the problems of coronal structure, energy balance, and evolution. For the reconstruction process, images of the solar corona observed from different angular positions within the ecliptic are needed, and these are not yet available. The purpose is to demonstrate the utility and the practicality of solar tomography with a series of computer simulations of the process, while exploring the sensitivity of the results to some of the parameters of the observing process, e.g., the number of observations, angular spacing, and signal to noise. The results show that tomography can be a powerful technique for determining the three-dimensional nature of active region magnetic fields, coronal loops, helmet streamers, coronal holes, and other structures in the corona.

Subject headings: methods: data analysis — Sun: corona

1. INTRODUCTION

Images of the solar corona taken from a single spacecraft typically measure the line-of-sight integral of the volumetric emissivity through the source. The resulting two-dimensional observations have an unavoidable ambiguity along the line of sight that can be removed only by making assumptions about the three-dimensional nature of the emission, e.g., assuming loop, or sheet structures. These ambiguities can be removed by observing the sun from different vantage points at the same time; i.e., solar tomography.

Tomography of the solar corona has been attempted before using a single spacecraft. This was done by allowing the Sun's natural rotation to change the viewing angle. Soft X-ray images from Skylab (L. Golub, private communication) and OSO data (Kastner et al. 1977) were used in an attempt to deduce the three-dimensional nature of coronal emission structures. The results were disappointing. The time evolution of solar features could not be deconvolved from spatial variations in the resulting reconstructions. In a separate study, Altshuler (1979) used an expansion method to deduce the corona density distribution in conjunction with the three-dimensional potential field extrapolations calculated from photospheric magnetograms. Again, however, temporal changes were large enough to cause considerable uncertainty in the resulting distributions. To resolve this problem, simultaneous observations from several vantage points are needed. The purpose of this paper is to consider, for the first time, how multi-spacecraft images can be used in the tomographic reconstruction process to determine the three-dimensional structure of the solar coronal features. Specific structures of interest are active region magnetic fields, coronal loops, helmet streamers, coronal holes, and other structures in the corona.

2. DISCUSSION OF THE RESULTS

The basic concept of tomographic reconstruction is fairly simple. For an optically thin emission source, each pixel in an image represents the line-of-sight integration of the volumetric emissivity of the plasma within the wavelength band of the observation. By obtaining several of these observations, from various angles, the underlying three-dimensional structure of the emission can be deduced. This principle has been used extensively in the medical community for the imaging of internal structure of the body with such techniques as computer-aided tomography (CAT) scanners and magnetic resonance imagers. Although we can draw much information from the medical literature regarding the basic theory of reconstruction, there has been no study to determine how these techniques can best be applied for solar observation. What is the best algorithm for reconstruction? What are the convergence properties of the method?

In most medical applications, many observation angles can be used to optimize the reconstruction. For solar observations the number of angular positions is likely to be more limited. Therefore, one might ask whether useful tomographic reconstructions can be done with a limited number of spacecraft? What is the optimum configuration of the observing spacecraft to maximize the resolution in the tomographic reconstruction? In addition, one would like to know how the results are affected by detector noise, i.e., what signal-to-noise ratio is necessary to obtain accurate reconstructions? To begin to develop the answers to these and other questions, a computer simulation of the observing and subsequent tomographic reconstruction process was developed.

2.1. Description of the Model

In the basic model, several identical spacecraft (the number is allowed to vary as discussed below) are assumed to be observing the corona from different angular positions (angular positions are also allowed to vary) in a 1 AU orbit in the ecliptic, similar to the arrangement shown in Figure 1.

In general the reconstruction process is three-dimensional, i.e., the intersections of the lines of sight for observations obtained from different vantage points must be obtained using three-dimensional geometry. However, in the plane containing the orbit of the spacecraft, only two-dimensional geometry is required to determine the intersections. This greatly simplifies the calculation without greatly modifying the basic physics of the process. For this reason, all simulations were done for this central ecliptic plane only, and the trends obtained from this
two-dimensional analysis are assumed to carry over to the three-dimensional case.

In this central plane an emission source incorporating both Gaussian and unresolved point source components was assumed. A plot of this function is shown in Figure 2. The widths and intensities of the various components making up the original source are shown in Table I.

These components could be regarded as the emission from loops passing through the ecliptic plane, as indicated schematically in Figure 1. Using this distribution as the source, synthetic observations were calculated for the various observation angles by essentially summing the emission along the line of sight to obtain a one-dimensional representation of the image for each observation angle, as indicated in Figure 3. These projected data were input then into the reconstruction algorithm, and finally the reconstructed distribution was compared with the initial source functions. The results of this theoretical study are summarized below.

2.2. Reconstruction Algorithm

All reconstructions presented in this paper were obtained using the algebraic reconstruction technique (ART). This technique is well known in medical imaging applications (Herman 1979; Barrett & Swindel 1981; Censor 1983). Because of its flexibility, ART has several advantages for solar applications:

1. ART is easy to implement computationally and does not require extensive computer resources to run. The models presented here can be easily run in a few minutes each on a workstation.

2. ART very naturally allows for unequally spaced observations. It is almost inevitable that spacecraft observing the solar coronal emission cannot be maintained at equal angular spacing without prohibitive cost to the mission. The reconstruction algorithms based on the convolution theorem and other Fourier transform techniques do not handle unequally spaced data well.

3. The number of observations can be varied easily, so that if the observations from one spacecraft are lost, the reconstruction process can easily be modified to account for it.

4. Unequal pixel size in the observations can be easily accounted for by a simple rebinning process before the recon-

![Figure 1](image1.png)

**Fig. 1.**—The basic tomography concept is to obtain observations of coronal structures from a number of different angular vantage points. In the plane containing all of the spacecraft, the problem reduces to reconstructing a two-dimensional distribution of emission.

![Figure 2](image2.png)

**Fig. 2.**—The original source distribution used in the simulations. The source distributions consists of two spatially broad sources with a Gaussian intensity profiles, and a total of four unresolved point sources.

![Figure 3](image3.png)

**Fig. 3.**—Observations measure the projected line of sight integral of the volumetric emissivity. Different vantage points provide different projection arrays. The variation of these arrays is related to the spatial variation of the emissivity distribution.

<table>
<thead>
<tr>
<th>Component</th>
<th>Amplitude</th>
<th>Width (pixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>G2</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>P1</td>
<td>1.0</td>
<td>...</td>
</tr>
<tr>
<td>P2</td>
<td>0.5</td>
<td>...</td>
</tr>
<tr>
<td>P3</td>
<td>0.3</td>
<td>...</td>
</tr>
<tr>
<td>P4</td>
<td>0.1</td>
<td>...</td>
</tr>
</tbody>
</table>

**Table 1**

**Intensity Components Used in the Original Source Distribution**
struction begins, so that if different instruments are used or if similar instruments are at different distances from the Sun the reconstruction process can still be carried out.

The implementation of the basic ART reconstruction procedure can be illustrated with a very simple example (after Swindell & Barrett 1977). The top portion of Figure 4 shows an assumed $3 \times 3$ source distribution and its associated projections. For this particular example, four projection angles are assumed. Given these projected arrays, one can reconstruct the two-dimensional distribution by the following steps:

1. Initialize all pixels in the reconstructed array to zero.
2. Choose any one of the projections to begin with, and spread the value in each pixel evenly along the line of sight.
3. Choose a second projection angle. Project the reconstructed array along the chosen line of sight. These values are shown in parentheses in Figure 4.
4. Spread the difference between the actual and the reconstruction projection evenly along the line of sight.
5. Continue this operation until all projections have been used.

As can be seen from Figure 4, with only four projections and this simple technique, a reasonably accurate reproduction of the original source distribution is obtained. Notice also that the projections in Figure 4 are not uniformly spaced in angle. The ART algorithm handles this without undue complication. Other algorithms, particularly those based on the convolution theorem, are adversely affected by unevenly spaced projections. This makes the ART method particularly well suited for solar observations, because even spacing will not be possible at all times.

![Fig. 4.—A simple illustration of the algebraic reconstruction technique (ART) for a $3 \times 3$ sample array.](image)

2.3. Spatial Resolution

The spatial resolution in the reconstructed image must be on the order of the resolution of the individual observations. For the simulations presented here, the size of the bins in the reconstructed array are exactly equal to the size of the pixels in the observation arrays. One could envision a situation where the resolution of the individual instruments are not equal. Perhaps the instruments are not identical, perhaps one or more instruments has suffered some environmental degradation, or perhaps the observing instruments are not equal distances from the Sun. In any of these cases one should not hope to reconstruct the emission distribution with better resolution than that obtained with the lowest resolution observation. Nevertheless for the reconstruction algorithm to work it is not necessary to assume that each observation is obtained with identical spatial resolution.

All reconstructions were done on a $20 \times 20$ grid. The solar diameter is almost $2000^\circ$. A typical field of view would be on the order of two solar diameters or $\sim 4000^\circ$. Therefore the reconstructions represented in this simulation have very coarse resolution indeed of $\sim 3^\prime$ pixel$^{-1}$. This resolution is consistent with the resolution of very large structures seen in full disk images. However, one need not use the entire image for the reconstruction. For example, if a flaring loop is observed at a resolution of $2.5^\prime$ pixel$^{-1}$, then the reconstruction volume would be on the order of nearly $1'$ on a side. The only restriction that must be addressed is that smaller volume is that all significant emission observed by the spacecraft instruments must originate from within the volume of reconstruction. The emission from a large flare can completely dominate the emission from other portions of the solar disk in some wavelength bands, so that this constraint should not be a problem if the wavelength of operation is properly chosen. This need not be the case in general, however.

2.4. Convergence of the Reconstruction Process

The basic theoretical framework for understanding the tomographic reconstruction process is very similar to concepts in radio interferometry. For a discussion of the analytic theory see Barrett & Swindell (1981), and Censor (1983). A simple argument illustrating the connection between the ART process and Fourier analysis is given by Davila & Thompson (1992). A theoretical discussion of the convergence properties of the ART technique and a proof that this method converges to the proper distribution is given by Kak (1984).

The ART process converges to a reasonably good representation of the original source distribution after one iteration, i.e., one cycle through all of the angular projection data. However, the fidelity of the reconstruction can be improved by performing additional iterations through all of the projections using the reconstructed array from the previous iterations as the initial a guess. How rapidly does such a process converge? This convergence process is illustrated in Figure 5.

For this simulation, four spacecraft equally spaced $45^\circ$ apart were assumed. The projection arrays were calculated as discussed above for the source distribution shown in Figure 3. These arrays were input into the ART algorithm to calculate the reconstructed distribution. To obtain a measure of the fidelity of this reconstruction, the average rms deviation per pixel of the reconstructed array relative to the original source was calculated after each iteration. A total of 150 iterations through all of the projection data were calculated. The results are shown in Figure 5. From this figure we see that the iteration procedure rapidly converges on the average to the
to vary from a minimum of 10° to a maximum of 180°, with models calculated at every 10°. The results of these calculations are shown in Figure 6.

It turns out, as one might guess, that the best spacing for a given number of spacecraft is to distribute them uniformly in angle. A minimum in the rms deviation is seen at an angular range near 135°. This indicates that the reconstructed array is most similar to the original array for spacecraft which are evenly spaced in angle. One way to understand this result qualitatively is that equal angular spacing implies a uniform coverage of Fourier space (Barrett & Swindel 1981; Davila & Thompson 1992). If the spacecraft are less than evenly spaced, the fidelity of the reconstruction is reduced. This is evident from the increase in the rms deviation as the angular range is decreased. All points shown on this plot are calculated after only one iteration, if additional iterations are allowed, the minimum at 135° is broadened. Notice also that for angles greater than 135° the rms deviations begin to increase. Again, this is not unexpected. If one considers four spacecraft and 180° angular range, the spacecraft at 0° and 180° obtain the same projection information. Therefore there are only three spacecraft contributing independent and useful data for the reconstruction, and the rms deviation increases. The effect of spacecraft number is investigated more fully in the following paragraphs.

The calculations shown here are only for a simplified two-dimensional case. For true three-dimensional structures, would a polar orbiting spacecraft be necessary? The answer to this question is available from analytic theory of the reconstruction process (Barrett & Swindel 1981). One can show that if sufficient samples are obtained, all of Fourier space is spanned by observations from a 180° angular range in one plane. Therefore, for the tomographic reconstructions, polar orbiters are not necessary, in principle, because all Fourier components of the distribution can be measured from the ecliptic.

2.5. Optimum Angular Spacing of the Observing Spacecraft

Intuitively, it seems clear that the fidelity of the reconstruction process should depend on the relative spacing of the observing spacecraft. For example, if all of the observing spacecraft are too near one another in angle, each spacecraft would measure essentially the same projection. Significant new information is not added by having more observations at the same angle. Similarly, if two observing spacecraft are 180° apart, they would observe exactly the same optically thin emission from any coronal source. One must then ask the question, what is the optimum angular configuration for a given number of spacecraft?

To address the question of optimum angular distribution the following approach was adopted. Consider a model with four observing spacecraft, uniformly spaced and contained within a given angular range, where the angular range is simply the angular separation between the first and last spacecraft. For example, if four observing spacecraft are used, and the angular range is 135°, then each spacecraft is 45° apart. Simulations of the observation and reconstruction process for various values of the angular range were performed, keeping the number of observations, iterations, etc., constant. For each reconstruction the pixel-averaged rms deviation between the original and reconstructed distributions was calculated. By looking for a minimum in the rms deviations, the optimum angular spacing of the four observing spacecraft was determined. This procedure was carried out for an angular range which was allowed
Specific instruments, however, might make polar observations desirable. For example, a coronagraph is sensitive to emission from a finite wedge only on the limb. The disk is completely blocked out by the occulting disk. For this instrument, then, structures near the solar equator are visible only from certain angular positions around the ecliptic, and polar structures are continuously visible. Conversely for a set of polar orbiting spacecraft, equatorial structures are continuously visible, while polar structures are visible only from restricted portions of the orbit. Therefore, although polar orbiters are not necessary in principle for the tomographic reconstruction of optically thin structures, in practice there may be some advantage to having observations from out of the ecliptic for some types of instrument.

2.6. The Optimum Number of Observing Spacecraft

What is the optimum number of spacecraft? This is a question which does not have a simple answer. It is clear that one would expect that as the number of observed projections is increased, the reconstruction process is better constrained, and therefore the fidelity of the reconstruction is increased until for sampling at the Nyquist limit the projection arrays can be used to reconstruct original source exactly. However, this requires orders of magnitude more projections than will be available for solar observation. Typically, if one wants to reconstruct an array with 1° resolution, then the projections must be taken with approximately this same spacing over the entire 180° angular range. Clearly this is not possible for solar applications. Nevertheless, useful information can be obtained with far fewer projections. In this section we explore the relationship between the number of spacecraft and the improvement in image reconstruction.

To accomplish this, models assuming various numbers of observing spacecraft were run. In each case, the spacecraft were spread evenly over the optimum angular range to obtain optimum results for that number of observations. For each configuration, models were computed for two to nine spacecraft; the synthetic observations were calculated, these were input into the reconstruction algorithm, and the average rms per pixel between the original and the reconstructed arrays was calculated. The results of the model calculations are shown in Figure 7.

As expected the fidelity of the reconstruction increases (rms deviation decreases) as the number of spacecraft is increased. We see that, for the models presented here, most of the rms improvement is obtained by having four observing spacecraft.

One should use some caution in the interpretation of these results, however. There are two reasons for this. First, as mentioned previously, the Gaussian components of the original distribution occupy by far the most pixels in the array. Therefore the rms deviation as calculated in this simulation is primarily weighted to the Gaussian source components. A small rms deviation tends to mean therefore that the Gaussian components are fairly well fitted, regardless of the reconstruction of the narrow unresolved point sources. However, just from visual inspection of the reconstructed arrays (see Fig. 8) the point source reconstruction seems to improve when the Gaussian reconstruction improves, although this relationship was never quantified. A second, and more fundamental, point is that all models were calculated on a 20 × 20 grid. The Nyquist limit for such a reconstruction is of order 20–40 projections. The maximum of nine spacecraft observations assumed in this simulation is a reasonable fraction of the Nyquist limit. It would be expected that a much more gradual fall-off of the rms deviation with spacecraft number would be obtained if 1000 × 1000 reconstructions were attempted.

In a sense this calculation provides the answer to the question at the beginning section, i.e., the number of spacecraft needed depends very strongly on the resolution desired in the reconstructed image, and hence on the science objectives of the particular mission considered. With a few spacecraft, we have seen that arrays with sizes of the order of 20 × 20 resolution elements can be reproduced. The corresponding spatial resolution considerations were discussed in § 2.3 and will not be repeated here.

To get a better idea of exactly how the quality of the reconstruction varies with spacecraft number, four typical examples are shown in Figure 8. The reconstructed array for two observing positions, 90° apart, is shown in the upper left panel of Figure 8. Note that almost all detail regarding the fine-scale structure of the emission is lost in this reconstruction. However, the centroid of each major emission region can be located with reasonable accuracy. This is essentially triangulation to locate the emission centroids, and it is valuable information for localized emission regions. The ART algorithm tends to smear sources along the lines of sight. If a strong and a weak source overlap along a given line of sight, the weak source can be swamped by emission from the larger component. This is the case for the point sources in this figure. The point sources are not discernible because of contamination from the Gaussian sources.

The reconstructed array for four observing positions, 45° apart, is shown in the upper right panel in Figure 8. Note that in this reconstruction, one can begin to discern the difference between the broad Gaussian sources and the unresolved sources, i.e., information regarding the distribution of emission around the centroid is beginning to be obtained. This is important added information, particularly for sources which are not highly compact.

The reconstructed array for six observing positions, 30° apart, is shown in the lower left panel of Figure 8. The fidelity

![Fig. 7.—The fidelity of the reconstruction increases as the number of observations is increased. The rms values for each of these points were calculated using the optimum angular configuration appropriate for that number of observation points.](image-url)
of the reconstruction is better for two or four observing spacecraft. The Gaussian components are well formed and all but the smallest point sources are clearly visible.

The reconstructed array for eight observing positions, 25° apart, is shown in the lower right panel of Figure 8. This reconstruction provides the most faithful representation of the original source distribution of any of the simulations shown here. The point sources are visible and the Gaussian sources are well represented.

2.7. Effect of Detector Noise

The primary source of error in the reconstruction process is likely to be detector noise. This is easy to understand, at least in a qualitative sense. In the absence of noise, one can show that a finite set of projections can exactly reproduce the source distribution if sufficient projection samples are taken. The required sample frequency turns out to be the Nyquist frequency (Barrett & Swindle 1981). However, if detector noise is added, the projection data is no longer self-consistent, i.e., there is no single solution which satisfies all of the constraints implied by the projection data. In practice, therefore, the array reconstructed from noisy data is always some “best-fit” solution to the projection data. The larger this uncertainty the less faithful will be the reconstruction.

To test this idea, a number of models with four observing spacecraft, optimally spaced, were calculated. The only difference in the calculations was the level of noise introduced into the projection data. For each model, the projection data was searched to find the brightest pixel, and the brightest pixel was used to determine the amplitude of the noise. Then Gaussian noise with fixed amplitude was added to each pixel in the projection data, and the tomographic reconstruction procedure was done. The results are shown in Figure 9. It is important to emphasize that the signal-to-noise values used in the figure represent the maximum signal-to-noise ratio in the image since the noise amplitude is defined relative to the brightest pixel in all cases. The results show that the reconstructions are degraded only slightly at attainable signal/noise ratios. A signal-to-noise ratio of order 100 in the brightest pixel resulted in reconstructions which were essentially indistinguishable from noiseless reconstructions.
The effect of noise is likely to be dependent on the reconstruction algorithm. Like most other image reconstruction algorithms, the details of the sensitivity of the ART method are not presently well understood. One can only look at a range of representative cases and draw conclusions from them. Because of this the results presented in this section should be taken only as indicators of the magnitude of the effect, details could differ for other reconstruction configurations and algorithms.

3. SUMMARY AND CONCLUSIONS

The results of the computer modeling indicate that tomography of large-scale coronal structures is possible with a moderate number of observing spacecraft. The most promising reconstruction algorithms are those which are based on discrete series methods, such as ART. These methods allow easy and natural accounting for unequal angular spacing of the observations and dissimilar pixel size. The method is relatively easy to implement and requires only modest computer resources. Methods derived from continuous descriptions of the process, like the convolution theorem, are not as flexible.

Reasonable reconstructions of large-scale objects can be accomplished with data from only three or four observation angles. Smaller-scale objects can also be reconstructed if the emission from these objects dominates along the line of sight to all observing instruments. The optimum angular spacing is to have the observations equally spaced in angle, but one must note that observations 180° apart provide the same information and are therefore redundant.

A signal-to-noise ratio of a few hundred in the brightest pixel is probably adequate for the reconstruction process. This is a level which should be easily attainable using modern, low-noise electronic detectors.

Based on the results presented in this paper, it must be concluded that tomography can be a powerful technique for determining the three-dimensional nature of active region magnetic fields, coronal loops, helmet streamers, coronal holes, and other structures in the corona.

REFERENCES

Altschuler, M. D. 1979, in Image Reconstruction from Projections, ed. G. T. Herman (New York: Springer), 105
Herman, G. T. 1979, Image Reconstruction from Projections (New York: Springer)