Role of Electric Currents in Magnetic Flux Tube Physics

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Abstract. By a multi-fluid approach the intensity and the topology of electric currents generated in flux tubes by azimuthal photospheric motions are derived, and their effects on flux tube physics are investigated. Two systems of currents flowing in opposite direction are created that are connected at the photospheric level by transverse currents. The electromagnetic forces produced by these currents lead to pressure enhancement and to vertical forces in the photosphere. As a result upward motions are present that lift the matter to an altitude characteristic of spicules outside the internal cylindrical current, and maintain a pinch effect at the chromospheric level in a still partially ionized gas. The partially ionized gas rising from the photosphere in the internal current shell is found to have two main effects: 1) the upflow of gas can brings to the chromosphere an energy flux comparable to the flux required for chromospheric heating, 2) the outflow of neutrals that takes place at the chromospheric level across lines of force leads to ion-neutral separation and may explain the observed abundance anomalies in the corona.

1. Introduction

In the deep photosphere, particles are coupled by collisions, and the physics of magnetic flux tubes is often described by the resistive one-fluid MHD approach. However a treatment of the atmosphere as an ensemble of three fluids, ions, electrons and neutrals gives a clear physical insight on the mechanisms of current generation in these flux tubes. Moreover, higher in the atmosphere significant effects arise due to the density decrease that leads to a decoupling of motions of ions and neutrals.

For an axially symmetrical magnetic field, the velocities of electrons, ions and neutrals in the photosphere have been computed by Hénoux and Somov (1991) by solving the equations describing the balance of horizontal forces. The model of the chromospheric part of the flux tube presented here is semi-quantitative, since, contrary to our photospheric modeling, vertical forces and vertical motions that play a significant role higher in the tube have not been coupled to the horizontal forces and motions. With this assumption a quantitative model of the photospheric flux tube can be obtained. The model allows
us to relate DC currents in flux tubes to the photospheric velocity field and demonstrates the significant role of these currents in the physics of flux tubes.

2. DC electric current generation

In a non-axially-symmetrical geometry, electric currents can be generated in separatrices or separators of the coronal magnetic field by shear motions either parallel or perpendicular to the photospheric neutral line. For axially symmetrical magnetic fields, azimuthal motions generate currents. In this case, two models of current generation can be distinguished.

1) One in which the plasma is fully ionized with infinite conductivity, such that the magnetic field is frozen and moves with the plasma. Consequently current generation requires a variation of $\omega$, the azimuthal plasma velocity along $s$, the flux tube length, i.e. $d\omega/ds \neq 0$. The most acceptable location for this mechanism is below the photosphere. However this model is widely used to explain qualitatively the generation of currents by twisting the photospheric part of magnetic loops.

2) One presented in this paper, in which currents are generated by azimuthal motions in a partially ionized atmosphere. Here currents can be generated even for azimuthal velocities constant along the loop length.

Assuming steady state, the azimuthal and radial components of the current density can be derived from the equation of fluid dynamics

$$\rho \vec{v} \nabla \vec{v} = \vec{F}$$

(1)

The corresponding equation for the horizontal components of the forces and velocities, in a thin flux tube of constant cross section leads to the following expressions for the azimuthal and radial current densities

$$j_\theta = (j_z B_\theta + \frac{\partial P}{\partial r})/B_z,$$

(2)

where the contribution of the terms of inertia can be neglected, and

$$j_r = -\frac{\rho}{B_z} \frac{V_r \partial(V\theta)}{\partial r}.$$  

(3)

Equation (3) shows that radial current generation requires an influx of matter and angular momentum inside the flux tube. The radial current density cannot be computed without knowing the plasma radial velocity.

3. Currents generated in a partially ionized medium

3.1. Equations

The solution of the equation of dynamics for the horizontal motions of a partially ionized plasma (Hénoux and Somov 1991, 1992) gives a set of four equations

$$j_\theta = \left( j_z B_\theta + \frac{\partial P}{\partial r} \right)/B_z,$$

(4)
\[ j_r = -\frac{n_n m_n V_{r,n}}{B_z} \frac{\partial(rV_{\phi,n})}{\partial r}, \]  
(5)

\[ V_{r,n} = \frac{1}{2\mu_0 \sigma} \left[ \frac{\partial B_z^2}{\partial z} (1 + \frac{\alpha_s}{\sigma B_z^2}) + \frac{\mu^2}{4\pi r^2} \frac{\partial J_z^2}{\partial r} \right] + \frac{j_r}{n_e e}, \]  
(6)

\[ r V_{\theta,n} = -n_n m_n V_{r,n} \left( \frac{1}{\alpha_s} + \frac{1}{B_z^2 \sigma} \right) \frac{\partial(rV_{\phi,n})}{\partial r}, \]  
(7)

where \( \sigma \) is the electric conductivity and \( \alpha_s = n = D1e(m_1 \nu_{1,n} + m_e \nu_{e,n}) \). The radial and vertical current densities are related by the particle conservation law

\[ \frac{\partial j_z}{\partial z} = -\frac{1}{r} \frac{\partial (r j_r)}{\partial r}. \]  
(8)

The radial variation of the vertical component of the field was taken to be identical to the one that corresponds to null azimuthal velocities and to a linear dependence on radial distance of \( V_{r,n} \), the neutral radial velocity. Then the set of equations (4) to (7) was solved iteratively. Since \( J_z \), the vertical electric current intensity, appears in equation (6), the current densities cannot be derived locally. A circuit model is necessary to relate \( J_z \) to the current densities.

### 3.2. Electric current circuit

Every layer \( l \) acts as a current generator in a circuit that extends above and below this layer. Two main circuit models are possible. One where the flux tube is at the foot of a magnetic loop, and the circuit extends to the other foot of this loop. Another where the flux tube opens and keeps its axial symmetry. In this case, currents transverse to the magnetic field are required in the upper coronal and lower convective part of the flux tube to close the circuit. In all cases the contributions of every layer to the regions of the circuit above and below it are proportional to the inverse ratio of the resistances of these parts of the circuit. These ratios are different for the two models since in the second case the perpendicular conductivity plays a role. In what follow, to estimate the resistance ratio, we used as a first approximation the ratio of the parallel conductivities integrated from the layer \( l \) to the corona or to the convective zone. As a matter of fact these boundaries were replaced by the lower and upper boundaries of the VAL C atmosphere (Vernazza et al. 1981) that was used to obtain the vertical dependence of the particle number densities. Consequently the contributions \( dj_z^a \) and \( dj_z^b \) to the current flowing above or below every layer \( l \) are given by

\[ dj_z^a = \left( \frac{\partial j_z}{\partial z} \right)_l \frac{R_a}{R_A + R_b} dz, \]  
(9)

\[ dj_z^b = \left( \frac{\partial j_z}{\partial z} \right)_l \frac{R_b}{R_A + R_b} dz, \]  
(10)

where \( (R_a = \int_l^N \sigma ds) \) and \( (R_b = \int_l^1 \sigma ds) \). The resulting current density in the layer \( k \) is

\[ j_z = \sum_{l=k}^{l=N} dj_z^a - \sum_{l=1}^{l=k} dj_z^b. \]  
(11)
and \( J_z = \int 2\pi r j_z \, dr \). Iterations are made between the two systems of equations (5) to (8) and (9) to (11).

3.3. Characteristics of the current system

At a given radial distance, the upper part of the ensemble of current generating layers will send (receive) currents predominantly into (from) the section of the circuit above it since its resistance is lower that the resistance of the section below it. This conclusion reverses for the lower layers of this ensemble. Therefore we expect the field aligned currents to change of sign at some depth in the atmosphere. Similar conclusion holds in the model of current generation in a twisted magnetic field frozen in a plasma. In this case the maximum of the magnetic twist is usually assumed to be located at photospheric level, and currents above and below this level flow in opposite directions. When the partial ionization of the plasma is taken into account, the height at which the vertical currents change of sign depends on the height dependence of the azimuthal velocity. In the numerical application presented here, where the azimuthal velocity at the periphery of the flux tube is constant and equal to 0.3 km s\(^{-1}\), the change of sign occurs at a height of 200 km above the level where the continuum optical depth \( \tau_{5000} \) is unity.

The radial currents are generated in low vertical magnetic fields. The radial current density amplitude shows a maximum and then decreases inwards. Accordingly vertical currents must flow to neutralize the radial current. Two systems of vertical currents are then generated that flow in opposite directions. These currents flow in two cylindrical shells near the boundary of the flux tube.

4. Effects of the currents in the photosphere

![Figure 1. Radial dependence of \( B_z \) the vertical component of the field](image)

The characteristics of the system of currents have been computed for a flux tube of radius 100 km with a vertical magnetic field \( B_z \) on the vertical axis of the tube.
equal to 1000 Gauss and an azimuthal velocity of 0.3 km s\(^{-1}\) at the boundary. The radial variation of the vertical component of the magnetic field \(B_z\) is plotted in Fig. 1. The Figs. 2a, b show the radial dependence of the radial and vertical current densities \(j_r\) and \(j_z\) at the altitudes of -25 and 50 km above \(\tau_{5000} = 1\), and Fig. 2c gives \(j_z\) at the altitudes of 150 and 350 km. Without radial and vertical currents the dependence of the radial velocity would be linear in the photosphere. However, the electromagnetic forces generated by the currents modify the \(V_{r,m}(r)\) law, and at a height of 350 km the radial velocity of neutral changes of sign and stop the influx of angular momentum. Consequently the layers at 350 km and above cannot act as current generator and the radial current density \(j_r\) is null at \(z = 350\) km. The radial currents are generated

![Diagram](image-url)

Figure 2. Radial dependence of: a - \(j_r\) the radial current density, b and c - \(j_z\) the vertical current density, d - \(P(r)/P(r_0)\) the ratio of gas pressure, at the depths of -25 and 50 km (a, b, d) and 150 and 350 km (c) above the level \(\tau_{5000} = 1\) in the VAL C atmosphere. (The current densities are in Ampère m\(^{-2}\))
near the boundary of the flux tube in low $B_z$ field regions as it can be seen by comparison of Fig. 2a,b,c with Fig. 1. The vertical current density $j_z$ profile shows a negative and a positive peak indicating the presence of two cylindrical shells of currents flowing in opposite directions. In each shell the vertical current density $j_z$ changes of sign at about 200 km as shown on Fig. 2c. As it can be seen in Fig. 2d, since the two cylindrical shells of current flowing in opposite directions are generating repulsive forces they create a depression between them. On the other hand, the most internal current shell of current produces a pinch effect and increases the gas pressure inside the flux tube.

4.1. Photospheric upflows in the flux tube inside the internal current shell

The pressure enhancement due to the internal current shell reaches 10 % and could generate upward flows and modify the pressure of the upper atmospheric layers. The amplitude of the upflow can be estimated by solving the system of fluid dynamics equations for vertical motions in presence of some local overpressure $\delta p$

$$\rho \frac{d\hat{v}_z}{dt} = -\frac{\partial}{\partial z} (p^* + \delta p) - \rho \bar{g}, \quad (12)$$

$$0 = -\frac{\partial \rho^*}{\partial z} - \rho^* \bar{g}, \quad (13)$$

where $z$ is the altitude in the solar atmosphere, and $p^*$ and $\rho^*$ are the pressure and density corresponding to hydrostatic equilibrium. Then we assume steady state ($\partial \hat{v}_z / \partial t = 0$) and no density changes, i.e. $\rho^* = \rho$. At a depth of about 50 km, the equations (12) and (13) lead to

$$\frac{\partial}{\partial z} (v_z^2) = 10^{12} \frac{\partial}{\partial z} \left( \frac{\delta p}{p^*} \right) + 5.5 \times 10^4 \left( \frac{\delta p}{p^*} \right) \quad (14)$$

At a height $z$ of 50 km, the relative pressure increase is $\delta p / p^* \approx 0.1$, and in the photospheric layers the vertical gradient $\frac{\partial}{\partial z} (\frac{\delta p}{p^*})$ of the relative pressure increase is negative or null. For a null value of this gradient, a lower limit of the vertical velocity gradient is found, which is

$$\frac{\partial}{\partial z} (v_z^2) = 5.5 \times 10^3 \text{ cm s}^{-2}. \quad (15)$$

Consequently, assuming a constant density, over a vertical distance of 100 km the vertical velocity can reach 2 km s$^{-1}$. Since both the relative pinch effect and the density decrease with height, significant upflows with velocities in the 10-20 km s$^{-1}$ range are expected at a height of about 250 km. Then an increase of the pinch in the chromosphere will slow down the flow.

4.2. Upflows between the two current shells

Between the two current shells, the dominant force is the vertical $\vec{j} \wedge \vec{B}_\theta$ force. This force is directed upwards in the low photosphere and downwards in the
Figure 3. Radial dependence of: a - $P/P(r_0)$ the ratio of the local gas pressure to the external gas pressure, b - $V_{n,m}$ the radial velocity of neutrals at three heights in the atmosphere, respectively 655, 705, 755 km. For each height, the two values for a scale factor give the inverse of the required increase of the radius of the internal and external cylindrical currents for having a positive gas pressure at center and between the two currents shells.
upper photosphere where the sign of $B_\theta$ reverses. However the dominant contribution comes from the dense lower photospheric layers where $j_r B_\theta$ reaches $6 \times 10^{-2}$ Newton at $z = 0$ km. This force is nearly equal to the gravity force. Consequently a significant velocity gradient is present and

$$\frac{\partial}{\partial z} (v_z^2) = 5 \times 10^4 \text{ cm s}^{-2}.$$ 

In the hypothesis of constant density a vertical velocity of about 7 km s$^{-1}$ is reached over a vertical distance of 100 km. These motions are similar to the upward motions in spicules. A velocity of about 60 km s$^{-1}$ is required at the base of spicules in order to rise matter to a height of about 7000 km s$^{-1}$. Velocities in the 40 to 60 km s$^{-1}$ range can be reached at the top of the photosphere since the density is decreased by more than one order of magnitude.

5. Effects of the currents at chromospheric heights

The horizontal force balance equation can be written as

$$\frac{\partial P}{\partial r} + \frac{1}{2\mu} \frac{\partial B_z^2}{\partial r} + \frac{\mu}{8\pi^2 r^2} \frac{\partial J_z^2}{\partial r} = 0. \quad (15)$$

The third term on the LHS of equation (15) gives the $j_r B_\theta$ force. The direction of this force changes between the two vertical current systems.

- 1) At the boundary of the flux tube in the external DC current shell, this force is directed outward and can be compensated only by a pressure gradient. The resulting minimum of pressure $P_m$ between the two current systems is approximately equal to

$$P_e - \frac{\mu}{8\pi^2} \frac{\left(J_z^M\right)^2}{r_0^2},$$

where $P_e$ and $J_z^M$ are the external pressure and the maximum value of the vertical current $J_z$. The external cylindrical current system must open and increase in diameter in order to reduce the amplitude of the outward electromagnetic $j_r B_\theta$ force.

- 2) In the internal current shell the $j_r B_\theta$ force is directed inwards. Integrating equation (15) from the axis of the flux tube to the location where the current $J_z$ is maximum leads to

$$\frac{\mu}{8\pi} \left(J_z^M\right)^2 = \pi r_M^2 \left[ \int_0^{r_1} \frac{B_z^2(x)}{2\mu} \ dx^2 + \int_0^{r_1} P(x) \ dx^2 - P_e \right], \quad (16)$$

where $r_M$ is the radial distance at which $J_z$ is maximum, $P_e$ is the external pressure and $x = r/r_M$. The magnetic field at the boundary of the flux tube is assumed to be negligible.

For low pressure forces, the magnetic field structure is force free, corresponding to a balance between the energy stored in the vertical and azimuthal components of the field. The first term on the RHS of equation (16) represents the energy of the vertical component and is proportional to $1/r^2$. Then, for a constant maximum current $J_z^M$, if plasma is injected from the photosphere into
the chromospheric part of the tube, this tube must widen in order to reduce this term. The equilibrium size of the flux tube will depend on the gas pressure, which is dependent on the flux and ionization state of the gas injected in the tube since, as discussed below, the non-ionized fraction of the plasma can escape across magnetic field lines.

![Graph](image1)

![Graph](image2)

Figure 4. Vertical dependence of the maxima of radial velocities of neutrals (dotted line) and ions (dashed line).

For a prescribed radial dimension of the internal current shell, the radial dependence of pressure necessary to satisfy the force balance equation is shown in Fig. 3a at three heights in the chromosphere. This figure corresponds to a flux tube of 100 km radius with an azimuthal velocity of neutrals high enough to generate a maximum of the vertical current that could confine a 1000 Gauss field in a so called force free situation. The radius corresponding to the force-free
solution is

\[ r_c = \mu J_z / (2\pi \sqrt{\left( \int_0^1 \frac{B_z^2}{\mu} x dx \right)}) \]

With the boundaries conditions used here \((B_z(0) = 1000 \text{ Gauss} \text{ and } V_\theta = 0.3 \text{ km s}^{-1})\), the radial dimension \(r_c\) is less than 100 km. Therefore, as illustrated in Fig. 3a, a significant pressure must be present in the tube to enlarge it to a radial size of 100 km.

Then the high pressure gradient required to balance the \(j_z B_\theta\) force leads to a high speed radial outflow of neutrals as shown in Fig. 3b. This flow of neutrals across lines of force is possible since the densities are small enough for ions and neutrals not being coupled by collisions. The height dependence of the maxima of outward radial velocities of neutrals, when present, and of the inward velocities of ions and neutrals at the same radial distance are represented in Fig. 4. In the photosphere ions and neutrals are collisionally coupled. Higher in the atmosphere neutrals can move across the field lines. The reversal in the sign of the neutral radial velocity takes place around the temperature minimum region.

The situation described in the preceding section requires that a significant upflow of gas balances the escape of neutrals at chromospheric level. The upward velocity \(V_u\) is such that \(V_u \approx V_r \times \Delta r / \Delta z \times n_{\text{chr}} / n_{\text{phot}}\), where \(\Delta r\) is the radial extension of the photospheric pressure increase and \(\Delta z\) is the vertical extension of the chromospheric layer from which neutrals escape with a velocity \(V_r\). \(n_{\text{chr}} / n_{\text{phot}}\) is the ratio of chromospheric to photospheric densities of neutrals. An outflow at a velocity of 10 km s\(^{-1}\) over a depth of 50 km at a height of 800 km implies a moderate upflow \(V_u \approx 2 \times 10^{-3}V_r \approx 20 \text{ m s}^{-1}\). The consequences of the possible upflows and outflows found in this Section and in Section 4 are examined below.

6. Discussion

A schematic representation of an open flux tube is given in Fig. 5, which shows the location and direction of the radial and vertical currents and the motions of the fluids of neutrals and ions. Excluding here the heating that currents can produce (Hirayama 1992), three main effects of the electromagnetic forces generated by DC currents flowing in this flux tube can be distinguished, i.e. coronal abundance anomalies, formation of chromosphere, acceleration of spicules.

6.1. Coronal abundance anomalies

As pointed out in Hénoux and Somov (1992), forced diffusion across magnetic field lines and lift of the plasma to the corona are the necessary ingredients for any model of FIP fractionation. The most quantitative work on coronal abundance anomaly was published by Von Steiger and Geiss (1989) and it was based on ion-neutral separation in a gas injected as a pressure pulse in a magnetic field.

Such conditions occur naturally in the current carrying flux tubes considered in our model, and the ion neutral separation takes place at the right place, i.e. in the chromosphere. Due to the pinch effect in the photosphere produced by the internal current shell, the partially ionized photospheric plasma rises
Figure 5. Cut of the current shells near the external boundary of an open flux tube. Thick arrows - DC current; thin arrows - velocity of neutrals (high First Ionization Potential (FIP) elements); double thin line arrows - velocities of ions (low FIP elements). Spicules rise between the two current shells and the ion-neutral separation takes place at the external border of the internal current shell contributing to the formation of a chromosphere. Notice the change with height of the sign of the azimuthal component of the magnetic field.
into the flux tube and is depleted at chromospheric level in neutral high FIP elements. Consequently the gas inside the internal current shell is enriched in low FIP elements in the chromosphere and above. The possibility to detect the resulting change in abundances are presumably limited to the coronal level since in at chromospheric level the internal current shell depleted in high FIP will be surrounded by the high FIP elements ejected between the two current shell. There must be a lower limit of the height at which the enrichment is high enough for the effect to be detectable and not compensated by the effects of the surrounding. Indeed such model is still qualitative and a quantitative study must be done that would include the modification of the ionization equilibrium by the increase of the re-combination emission associated with the rise in ionization inside the flux tube.

6.2. Formation of chromospheres

The decoupling between ions and neutrals takes place at chromospheric heights and starts around the temperature minimum level. This suggests that the chromosphere - defined as a rise of the ionization degree with height - could result from the ion-neutral separation in concentrated magnetic flux tubes. Energy can be brought into the chromosphere as ionization energy carried by ionized low FIP elements. The energy flux into the chromosphere in the internal current shell is then $F = \bar{\chi_i} n_i V_u$, where $\bar{\chi_i}$ is the mean value of the product of $\chi_i$, the ionization potential, by $n_i$ the number density of low FIP elements. Taking $\bar{\chi_i} n_i = 7.8 \text{eV} \times 10^{13} \text{cm}^{-3}$ the energy flux required to heat the chromosphere is obtained for upflow velocities of 300 m s$^{-1}$ at photospheric level. Such velocity is low compared to the so called macro-turbulence used to explain photospheric line profiles and can be generated by pinch effect as discussed in Section 4.

6.3. Spicules

In open magnetic flux tubes, in the low pressure region between the two current shells, the $j_r B_\theta$ force can accelerate the plasma to high velocities in the chromosphere. Since the plasma motion is not slow down by pinch effects in the chromosphere, these velocities are high enough for the plasma to reach the altitude of about 7000 km reached by spicules, Hirayama (1992) suggested a possible role of Joule heating on spicules formation. We think that the $j_r B_\theta$ electromagnetic force is the best candidate to explain spicule formation. The amplitudes of the upward force and of the upward velocity depend on the flux tube radius and of the azimuthal velocity at the periphery of the tube. Since the values used in this study are moderate, higher values can be used, and there is no doubt that the inflow of angular momentum into flux tubes can generate the required high upwards velocities for spicule formation. However, since spicules are transitory phenomena, the $j_r B_\theta$ force must have a transitory character. This may come naturally from the transitory character of the azimuthal velocities.

References