Dynamics of Flux Tube Ensembles

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Abstract. We study some basic physical processes in the solar atmosphere provided by collective phenomena in the random ensembles of magnetic flux tubes. It is shown that the dynamics of the medium and its evolution depend crucially on the distribution of magnetic structures in space and their physical parameters. Two different types of magnetized regions are discussed. One type possesses closely packed bundles of magnetic flux, and can correspond, for example, to sunspots, plages, or to those regions in high layers of the solar atmosphere where different magnetized domains are composed of a dense conglomerate. The second type of region has a small magnetic filling factor so that magnetic field is concentrated in random ensembles of widely spaced flux tubes. It is shown that the dynamics of these two different ensembles, and, in particular, their dispersion properties, their response to the propagation of acoustic waves, the specific mechanisms of the energy transfer and its release, are completely different. Therefore, these studies may be useful for the diagnostics of the visible layers of the solar atmosphere and for magnetic helioseismology.

1. Introduction

The study of collective phenomena in multiphase media is a general problem important for various physical objects, where parameters of the medium are random functions of coordinates. This problem is of particular importance for the physics of the solar atmosphere, which can be described as strongly inhomogeneous plasma containing ensembles of magnetic flux bundles randomly distributed in space and over their physical parameters. The collective phenomena in these ensembles determine the dynamics of the medium, and, in particular, the response of the medium to the propagation of acoustic waves, the specific mechanisms of energy transfer and its release, and others.

Rich observational data on acoustic wave propagation in different magnetic regions of the solar atmosphere obtained in the past few years have opened a new discipline of acoustic spectroscopy, and show clearly that the effects of the wave propagation are different in differently magnetized regions.

We choose here two different effects which are observed in regions of different magnetic filling factors, and which, we believe have a reasonable explanation within the framework of our theoretical studies.

One is the fact that sunspots and plage regions are observed to absorb large fractions of acoustic wave power incident upon them. It is important that there
are clear and detailed differences in the observed regularities in sunspots and plages (Braun, Duvall and LaBonte 1987, 1988, 1990, Braun and Duvall 1989).

Second, high frequency acoustic halos are observed to extend well beyond the boundary of plages into quite Sun. These halos are believed to be a true solar feature representing the emission of waves (Braun et al. 1992, Fernandes et al. 1992, Brown et al. 1992, Toner and LaBonte 1993).

The observed difference in the response of differently magnetized regions and the understanding of the origin of this difference can give a possible basis for the diagnostics of the magnetic structures of solar atmosphere and their role in helioseismology. Thus, the theory should adequately be able to distinguish the regions with different magnetic filling factors, and describe the effects provided by the specific distribution as well as the randomness of magnetic flux tubes.

In our studies we proceed from the observational fact that the randomly magnetized solar atmosphere can be divided roughly into two different types of regions (Fig. 1). One type has a small magnetic filling factor so that ensembles of

Figure 1. Classes of ensembles of magnetic structures: (a) widely spaced magnetic flux tubes; (b) dense conglomerate of random magnetic flux tubes

widely spaced bundles of magnetic flux are embedded in almost nonmagnetized plasma. The characteristic transverse dimension $R$ of these structures (magnetic flux tubes) is much smaller than the characteristic distance $l$ between them: $R \ll l$. Note that these tubes may possess a wide variety of physical parameters (radius, magnetic field strength, inclination, plasma density inside and outside them, etc.), as well as distances between them. This type of region covers almost 90% of the solar surface outside sunspots (see, for example, Stenflo 1989).

The second type of region possesses random bundles of magnetic flux in the form of dense conglomerates. This kind of closely packed ensemble of magnetic flux tubes occurs, for example, in sunspots (both umbra and penumbra), plages, and in those regions of high layers of solar atmosphere which can be described as a mosaic of random domains where all the parameters of the medium (magnetic field, plasma density, temperature, etc.) change from one domain to another by an order of magnitude.

The dynamics of these two different types of ensembles, and, in particular, their response to the propagation of acoustic waves are quite different. The dispersion properties and the observational spectroscopy of these regions, as
well as the mechanisms of energy transfer from lower to upper layers of the atmosphere, are also different.

The first paper that addressed the problem of wave propagation in an ensemble of widely spaced magnetic flux tubes was that by Ryutov and Ryutova (1976). They introduced a broad distribution function over the flux tube's parameters and deduced a set of equations describing the interaction of sound waves with a random ensemble of widely spaced flux tubes. It was shown that energy transfer from sound waves to the medium is provided by different physical mechanisms depending on the magnetic filling factor and the distribution of flux tubes. First of all, the energy of sound waves can be absorbed by magnetic flux tubes even in the absence of the usual dissipative effects, due to an effect similar to Landau damping: resonant excitation of oscillations propagating along the flux tubes. After that, over a time considerably longer than the Landau damping time, the resonant flux tubes radiate their energy in the form of secondary acoustic (or MHD) waves in the higher layers of the atmosphere. In addition to resonant absorption, resonant scattering of sound waves can also take place. In this case the energy of the primary sound wave is transferred directly to the energy of secondary acoustic waves without a preliminary accumulation of energy in natural flux tube oscillations. A consideration of sound waves with arbitrary amplitude (i.e. the nonlinear regime) allows one to find the maximum energy which can be transferred from sound waves to the medium and the corresponding nonlinear frequency shift.

Beginning in 1982, Parker (1982, 1985) devoted a series of papers to the study of the dynamics of the ensemble of widely spaced identical magnetic flux tubes, and obtained the mean-field equation for plasma containing an ensemble of magnetic flux tubes similar to those obtained by Ryutov and Ryutova (1976). Some methods developed by Parker were used later by Bogdan and his collaborators (Bogdan and Zweibel 1987, Zweibel and Dappen 1989, Bogdan and Cattaneo 1989) who have studied the scattering of acoustic waves by ensembles of identical flux tubes. The results obtained in these studies are of a general interest, but cannot be applied as diagnostics of randomly magnetized solar atmosphere. First, the model of identical flux tubes does not allow to distinguish regions with different magnetic structures; second, just the randomness of solar magnetic structures leads to qualitatively new effects (such as the excitation of flux tube oscillations which carry energy from lower layers to the upper layers of atmosphere, Landau-like damping, a nonlinear frequency shift, an echo effect, etc.) which are absent in a model of identical flux tubes. Note that any result described in a series of papers by Bogdan et al. can be obtained in the framework of our theory if we simply simply replace our general distribution function by a delta-function (Ryutova and Priest 1993a).

The interaction of MHD waves with a random ensemble of closely packed magnetic flux tubes was studied by Ryutova and Persson (1984). They obtained a general set of averaged equations describing large-scale motions of a strongly inhomogeneous medium. It was shown that such a medium behaves as a sink for incoming wave energy: a strong enhancement of the dissipation of long-wave oscillations, with a characteristic wavelength $\lambda$, occurs due to the presence of small-scale inhomogeneities. The characteristic scale of inhomogeneities $R$ was
assumed to be much less than the wavelength:

\[ R \ll \lambda \]  \hspace{1cm} (1)

The efficiency of enhanced dissipation is of the order of \( \lambda^2/R^2 \).

LaBonte and Ryutova (1992) proposed a mechanism to explain the enhanced absorption, similar to that of Ryutova and Persson, and based on the presence of highly varying physical parameters. Ryutova, Kaisig, and Tajima (1991) studied the propagation of nonlinear waves in closely packed ensembles of magnetic flux tubes and found that the energy input in such systems is determined by different scenarios which depend on the statistical properties of the medium. There is an interplay between dispersion, nonlinearity and dissipation, and so the energy of the primary acoustic wave can be transferred to the medium either by the formation of shocks, or by a preliminary storage of energy in a series of solitons and its subsequent release; or the process of enhanced absorption of wave energy can stop at the linear stage due to the usual dissipative effects.

In the present paper we describe qualitatively some basic features of the interaction of acoustic waves with a random ensembles of flux tubes. We consider both, an ensemble of widely spaced flux tubes which corresponds to small magnetic filling factor of the medium (Ryutov and Ryutova 1976, Ryutova and Priest 1993a,b), and closely packed flux tubes where the magnetic filling factor of the medium is close to unity (Ryutova and Persson 1984, and LaBonte and Ryutova 1993). It is important to note, that in both cases, the magnetic filling factor can change considerably, and, with respect to each, the theory is able to distinguish the differences inside each model - this fact from our point of view is a necessary element to use the theoretical predictions for diagnostic goals.

2. The Ensemble of Widely Spaced Flux Tubes

The interaction of acoustic waves and unsteady wave packets with an ensemble of magnetic flux tubes reveals some simple and important features, which, in principle, are observable. A most important role in these effects is played by resonant interaction - both absorption and scattering of the sound wave by flux tubes. We focus on the case when the incident wavelength \( \lambda \) is much larger than the separation \( d \) between tubes, which is in turn much larger than the tube radii \( R \).

The key assumption in the theory is the introduction of a broad distribution function \( f(\Xi) \) for magnetic flux tubes, in order to reflect the great variety of observed magnetic structures and to set up an adequate theory for wave-propagation in such an inhomogeneous medium. \( \Xi \) is the set of flux tube parameters completely determining the individuality of separate flux tubes, namely their radius \( R \), plasma density ratio inside and outside the flux tube \( \eta = \rho_i/\rho_e \), magnetic field \( B \), and flux tube inclination \( n \), where \( n \) is the unit vector along the flux tube axis. The distribution function is normalized to the magnetic filling factor of the medium (containing many flux tubes):

\[ \alpha = \int_0^\infty f(\Xi) d\Xi \]  \hspace{1cm} (2)
As shown by Ryutov and Ryutova (1976) and Ryutova and Priest (1993a) the interaction of an acoustic wave with a random ensemble of flux tubes can result in the excitation of oscillations propagating along those particular flux tubes for which the following resonance condition is satisfied:

\[ \omega = (k \cdot n) v_{ph} \]  \hspace{1cm} (3)

where \( \omega \) and \( k \) are the frequency and wave vector of the acoustic wave, \( v_{ph} \) is the phase speed of flux tube oscillation, and \( n \) is a unit vector along the tube axis. Here \( v_{ph} \) corresponds either to a kink mode (with azimuthal wave number \( m = \pm 1 \)) or to a sausage mode (with \( m = 0 \)) and, by carrying the required information on flux tube parameters, it completely determines its individuality. The phase speeds for kink (Ryutov and Ryutova 1976) and sausage modes (Defouw 1976) are, respectively,

\[ v_{ph}^{m=\pm 1} = \sqrt{\frac{\eta}{1 + \eta}} c_A \]  \hspace{1cm} (4)

and

\[ v_{ph}^{m=0} = \frac{c_t c_A}{\sqrt{c_t^2 + c_A^2}} \]  \hspace{1cm} (5)

where \( c_t \) and \( c_A \) are the sound and Alfvén speeds inside the flux tube. Thus, the condition (3), which contains the frequency and wave vector of the incident wave, the angle of its propagation with respect to the flux tubes, and the physical parameters of the flux tubes, is just the condition under which the the energy of some acoustic wave is transferred to the energy of the oscillation of the particular flux tube. The mechanism of the energy transfer is essentially a nondissipative one and is similar to Landau damping in rarefied plasma. The condition (3) is analogous to the Cherenkov condition in the standard Landau resonance.

The damping rate due to resonance absorption of sound wave and the excitation of kink modes propagating along the flux tubes is proportional to the magnetic filling factor \( \alpha \) (Ryutov and Ryutova 1976)

\[ \nu^{m=\pm 1} = Im(\omega) \simeq \alpha \pi k c_{se} \sin^2 \theta \]  \hspace{1cm} (6)

\( c_{se} \) being the sound speed outside flux tubes. The damping rate corresponding to resonance absorption of sound waves due to the excitation of sausage oscillations has the form (Ryutova and Priest 1993a):

\[ \nu^{m=0} \simeq \alpha \frac{\pi k c_{se}}{4 \cos^2 \theta} \]  \hspace{1cm} (7)

Thus, under the resonance conditions (3) an acoustic wave with its own frequency and wave vector in a whole acoustic spectrum chooses among a great variety of magnetic structures those flux tubes whose parameters satisfy these resonance conditions, and gives them their energy due to the excitation of the corresponding tube oscillations (see Fig. 2a).

As was shown by Ryutov and Ryutova (1976) the oscillating flux tube can be a source of secondary acoustic (or MHD) waves; the oscillations of flux tubes once excited can be radiatively damped through the emission of secondary waves.
Figure 2. Propagation of an acoustic wave through an ensemble of flux tubes: (a) some flux tubes are nonresonant, while others are resonant with respect to kink or sausage modes; (b) radiation of secondary acoustic waves with random phases by oscillating flux tubes.

The corresponding damping rates are as follows (Ryutov and Ryutova 1976, Ryutova 1981):

\[ \nu_{rad}^{m=\pm 1} = \frac{\pi \omega}{1 + \eta} \left( \frac{kR}{2} \right)^2 \left[ \frac{2}{\gamma(1 + \gamma)} - 1 \right] \]  
\[ \nu_{rad}^{m=0} = \frac{\pi \omega}{2} \left( \frac{kR}{2} \right)^2 \frac{c_s^6}{c_{se}^2 (c_{si}^2 + c_A^2)^2} \]  
\( \gamma \) being a specific heat ratio.

The process of the interaction of monochromatic sound waves with the random ensemble of flux tubes can now be easily described: under the Cherenkov condition (3) the resonant flux tubes absorb the energy of sound wave in a time

\[ \tau_L = \nu_L^{-1} \]  

Excited oscillations then propagate along the flux tubes, and in a time

\[ \tau_{rad} = \nu_{rad}^{-1} \]  

the absorbed energy is reradiated as secondary acoustic waves. The time for radiation of secondary acoustic waves is estimated as (Ryutov and Ryutova 1976):

\[ \tau_{rad} \approx \frac{1}{\omega(kR)^2} \]  

For those regions of the solar atmosphere where the magnetic filling factor \( \alpha \) is small, expression (12) is much larger than the Landau-like damping time,
Thus, the energy of the incident acoustic waves remains for a long time in the form of flux tube oscillation energy.

It is clear from expression (12) that the different flux tubes radiate secondary acoustic waves at different heights and over different times: for example, thicker tubes radiate sooner than thinner tubes; and, of course, the radiated waves have random phases (Fig. 2 b):

\[ \Delta \omega \simeq \nu_L \]  

(13)

All these simple features of the interaction of a monochromatic acoustic wave with a random ensemble of flux tubes produce a remarkable phenomena in the propagation of unsteady wave packets through such tubes (Ryutova and Priest 1993b). These phenomena manifest themselves in clear morphological effects, which, in principle, can be observed. For example, the interaction of an unsteady wave packet with a random ensemble of flux tubes can lead to significant spreading of the energy input region over scales much larger than the size of the initial wave packet. Mainly two sets of parameters determine the character of the above effect and the efficiency of the energy input, namely (a) the distribution of flux tubes in space and over their physical parameters (including their noncollinearity) and (b) the "size" of the particular wave packet compared with the Landau damping length (the scale of the resonance absorption of the wave packet). In other words, the character of the effects is determined by the statistical properties of the medium. We can consider two limiting cases. (1) the case of a large wave packet whose size \( D \) is much larger than the Landau damping length \( L_L = c_{se}/\nu_L \):

\[ D \gg \frac{c_{se}}{\nu_L} \]  

(14)

and (2) the case of a short wave packet when \( D \) is much smaller than \( L_L \):

\[ D \ll \frac{c_{se}}{\nu_L} \]  

(15)

Under condition (14) a wave packet interacting with an ensemble of flux tubes is damped away without a considerable displacement: since the Landau damping length is much less than the size of the wave packet, all the resonant flux tubes are excited in the initial area of the wave packet. At a time which is larger than the Landau damping time and less than the time of the radiation of secondary acoustic waves

\[ \nu_L^{-1} \ll t \ll \nu_{rad}^{-1} \]  

(16)

the wave packet is damped away, but the excited flux tubes have not yet radiated secondary acoustic waves. In other words, its energy remains in the form of natural oscillations of resonant flux tubes imitating the initial area of the wave packet. The excited perturbations (kink or sausage mode) propagate along the flux tubes carrying the accumulated wave packet energy to higher layers of the atmosphere with a speed whose projection on the direction of wave packet-propagation is approximately \( c_{se} \). After a time

\[ t \simeq \nu_{rad}^{-1} \]  

(17)
secondary acoustic waves are radiated. Since \( \nu_{\text{rad}} \) depends on the radius of the flux tube (12), different flux tubes radiate at different times; thicker flux tubes, for example, radiate earlier than thinner ones. Similarly, the height of the energy input is different for different flux tubes. This fact leads to a significant spreading of the region where the energy of the initial wave packet is transferred to the medium. The location of emission of secondary acoustic waves, differs, of course, from the expected position of the wave packet in the absence of flux tubes. Presence of noncollinear flux tubes results in formation of large emission regions (the “acoustic halos”) which extend beyond the original location of magnetic flux tubes and greatly exceed the size of the initial wave packet.

For a “short” wave packet qualitatively the picture is similar to that for a “large” wave packet, but is much more complicated and, probably more important for diagnostic goals. Detailed qualitative theory of the interaction of a short wave packet with the random ensemble of flux tubes can be found in Ryutova and Priest (1993b). The condition (15) means that, during the traveling of the short wave packet through the ensemble of flux tubes, both excited and nonexcited resonant flux tubes exist simultaneously.

Indeed, the wave packet traverses any particular flux tube within a time

\[
t \sim \frac{D}{c_{se}} \gg \nu_{L}^{-1}
\]

which is short with respect to Landau damping time. Thus, the wave packet excites resonant flux tubes on its way and propagates further, leaving a trace of excited flux tubes, which in turn radiate secondary acoustic waves. In principle, the first excited flux tubes can already radiate their energy before the wave packet is finally damped away. In this case secondary acoustic waves coexist with the initial wave packet. The particular scenario of wave packet dynamics and the final region of the energy emission depend on the specifics of flux-tube distribution. We believe that the effects described above are directly related to observed acoustic halos which consist in the emission of high frequency acoustic waves in neighboring regions with different magnetic regions (Braun et al. 1992, Fernandes et al. 1992, Brown et al. 1992, Toner and LaBonte 1993).

3. The Ensemble of Closely packed Flux Tubes

The dispersion properties and the dynamics of the ensemble of closely packed flux tubes are essentially different from those of widely spaced flux tubes. As mentioned in the introduction, strongly inhomogeneous media where random magnetic structures form a dense conglomerate behave as a sink of the energy of the acoustic and MHD waves incident upon them (Ryutova and Persson 1984, Ryutova et al. 1991).

According to observational data (Braun, Duvall and LaBonte 1987, 1988, 1989, 1990) magnetic regions on the Sun surface absorb a large fraction of p-mode wave power.

We propose a mechanism for enhanced absorption of p-modes based on the idea that in sunspots and plages the magnetic field has a fine filamentary structure (LaBonte and Ryutova 1993). We consider the sunspot and plages to be a dense conglomerate of closely packed magnetic flux tubes of typical radius
$R$, which is much smaller than characteristic size of sunspot $L$, $L \ll R$, and, also, $R \ll \lambda$. Physical parameters such as magnetic field strength and plasma density vary from one flux tube to another by the order of unity. At the same time we shall distinguish sunspots and plage, making a natural assumption that the magnetic filling factor in sunspots is close to unity, while in plages it is less than unity. We assume a Gaussian profile of the magnetic field in an individual flux tube and define the areal filling factor to be

$$\phi = \frac{R^2}{l^2}$$

where $l$ is the average distance between the centers of neighboring flux tubes.

The physical mechanism responsible for the enhanced absorption of acoustic waves propagating in such an inhomogeneous medium is easily understood. The perturbations of all parameters in a propagating wave will be different inside different flux tubes. In a dense conglomerate, flux tubes have common boundaries. Near those boundaries, strong local gradients of all physical parameters will appear. The equations of motion then contain a large vortex component of the perturbed quantities. The characteristic scale of the perturbations is naturally the size of the background inhomogeneities $R$. The presence of the strong small-scale gradients results in the enhanced absorption of the wave energy with the properties which are in a complete agreement with the observed regularities. There are two sub-regimes corresponding to higher and lower wavenumbers with respect to some critical value $k_c$, which is determined by plasma parameters. In the viscosity-dominant case, for example,

$$k_c = \frac{\nu}{R^2 c_{se}}$$

where $\nu$ is the kinematic viscosity coefficient. The damping rate of p-modes with low wavenumbers $k < k_c$ scales as $k^2$, that is, rises from zero, and reaches some value near the critical wavenumber. Soon after reaching the critical value the damping rate saturates, and becomes independent of the wavenumber, which is in a complete agreement with observational data. The spatial damping rate in the subcritical regime for $k < k_c$ has the form:

$$\text{Im}(k) \simeq k_c \left( \frac{7}{6} k^2 R^2 + \frac{1}{2 k_c^2} \frac{\phi}{\Pi^2} \right)$$

(21)

Here $p_m = B^2/4\pi$ and $\Pi = p_{gas} + p_m$ are magnetic and total pressures inside the magnetic flux tube, the average is an ensemble average over a scale much larger than the radius of inhomogeneity and much less than the wavelength.

The first term in equation (21) corresponds to usual losses in a homogeneous medium,

$$\text{Im} k_h = \frac{7}{6} \frac{k^2}{c_{se}} \nu,$$

(22)

while the second term describes the presence of small scale inhomogeneities. It is obvious that even in the subcritical regime the second term in equation (21) exceeds the first one (cf. equation (1), $k^2 R^2 \ll 1$). The region of $k \geq k_c$
corresponds to more efficient damping which, as mentioned above, soon reaches the saturation and becomes independent of $k$:

$$q_0 = \text{Im}(k) = k_c \frac{1}{2} \frac{\phi < p_m^2 >}{< \Pi^2 >}$$

(23)

Thus, the enhancement factor $f$ in a saturation regime is quite large:

$$f = \frac{3}{7} \frac{1}{k_c R^2} \frac{\phi < p_m^2 >}{< \Pi^2 >} \gg 1.$$  

(24)

To unify the description of both limiting cases we derived the following interpolation formula for the total absorption coefficient $\alpha_T$:

$$\alpha_T = 1 - \exp \left( -q_0 L \frac{k^2}{k^2 + k_c^2} \right)$$

(25)

where $L$ is the sunspot dimension, and $q_0$ is the damping rate in saturation regime (equation (23)). This interpolation formula was used to match the theoretical curves with observational data for six sunspots. We give some examples in Figs. 3-4.

The expressions (19)-(25) give a simple and natural explanation of regularities in the observed properties of p-mode absorption in sunspot and plage regions:

1) The absorbing regions are spatially coincident with sunspots and plage seen on the solar surface, with some differences in detail. The theory gives the enhanced absorption provided by the presence of magnetic field structures, in other words, the absorbing regions should be distinct from the background. For a quantitative estimate, we use the observed quantities for $k$ and $R$, take the filling factor $\phi = 1$ as appropriate for a sunspot, and assume that $< p_m^2 > / < \Pi^2 > \approx \frac{1}{2}$, a value consistent with the models of Maltby et al (1986) for a sunspot with a photospheric magnetic field of 2 kG. This gives an enhancement factor of

$$f = 3 \times 10^2.$$

(26)

The lower filling factor in plage compared to sunspots explains the reduced absorption seen in the observations, despite the larger size of the plage.

2) The fraction of the incident p-mode power absorbed is zero at low wave numbers ($k < 0.1\text{Mm}^{-1}$), then rises to a high value at high wave numbers ($k < 0.4\text{Mm}^{-1}$). The fraction absorbed remains constant at the high value to the observational limit ($k \propto 1\text{Mm}^{-1}$).

From equation (21)-(23) we see that at wave numbers below the critical value $k_c$, the damping rate is proportional to $k^2$. Soon above the critical value the absorption saturates and at large enough wave numbers ($k = 1\text{Mm}^{-1}$) becomes constant. A plot of absorption coefficient versus wave number as calculated from our model for six different sunspots are compared with the observational data and give a good agreement (we give here examples of three sunspots and one giant active region).

3) The onset of absorption occurs at lower wave numbers and the absorbed fraction reaches a higher value in larger sunspots than in smaller sunspots. Typical isolated sunspots absorb 40% of high wave number ($k < 0.4\text{Mm}^{-1}$) p-mode
Figure 3. The absorption coefficient $\alpha_\tau$ vs. wavenumber $k$ calculated from our theoretical model for two sunspots are compared with the observational data. The cross indicates the value of critical wavenumber $k_c$: (a) 1986 October 25, $L = 32$ Mm; (b) the giant active region of 1989 March 10, $L = 120$ Mm.

Figure 4. The theoretical curve and the observed absorption coefficient for the 1988 November spot: (2) vs. spherical harmonic degree of the mode 1, and (b) vs. the wave frequency; the theoretical curve is calculated for the saturation regime which corresponds to the critical $l_c = 309.73$. The observational point corresponds to $l_c = 308$ (Braun et al. 1992).
power incident upon them. Giant spots absorb up to 70%, while small spots absorb only 20%.

Calculated values for the critical wave numbers $k_c$, covering the cases from small spot saturating at 30% (1986 November 20) to the giant active region (1989 March 10) saturating at 70%, show a good agreement with the observational value of wave numbers corresponding to the saturation region (Fig. 3 a, b). Note that the deviation of critical wave numbers for different spots from the mean value of $k = 0.4\text{Mm}^{-1}$ (both in theory and in observations) is quite small. This means that, generally speaking, the physical parameters of medium (mean viscosity $\nu$, sound speed $c_{se}$, the scale of inhomogeneities $R$, etc.) are quite similar in different sunspots. We used the estimate for the kinematic viscosity coefficient based on the observed limit of turbulent velocities in sunspots (Beckers 1976).

4) The absorption level increases with sunspot size. The local damping rate has no dependence on the sunspot size (cf. equation (23)), and the total absorption of a spot scales simply with the path length through the spot (cf. equation (25)).

5) The absorbed fraction is larger in spots than in plages. At the same time, the absorption per unit magnetic field appears to saturate in sunspots compared with plages. The dependence of the enhancement factor (24) on the magnetic field strength is $f \propto B^4$ for weak magnetic fields; at large values of $B$, when $\langle p_m \rangle$ becomes of the order of $\langle \Pi_{tot} \rangle$ the enhancement factor saturates just because it becomes independent on the magnetic field strength. In plages, where the magnetic filling factor is less than unity, the average magnetic pressure is always less than the sum of magnetic and gas kinetic pressure, which explains the observed difference.

6) The dependence on the temporal frequency shows a broad peak in $p$-mode absorption centered approximately at 3 mHz.

In linear theory, frequency scales linearly with the wave number and we should expect the dependence of the absorption on frequency similar to that on wave number. However the presence of inhomogeneities results in the dispersion of the wave (Ryutova, Kaisig, Tajima 1991), and the frequency dependence of enhanced absorption requires a separate study. As a first try we computed the absorption coefficient versus wave frequency dependence for the 1988 November spot at the saturation regime as a function of the wave number. The comparison with observations (Fig. 4) shows the order of magnitude agreement, and, as we are dealing with the saturation regime (above $k_c$) the calculated curve is quite smooth.

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References


Group Discussion

Vekstein: Can you suggest any mechanism for the formation of closely-packed flux tubes from an initially tenuous ensemble of flux tubes with a small averaged filling factor $\phi \ll 1$?

Ryutova: I doubt very much if there is any mechanism which can bring together the thin magnetic flux tubes in those regions where they are too far removed from each other, that is for regions $\phi \ll 1$. The situation is different for those regions where $\phi < 11$, but the density of magnetic elements still is quite high, for example, in plage regions or higher layers of the atmosphere where magnetic field become more diffuse and magnetic structures are more close. In this case the long-range interaction, similar to those long-range van der Waals attraction, can bring together magnetic flux tubes leading to formation of a dense conglomerate.