Vertical Structure and Theoretical Spectra of Accretion Disks

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Abstract. The present status of vertical structure models for accretion disks is briefly summarized. Particular emphasis is given to examining the influence of viscosity and the total column mass of the disk on its vertical structure and predicted spectrum.

1. Introduction

Accretion disks are ubiquitous in the Universe. They are believed to provide the source of radiation of many objects, ranging from active galactic nuclei (AGN), cataclysmic variables (CV), other types of close interacting binary systems, to young pre-main sequence stars (T Tauri type stars). Therefore, studying accretion disks is very important in many branches of modern astrophysics.

The best studied accretion disk objects are CVs, which are systems composed typically of a compact degenerate object (white dwarf in the case of dwarf novae and novalike variables; or a neutron star or a black hole, in the case of low-mass X-ray binary systems), and a late-type dwarf star filling its Roche lobe. The matter lost by the companion star via Roche lobe overflow is captured in the gravitational field of the compact object, but because of an excess of angular momentum it is not immediately accreted on the compact star, but rather forms an accretion disk. The case where both components of the close binary system are “ordinary” (i.e. non-degenerate) stars is known as Algol-type, or eventually W Serpentis (hyperactive Algol-type) stars (see Plavec & Hubeny, this proceedings).

All the information about a distant accretion disk comes from studying its electromagnetic radiation. It is very important to realize, however, that unlike most laboratory situation, radiation is not only the probe of the medium, but also one of the most important energy balance agents. In other words, radiation not only reveals the structure of the disk, but in fact determines it. All diagnostic approaches that treat disk radiation only as a probe are therefore, at least in principle, uncertain.

Exactly the same situation applies in the case of studying stellar atmospheres. This is already a rather mature field. Thanks to a concentrated effort of several research groups during several last decades, new fast numerical methods have been developed that treat the interaction between radiation and matter to a high degree of sophistication, so that we are now able to produce model atmospheres and synthetic spectra of stars with unprecedented degree of real-
ism and accuracy (Mihalas 1978; several review papers in Crivellari, Hubeny & Hummer 1991).

Indeed, theory of stellar atmospheres served as an important guide for modeling accretion disks (Kift & Hubeny 1986, Shaviv & Wehrse 1986; Hubeny 1989; 1991; Wehrse & Shaviv 1991). However, taking the analogy between accretion disks and stellar atmospheres too literally may be dangerous. One of the aims of this paper is to demonstrate this warning on actual model examples.

2. Modeling Vertical Structure

Why we particularly stress vertical structure? Accretion disk is a complicated 3-D structure and, strictly speaking, should be treated as such. However, this is a formidable task. Moreover, basics physics of viscous energy dissipation is still poorly known, so it is not clear whether even most sophisticated 3-D models, which still treat viscosity by means of various ad hoc parameters, would be more than a mathematical exercise.

The problem many be significantly simplified by disentangling the radial and vertical structure. This approximation is possible because for many disks of astrophysical interest the radial extent of the disk is far larger than the vertical extent. In other words, the disk is geometrically thin. Therefore, models are constructed as an at least two-step process. First, assuming a vertically homogeneous (typically some kind of vertically averaged) structure, and assuming an axially symmetric disk, the problem is reduced to a 1-D problem in the radial direction. Solving simultaneously equations of continuity, angular momentum balance, and energy balance one can in principle determine the total column mass, angular velocity, radial velocity, and the total dissipated energy, as a function of radius (Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974; Pringle 1981; Frank, King, & Raine 1992). In order to be able to formulate the energy balance equation without understanding the physics of turbulent viscosity, Shakura & Sunyaev (1973) have introduced the famous $\alpha$-prescription for viscosity, which simply parametrized an unknown kinematic viscosity coefficient, $\nu$, as

$$\nu = \alpha c_s H,$$  \hspace{1cm} (1)

where $c_s$ is the sound speed, and $H$ the typical scale-height of the disk in the vertical direction. $\alpha$ is nothing else than an ad hoc scaling parameter. Alternatively, the coefficient of kinematic viscosity may be expressed through the Reynolds number,

$$\nu = r v_\phi / Re,$$  \hspace{1cm} (2)

where $r$ is the radial distance from the symmetry axis, and $v_\phi = r \Omega(r)$ the circular velocity; $\Omega$ being the angular velocity.

As is customary in the theory of stellar atmospheres, one expresses the total energy dissipated (and therefore radiated away) from the unit disk face area, $D(r)$, through the effective temperature, $T_{\text{eff}}$,

$$\sigma T_{\text{eff}}^4 = D(r),$$  \hspace{1cm} (3)

where $\sigma$ is the Stefan-Boltzmann constant. Assuming further a stationary, Keplerian disk, $\Omega(r) = \Omega_K(r) = (GM_*/r^3)^{1/2}$, ($M_*$ being the mass of the central
star), the effective temperature is given by
\[ T_{\text{eff}} = \left( \frac{3G}{8\pi \sigma} \right)^{1/4} \left( M_\star \dot{M} R_\star^{-3} \right)^{1/4} x^{-3/4} (1 - x^{-1/2})^{-1/4}, \]
(4)
where \( x = r/R_\star \) and \( \dot{M} \) the mass flux. This is the most important result of this so-called “classical” theory – the effective temperature is a given function of the radial distance, and does not depend on the viscosity coefficient.

The second basic equation is the expression for the total column mass, \( \Sigma \), as a function of radius,
\[ \nu \Sigma = (\dot{M}/3\pi)(1 - x^{-1/2}). \]
(5)
Adopting the Reynolds number prescription for viscosity we obtain
\[ \Sigma = (3\pi G)^{-1} \dot{M} M_\star^{-1/2} R_\star^{-1/2} Re x^{-1/2} (1 - x^{-1/2}). \]
(6)
Equation (4) is the basis of the spectroscopic diagnostics of accretion disks. Early studies simply assumed that the predicted spectrum as a function of \( r \) is simply a blackbody spectrum corresponding to \( T_{\text{eff}}(r) \). A better approximation was to assume that the emergent flux is given by the flux computed for a normal stellar atmosphere with the same \( T_{\text{eff}}(r) \) (Mayo, Wickramasinghe, & Whelan 1980; Wade 1984). Both latter approaches were carefully examined by Wade (1988) who concluded that neither one describes the disk spectrum properly. The obvious improvement is to calculate the spectrum consistently, solving the corresponding radiative transfer equation. And here comes the importance of vertical structure: having previously computed the radial structure, we may solve in detail the equations governing the vertical structure (basically, the hydrostatic equilibrium and energy balance equations), together with the radiative transfer equation, which then yields a far more accurate model structure and predicted emergent radiation.

The underlying assumption of all approaches of this type is that the disk is represented by a set of mutually non-interacting annuli (rings), each behaving as an independent radiating slab. The fact that most approaches further assume a Keplerian disk with the classical \( T_{\text{eff}} \) versus \( r \) law (eq. 4) is not essential here. Any run of the effective temperature with \( r \) may in principle be used, as for instance the recently calculated radial structure of the boundary layer and inner disk by Narayan & Popham (1993), and Popham et al. (1993). However, it should be kept in mind that disentangling the vertical and radial structure is generally inapplicable at both the inner and the outer boundary of the disk, and also for geometrically thick disks.

Modeling vertical structure was pioneered by Meyer & Meyer-Hofmeister (1982), who have however used an approximate treatment of radiation (diffusion approximation), and later calculated more along the lines of constructing model stellar atmospheres (Krifz & Hubeny 1986; Shaviv & Wehrse 1986). Later developments were reviewed e.g. by Hubeny (1989, 1991), Wehrse & Shaviv (1991), and Wehrse (this proceedings).

Most of the above references stress two important conclusions. (i) Stellar atmospheres theory provides us with a methodological guide, and in particular with suitable numerical methods, to calculate models of vertical structure...
of accretion disks; but, at the same time, (ii) there are potentially important differences between stellar atmospheres and accretion disks. Very briefly, there are three basic reasons for such a difference, namely (1) gravity varies with the vertical coordinate for disks, but is constant for (plane-parallel) stellar atmospheres; (2) total radiation flux is constant for stellar atmospheres but varies for disks, and (3) the total optical thickness of the disk does not have to be infinite, and moreover is not a priori known.

Nevertheless, most workers in the field still feel that somehow the stellar atmosphere models are capable of providing a satisfactory approximation for the disk radiation, or at least that the effective temperature is the dominant quantity that determines the emergent radiation flux. While it may indeed be so in some cases, there are important situations where the computed emergent flux differs substantially from the stellar atmospheric flux, or, likewise, where the effective temperature is not the only quantity determining the emergent radiation.

The aim of this paper is to clarify this question. To this end, I have calculated a series of models for a single ring of a disk, each having the same effective temperature, while other parameters were varied. The classical theory would then predict the same emergent spectrum for all these rings, while models which calculate the vertical structure self-consistently, predict generally different emergent radiation. This exercise thus serves a dual purpose. First, it shows for what range of the parameter space are the self-consistent models of vertical structure important from the point of view of spectroscopic diagnostics, and, second, what are the most important parameters influencing the emergent radiation.

The modeling procedure consists in solving equations of vertical hydrostatic equilibrium, energy balance, and radiative transfer by applying the method of complete linearization, first developed for stellar atmospheres by Auer & Mihalas (1969; see also Mihalas 1978), and later modified for accretion disks by Kriz & Hubeny (1986). The starting solution, the so-called LTE-grey model, is calculated as described by Hubeny (1990). The computer program, TLUSDISK, is in fact a variant of my program TLUSTY (Hubeny 1988) designed for computing non-LTE model stellar atmospheres, upgraded recently by using various acceleration schemes (Hubeny & Lanz 1992).

3. Influence of Mass Flux and Viscosity on the Computed Structure and Spectrum

As explained above, the basic strategy is to compute models with the same effective temperature for various combinations of the stellar mass and radius, the mass flux, and the Reynolds number (viscosity). The structure, and therefore the emergent radiation, is generally different because the basic parameters enter the expressions for $T_{\text{eff}}$, $\Sigma$, and others, in different combinations.

As a basic model, I select a ring for a disk with $M_* = 1M_\odot$, $R_* = 5 \times 10^8$ cm (i.e. the corresponding white dwarf radius), and the relative radial distance $x = 1.3611$ (i.e. $x = (7/6)^2$, the distance where the canonical model predicts the maximum effective temperature), and $M = 10^{-11}M_\odot$ yr$^{-1}$. The corresponding effective temperature is, as follows form eq. (4), $T_{\text{eff}} = 16,806$ K. This model schematically represents a typical CV system in quiescence. For simplicity, I
assume a simple H-He composition, with \( N(He)/N(H) = 0.1 \). Although it is possible to compute non-LTE models, most models presented in this section are LTE models. Since the models presented here resemble model atmospheres for white dwarfs, NLTE effects are not very important.

### 3.1. Influence of mass flux

First we study the influence of varying the mass flux. We take the mass and radius of the central star fixed, \( M_* = 1 M_\odot \), and \( R_\star = 5 \times 10^8 \); and also the Reynolds number is fixed, \( Re = 5000 \), which is the value used by Lynden-Bell & Pringle (1974), and Kříž & Hubeny (1986). We then vary the mass flux, and consequently also the radial distance, \( z \), in order to obtain the same effective temperature. Table 1 summarizes the basic parameters of the individual models. The mass flux, \( \dot{M} \), and the relative radial distance, \( z \), are input parameters, while the total (Rosseland) optical semi-thickness of the disk, \( \tau_{\text{tot}} \), is a quantity that follows from the model calculations.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \dot{M} \ [M_\odot \ yr^{-1}] )</th>
<th>( z )</th>
<th>( \tau_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>( 10^{-11} )</td>
<td>1.36</td>
<td>2.4</td>
</tr>
<tr>
<td>C1</td>
<td>( 10^{-10} )</td>
<td>4.54</td>
<td>40</td>
</tr>
<tr>
<td>D1</td>
<td>( 10^{-9} )</td>
<td>10.7</td>
<td>285</td>
</tr>
<tr>
<td>E1</td>
<td>( 10^{-8} )</td>
<td>24.1</td>
<td>1040</td>
</tr>
</tbody>
</table>

The computed structure and emergent flux for these models are displayed in Figure 1. With increasing mass flux, the total column density increases (see eq. 6), and so does the total optical thickness of the disk. Models C1, D1, and E1 are optically thick, while model B1 is marginally optically thin. The larger the total optical thickness of the disk, the higher is the central temperature (see Hubeny 1990 for an analytic expression, which is analogous to the classical grey atmospheric temperature distribution). Figure 1 also nicely demonstrates another feature of the analytical temperature distribution, namely that the surface temperature is virtually independent of the central temperature as soon as the disk is sufficiently optically thick.

As a consequence of increasing central temperature with increasing total column mass, the geometrical thickness of the disk increases (see Fig. 1c) since the scale height is dominated by the central temperature. However, although the structure of the three optically thick models is rather different, the emergent radiation is virtually the same. This demonstrates the important conclusion, that once the model is optically thick, it is primarily sensitive to the effective temperature, and much less to anything else. Consequently, this also indicates that the spectra of optically thick disks are not very different from the model stellar atmosphere spectra. An analogous conclusion has also been recently reached by Long et al. (1994) who have carefully studied the observed and theoretical spectrum of the bright novalike variable IX Vel.

It is well known from the theory of stellar atmospheres that the second basic model parameter is the surface gravity. In the case of accretion disks,
Figure 1. Effects of changing the mass flux for models with $Re = 5000$. The computed structure and emergent flux for models B1, C1, D1, and E1. Panel (a): the temperature; panel (b): the density; (c) the geometrical height $z$, as a function of Lagrangian column mass, $m$. Panel (d) displays the emergent radiation flux for all four models (continuum + hydrogen lines only). Notice, however, that the fluxes for models C1, D1, and E1 are indistinguishable on the plot.
the gravity varies from zero at the midplane to a certain value given by the
total geometrical height of the disk. In order to be able to compare disk models
directly to stellar atmospheric models we have to introduce some kind of effective
gravity. Although I am not concerned with this particular question here, it can
be easily demonstrated that the predicted continuum flux is not very sensitive
to effective gravity. Some effects may however be found for the hydrogen Lyman
and Balmer line profiles (see also Wade, Hubeny, & Polidan – this proceedings).

3.2. Influence of viscosity

Next, we construct a parallel set of models, denoted B2, C2, D2, and E2, which
are characterized by the same parameters as their counterparts B1, C1, D1, and
E1; the only difference is a different Reynolds number, $Re = 1000$. The basic
model properties are summarized in Table 2.

Table 2. Computed disk ring models for $Re = 1000$

<table>
<thead>
<tr>
<th>Model</th>
<th>$\dot{M} [M_\odot \text{ yr}^{-1}]$</th>
<th>$x$</th>
<th>$\tau_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>$10^{-11}$</td>
<td>1.36</td>
<td>0.09</td>
</tr>
<tr>
<td>C2</td>
<td>$10^{-10}$</td>
<td>4.54</td>
<td>5.1</td>
</tr>
<tr>
<td>D2</td>
<td>$10^{-9}$</td>
<td>10.7</td>
<td>11</td>
</tr>
<tr>
<td>E2</td>
<td>$10^{-8}$</td>
<td>24.1</td>
<td>129</td>
</tr>
</tbody>
</table>

Figure 2 compares the temperature distribution of models B1 and B2 (left
panel), and C1 and C2 (right panel). We see that temperature distribution for
optically thin models (B) differs substantially, while it is very similar for optically
thick models (C) (besides the temperature in the central parts of the disk, which
is however inconsequential as far as the predicted spectrum is concerned).

The difference between models is even more dramatically demonstrated in
Figure 3 where the emergent radiation flux is plotted. The Balmer jump appears
in emission for very optically thin model (B2; $\tau_{\text{tot}} = 0.09$); likewise the higher
Balmer lines show a complicated structure (emission in a near wing with a
central reversal). Also, the UV continuum is markedly different between B2 and
marginally optically thick model C2 ($\tau_{\text{tot}} = 5.1$). In order to illustrate the latter
point better, I have computed a full synthetic spectrum in the UV region, which
is displayed on Figure 4. In addition to the quite different continuum shape
shown already on Figure 3, there is also a difference in predicted line profiles.
In many cases, emission lines for the optically thin model B2 appear as a mirror
image of corresponding absorption lines for the marginally optically thick model
B1.

Since all the input parameters for the model but the Reynolds number are
the same, the above results indicate the possibility that the value of Reynolds
number may be constrained by the observations, in case the disk is optically
thin. I stress, however, that all calculations presented here are done for a single
ring. In reality, one has to construct models for all rings of a disk which may
have considerably different properties, so any differences found by examining a
single ring are likely to be less visible when the whole disk model is constructed.
On the other hand, the observational techniques were recently developed that
Figure 2. Comparison of the computed temperature structure for models B1 and B2 (left panel), and C1 and C2 (right panel). The models differ by the assumed value of Reynolds number, $Re = 5000$ for models labeled 1, while $Re = 1000$ for models labeled 2.

Figure 3. Emergent radiation flux (continuum + hydrogen lines only) for models B2, C2, D2, and E2. The flux for model E2 is indistinguishable from that for model D2.
enable us to extract observed spectra of individual rings (e.g. Rutten et al. 1993; Wood, this proceedings). Therefore, the details of the structure and the predicted spectra even for individual disk rings are now becoming diagnostically quite important.

4. Conclusions

The presented simple exploratory calculations demonstrate that for a constant effective temperature, the total optical thickness of the disk is the most important parameter that controls the shape of the emergent spectrum.

For optically thick disk rings the computed spectrum the virtually the same regardless of viscosity, geometrical height, central temperature, etc. In contrast, for optically thin disks ($\tau < 5$) the spectrum is very sensitive to the actual thickness. Since the optical thickness is a rather sensitive function of the adopted viscosity coefficient, the predicted spectrum reflects the changes in viscosity. Although the results presented here concern only a single ring in a disk, they indicate that a comparison between observations and carefully constructed models of vertical structure of accretions disks may provide observational constraints on the value of disk viscosity.

Acknowledgments. I am grateful to Mirek Plavec for helpful discussions and for critical reading of the manuscript.
References

Hubeny, I. 1988, Comp. Phys. Commun. 52, 103