MAGNETOCONVective PATTERNS

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Abstract. Two different modelling procedures are discussed. In the first, the magnetic field is
assumed to be so weak that flux tubes are advected passively. Kinematic modelling then provides
a means of describing two-dimensional behaviour at the solar surface and of estimating the rate at
which flux diffuses owing to supergranular motion. At the other extreme, strong fields control the
pattern of convection and a variety of two- and three-dimensional spatiotemporal structures can be
identified in the mildly nonlinear regime. Streaming instabilities and pulsating waves are described
as an example.

Key words: magnetoconvection, diffusion, sunspots

1. Introduction

The first studies of magnetoconvection were motivated by the need to understand
energy transport in sunspots, but it soon became clear that the interaction between
magnetic fields and convection is a fascinating subject in its own right. The com-
petition between the destabilizing effect of a superadiabatic temperature gradient
and the stabilizing effect of the magnetic field produces a remarkably rich variety of
behaviour: linear theory allows steady convection, stationary oscillations and trav-
eling waves; weakly nonlinear theory describes transitions from these solutions to
modulated waves, pulsating waves and chaos, while idealized numerical experiments
exhibit an even greater variety of behaviour. Intensive investigation of these prob-
lems had made magnetoconvection a paradigm for the study of double convection.
At the same time there have been ambitious simulations of turbulent compressible
convection and its effect on an imposed magnetic field. We have yet to connect these
two approaches and to follow the transition from the relatively ordered pattern of
mildly supercritical convection to the short-lived structures that appear in the fully
turbulent regime.

The most striking feature of convection in the solar photosphere is the instan-
taneous order of the coherent structures that correspond to granules, mesogranules
and supergranules. To be sure, these structures are ephemeral and only survive for
a turnover time. Nevertheless, they suggest that we can gain some understanding of
magnetoconvection in the Sun by developing models with planforms that are well-
defined. In this paper I shall discuss two different approaches to this problem. First,
I shall describe some simple kinematic models of the interaction between convec-
tion and magnetic fields at the solar surface. The three-dimensional flow is reduced
to a two-dimensional pattern which allows us to compute the rate at which weak
fields diffuse across the surface. Of course, the flow depends on subphotospheric
dynamics and in Section 3 I shall review recent progress in modelling two- and


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three-dimensional magnetoconvection and demonstrate some of the patterns that arise. Then, in the final section, I shall point the way ahead.

2. Kinematic models

![Graphs](image)

Fig. 1. Velocity field for a single axisymmetric source, given by equations (3) and (4) with $V = R = H = 1$. (a) Radial velocity $v(r)$ and (b) vertical velocity $w(r)$. (After Simon and Weiss, 1989.)

The aim of the work described in this section, carried out in collaboration with George Simon and Alan Title, has been to explore the effect of cellular convective patterns on weak fields at the solar surface. The assumption is that small flux tubes are passively advected by two-dimensional horizontal flows, so that their motion can be represented by following test particles, or corks. These particles travel with the local fluid velocity, which is derived from simple analytical expressions. An individual mesogranule or supergranule is represented by an axisymmetric source: so the horizontal outflow

$$\mathbf{u} = v(r)\hat{r} = -\nabla \phi(r)\hat{r},$$  \hspace{1cm} (1)

where $r = r\hat{r}$ is the position vector with respect to the centre of the source. A good fit to observed mesogranules is provided by the gaussian potential

$$\phi(r) = \frac{1}{2}VR \exp[-(r/R)^2],$$  \hspace{1cm} (2)

whence

$$v(r) = V(r/R) \exp[-(r/R)^2],$$  \hspace{1cm} (3)

while the associated vertical velocity $w$ depends on the divergence of the horizontal flow, so that

$$w = \frac{2VH}{R^3} (R^2 - r^2) \exp[-(r/R)^2],$$  \hspace{1cm} (4)
where $H$ is an appropriate scale height (Simon and Weiss, 1989). The variation of $v$ and $w$ with $r$ is shown in Figure 1.

An assembly of sources interact with one another to produce a tesselated cellular pattern. If corks are distributed randomly over the plane they will be rapidly swept into a linear network and then migrate towards the nodes, which correspond to sinks located at stagnation points of the two-dimensional flow. Simon and Weiss (1989) showed that mesogranular velocity fields, derived from Spacelab 2 observations by correlation tracking, could be accurately represented by suitably calibrated sources and that the resulting cork patterns agreed with the observed magnetic fields. Subsequently, Simon, Title and Weiss (1991) used synthetic velocity fields generated from axisymmetric sources to investigate the relationships between exploding granules, mesogranules and supergranules. Figure 2 shows three stages in the development of an irregular magnetic network as the result of velocities corresponding to three fixed supergranules and a collection of ephemeral mesogranules, which are themselves transported by the supergranular flow.

The same approach can be used to study the diffusion of weak magnetic fields across the solar surface. Leighton (1964) realised that supergranules led to a random walk of magnetic flux elements (and incorporated this diffusion in his dynamo model). More recently, Sheeley and his collaborators have demonstrated that the spread of flux from active regions can be described by a combination of turbulent diffusion, with a diffusion coefficient $D = 600 \text{ km}^{2}\text{s}^{-1}$, and a poleward meridional flow of $10 \text{ m s}^{-1}$ (see Sheeley's paper in this volume). We first studied the effect of a fixed velocity pattern, where the mean square displacement $\langle d^2 \rangle$ of randomly distributed corks at first increases quadratically with time as they move towards cell boundaries, then slows down as they move along the boundaries and finally saturates as they accumulate at sinks. Initially, the diffusion coefficient $D = \langle d^2 \rangle / 4t$ rises linearly but it then attains a maximum and falls off inversely with time. If
we suppose that the whole pattern is replaced with a new, uncorrelated pattern after a lifetime $T$, then we expect an assembly of close-packed sources to yield a diffusion coefficient $D = \alpha^2 R^2 / 4T$, where the coefficient $\alpha$ is of order unity. Taking $R = 15$ Mm, $T = 30$ hr we obtain $D = 520 \alpha^2$ km$^2$ s$^{-1}$, which is in the right ballpark.

![Graph](image)

**Fig. 3.** Diffusion of magnetic fields through supergranular motion at the solar surface. (a) The supergranular diffusion coefficient $D$ as a function of the cell lifetime $T$, for $R = 14000$ km, $V_{\text{max}} = 0.5$ km s$^{-1}$. (b) The dimensionless diffusivity $D^* = DT/R^2$ as a function of the dimensionless velocity $V^* = VT/R$. (After Simon et al., 1994.)

In a more elaborate numerical simulation the individual supergranules were randomly distributed, with a mean lifetime $T$; as each supergranule died, it was replaced with a new one located so as not to overlap a pre-existing source. After a time $t \gg T$ the diffusion coefficient fluctuated about a mean value $D$. Figure 3a shows $D$ as a function of $T$: as expected, $D \propto V^2 T$ for $T \ll R/V$, while $D \propto R^2 / T$ for $T \gg R/V$. The results can be represented in dimensionless form by plotting $DT/R^2$ against $VT/R$, as shown in Figure 3b. For a typical supergranule we take $V = 1.63$ km s$^{-1}$, corresponding to peak speeds of around 0.5 km s$^{-1}$ in the array of interacting sources; then we find that $D \approx 500 - 700$ km$^2$ s$^{-1}$, in agreement with the value used by Sheeley. This work will be described in a paper by Simon, Title and Weiss (1994).

Other problems remain to be addressed. Our models tend to produce isolated clumps rather than the linear network observed in the quiet Sun. To maintain that, we require continually emerging bipoles. In places the measured diffusion coefficient is much smaller, showing that the magnetic field exerts forces which affect its motion. Consideration of the Lorentz force then leads us into dynamical investigations.

3. Dynamic models

At the other extreme to kinematic models are numerical experiments on convection in a strong magnetic field, as in the umbra of a sunspot. I shall describe some
Fig. 4. Two-dimensional pulsating waves in a vertical magnetic field, followed over a half-period. Left, streaklines of the velocity field and, right, magnetic field lines. (After Proctor \textit{et al.}, 1994.)
recent work on idealized configurations by Paul Matthews, Michael Proctor, Alastair Rucklidge and myself at Cambridge. We have carried out a systematic study of two- and three-dimensional magnetoconvection for a fully compressible, weakly stratified fluid in the presence of an imposed vertical magnetic field. Earlier two-dimensional results have already been described (Hurlburt et al., 1989; Weiss et al., 1990; Weiss, 1991). One feature of the new results is the amazing variety of oscillatory and wavelike solutions that appear even when the system is only mildly supercritical. So it is first necessary to classify the zoo of possible solutions (Proctor and Weiss, 1993). This becomes especially important as we struggle to explain the structure of sunspot penumbrae, where the mean field is no longer vertical.

To illustrate these effects I shall focus on streaming instabilities; they lead to a new type of solution which we call pulsating waves (Proctor and Weiss, 1993; Matthews et al., 1993). Figure 4 illustrates the behaviour of a vigorously pulsating wave in two dimensions (Weiss, 1989; Proctor et al., 1994). The sequence of events is as follows: convection rolls develop and tilt (say) to the right; transport of linear momentum leads to a streaming flow, rightwards at the top of the layer and leftwards at the bottom, which enhances the tilt and eventually suppresses convection. The field lines are dragged over until the Lorentz force halts the shear and reverses the streaming motion. Then convection starts again and the whole process is repeated with the opposite sense of streaming. The solution is strictly periodic and has a symmetry such that after half a period it is exactly reflected about a vertical plane.

Fig. 5. Schematic bifurcation diagram, showing the kinetic energy of convection as a function of the Rayleigh number $R$, with branches of steady, travelling wave, modulated wave and pulsating wave solutions. Filled (hollow) circles indicate local (global) bifurcations. (After Proctor et al., 1994.)

It is also important to establish the bifurcation structure associated with this pattern of behaviour. Careful investigation shows that, as the superadiabatic gradient (represented by a Rayleigh number $R$) is increased in this regime, convection sets in at a stationary bifurcation, leading to a stable steady solution. This solution has symmetry about vertical planes centred on rising and falling plumes but that symmetry is soon broken, giving rise to (leftward or rightward) travelling waves.
Fig. 6. Alternating rolls in three dimensions. Streaklines are projected onto the walls of the box. Streaming instabilities lead to the development of pulsating waves which rotate anti-clockwise through 90° in a quarter-period. (After Matthews et al., 1994.)
The travelling waves in turn undergo an oscillatory instability, producing modulated waves. The latter involve tilting and streaming but lack the spatiotemporal symmetry of pulsating waves, which is only acquired at a more subtle global bifurcation. Thus the full structure is as shown in Figure 5. As convection grows yet more vigorous, the spatiotemporal symmetry is broken and the motion becomes chaotic.

In three dimensions the streaming instability becomes more complicated still (Matthews et al., 1994). Rolls again tilt to produce streaming perpendicular to their axes but the shear does not hinder convection in an orthogonal plane. So the new set of rolls has axes that are rotated by 90°. Thus the pulsating waves develop into alternating rolls whose axes rotate persistently in the same sense. Figure 6 shows an example where the axes rotate anti-clockwise, followed over a quarter-period.

The solutions presented in Figures 4 and 6 show how regular structures can persist into the nonlinear regime. In three dimensions behaviour rapidly becomes more complicated and our emphasis now is on analysing the spatiotemporal structures that appear as we press further into the nonlinear regime.

4. Future prospects

I have selected a few examples to illustrate approaches to modelling solar magnetoconvection. There are many other problems to be tackled. For example, Matthews et al. (1992) looked at weakly nonlinear behaviour in obliquely inclined fields and fully nonlinear solutions have recently been obtained (Hurlburt, Matthews and Proctor, 1994). Watson (1994) has studied two-dimensional magnetoconvection in a trapezoidal container (as a simple model of the varying inclination in a sunspot) and has found a rich but different bifurcation structure. In a different context, earlier studies of instabilities driven by magnetic buoyancy have recently been extended from two (Cattaneo and Hughes, 1988) to three dimensions. In this configuration the magnetic field is initially horizontal and confined to an isolated layer, embedded in a stably stratified region. The new results indicate that the instability first develops with no variation along the direction of the field but a secondary instability leads to stationary three-dimensional structures which develop into isolated loops (Matthews, Hughes and Proctor, 1994). This calculation is the first dynamical model to indicate how Ω-loops can be formed at the base of the convection zone, so that they can eventually emerge in active regions at the photosphere (Zwaan, 1992).

The idealized problems that I have described show how mildly nonlinear behaviour can be analysed and understood. At the other extreme are more ambitious studies of turbulent magnetoconvection, which relate more closely to what we see at the surface of the Sun. These are discussed in the paper by Nordlund in this volume. The challenge now is to forge a link between these two competing approaches. It remains my hope that it will be possible to extract a simplified description of behaviour that is spatially and temporally chaotic and that we shall be able to relate this description to the intermittent structure of magnetic fields in the turbulent regime.
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References
