RECIPE FOR SOLAR POLARIMETRY

EGIDIO LANDI DEGL'INNOCENTI
Dipartimento di Astronomia e Scienza dello Spazio, Università di Firenze, Largo E. Fermi 5, 50125 Firenze, Italia

Abstract. Some of the most widely used methods for deducing the magnetic field vector from spectropolarimetric observations of the solar atmosphere are reviewed. Particular emphasis is given to those methods that are sufficiently simple and model-independent to fully deserve the name of “recipes for solar polarimetry”. In the last section a simple formula is proposed for the diagnostic of stochastic magnetic fields.

Key words: polarization, magnetism, solar physics

1. Introduction

Spectropolarimetric profiles of lines that are formed in the active solar atmosphere show clear signatures of the presence of magnetic fields. Obvious as it may seem, this statement keeps in itself one of the most compelling and difficult tasks of solar physics: to deduce the value of the magnetic field vector from the signatures observed in such profiles.

Although the story started almost a century ago with the first observation of a magnetic field in a sunspot (Hale, 1908), progress in this research field has been rather slow, mainly for the fact that the solar atmosphere has revealed to be more structured than it might ever been thought. This implies that, in interpreting any polarimetric observation of the solar atmosphere, the simple approximation of considering the magnetic field constant along the line of sight and uniform over the observed area is, in general, a very rough schematization of the real physical situation.

Moreover, it can even be questioned a-priori whether the magnetic field in the solar atmosphere can be simply described as a purely deterministic physical quantity, and not, more precisely, as a stochastic quantity having, at any point in space and at any time, a probability-distribution around a suitable average value. The existence of turbulent velocity fields, as diagnosed by the analysis of spectral line profiles in magnetic (and non-magnetic) regions, quite naturally suggests the idea that the magnetic field may also be found in a turbulent form, thus requiring a stochastic description.

For measuring solar magnetic fields, simple recipes have indeed been developed, or, in other words, simple methods (or series of operations) have been devised to obtain a “measurement” of the magnetic field from observations. Unfortunately, many among these methods can only be applied when the simplifying assumptions specified above (B constant along τ and uniform over the field of view) are fulfilled. In some cases the second assumption can be relaxed, although one has to stick to a
simplified two-component model with a magnetic region covering a fraction $\alpha$ of the field of view and a non-magnetic one covering the remaining fraction $(1 - \alpha)$.

An important characteristic of the different methods is their sensitivity to the thermodynamic parameters defining the physical situation of the atmosphere where the spectral line is formed. Methods that imply the smallest number of assumptions on the atmosphere (or model-independent methods) are obviously to be preferred.

In the following we will just review some of the simplest and most widely used "recipes" for solar polarimetry, restricting ourselves to the diagnostic of magnetic fields based on the Zeeman effect and mainly focusing on those methods that are (to a large extent) model-independent. In the last section we will present some newly developed results concerning the diagnostic of stochastic magnetic fields.

This review is by no means complete. An excellent review that covers the same topic has been recently written by Solanki (1993). Older reviews are those of Stenflo (1978, 1985), Landi Degl’Innocenti (1985), and Semel (1986).

2. Simple Formulae for Magnetic Field Diagnostic

The most direct way of measuring the intensity of the magnetic field is the one of deriving the Zeeman splitting from the separation of the $\sigma$ (and $\pi$) components as observed in the intensity profile of suitable spectral lines. This method is simple and fast and it does not even require polarimetric analysis of the radiation, but it can be applied only to spectral lines that are found in the infrared (around 10 $\mu$m). Unfortunately, in this spectral domain the spatial resolution of the present observing facilities is still rather poor. Moreover, lines that are found in this region of the spectrum are formed in rather high layers of the solar atmosphere.

Apart from this simple method, the most traditional recipes for the diagnostic of solar magnetic fields at the photospheric level are those based on the weak-field approximation. If the amplitude of the magnetic field is such that the Zeeman splitting is everywhere (at any $\tau$) smaller than the Doppler broadening, the magnetic field vector can be recovered through the following equations (cf. Landi Degl’Innocenti and Landi Degl’Innocenti, 1973, Jefferies et al., 1989, Landi Degl’Innocenti, 1992)

$$B \cos \theta = -8.57 \times 10^4 \frac{1}{g} \left( \frac{5000}{\lambda_0} \right)^2 \frac{V(\lambda)}{\left( \frac{dI}{d\lambda} \right)}$$

$$\tan 2\chi = \frac{U(\lambda)}{Q(\lambda)}$$

$$B \sin \theta = 1.71 \times 10^5 \left( \frac{5000}{\lambda_0} \right)^2 \sqrt{-\frac{1}{g^2 - \delta} Q^*(\lambda_0)} \left( \frac{d^2I}{d\lambda^2} \right)_{\lambda=\lambda_0}$$

$$B \sin \theta = 1.71 \times 10^5 \left( \frac{5000}{\lambda_0} \right)^2 \sqrt{-\frac{1}{g^2 - \delta} Q^*(\lambda) \left( \frac{\phi'}{\phi''} \right)} \left( \frac{dI}{d\lambda} \right)_{\lambda=\lambda_w}$$

$$B \sin \theta = 9.89 \times 10^4 \left( \frac{5000}{\lambda_0} \right)^2 \sqrt{\frac{1}{g^2 - \delta} Q^*(\lambda_w) (\lambda_w - \lambda_0)} \left( \frac{dI}{d\lambda} \right)_{\lambda=\lambda_w}.$$
In these equations \( B, \theta, \) and \( \chi \) are, respectively, the amplitude (in Gauss), the polar angle, and the azimuth angle defining the magnetic field vector; \( \lambda_0 \) is the wavelength corresponding to line center (to be expressed, as all other wavelengths, in Å), \( \lambda_w \) is a wavelength in the line wings, \( \tilde{g} \) is the effective Landé factor of the line, \( \phi \) is the Voigt profile describing the line absorption coefficient for \( B = 0 \) (an apostrophe means derivation with respect to wavelength), \( I(\lambda), Q(\lambda), U(\lambda), \) and \( V(\lambda) \) are the (wavelength dependent) Stokes parameters, \( Q^*(\lambda) \) is the \( Q \)-Stokes parameter defined in the “preferred frame” (the one where the positive \( Q \) direction lies in the plane containing the line of sight and the magnetic field), and, finally, \( \delta \) is a correction that has to be applied when considering anomalous Zeeman patterns \( (\delta = 0 \) for Zeeman triplets) and is given by

\[
\delta = \frac{1}{80} g_d^2 (16s - 7d^2 - 4)
\]

where \( g_d, s, \) and \( d \) are given in terms of the Landé factors and \( J \)-quantum numbers of the upper and lower level by the expressions

\[
g_d = g_u - g_l
\]
\[
s = J_u(J_u + 1) + J_l(J_l + 1)
\]
\[
d = J_u(J_u + 1) - J_l(J_l + 1).
\]

Eqs. (1)-(5) require that the magnetic field is weak. Moreover, each of them is derived by supposing that some of the magnetic or thermodynamic parameters are \( \tau \)-independent. The situation is summarized in Table 1, where a star means that the corresponding parameter has to be \( \tau \)-independent for the equation to hold (the quantity \( w_M \) appearing in the table represents the component of the macroscopic velocity along the line of sight). Note that, apart from the weak-field approximation and the other restrictions summarized in Table 1, Eqs. (1)-(5) are model-independent, in the sense that they are valid for any stellar atmosphere and also for non-LTE lines (neglecting however atomic polarization effects). For two-component atmospheres they are still valid if by \( I(\lambda) \) we mean the contribution to the intensity profile emerging only from the “magnetic atmosphere” (unfortunately, this contribution is smeared in the observations by the “non-magnetic” contribution).

<table>
<thead>
<tr>
<th>Equation</th>
<th>( B \cos \theta )</th>
<th>( B \sin \theta )</th>
<th>( \chi )</th>
<th>( w_M )</th>
<th>( \Delta \lambda_D )</th>
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Other traditional formulae for the diagnostic of magnetic field vectors are those based on a series of drastic assumptions on the various physical parameters affecting the Stokes profiles emerging from the solar atmosphere. To obtain such formulae, one writes the LTE transfer equation for the Stokes parameters in the matrix form

\[
\frac{d\mathbf{I}}{d\tau} = \mathbf{K}(\mathbf{I} - \mathbf{B})
\]  

(6)

where \( \mathbf{I} = (I, Q, U, V)^\dagger \) is the Stokes vector, \( \mathbf{K} \) is the propagation matrix (see e.g. Landi Degl’Innocenti, 1976 for its definition), and \( \mathbf{B} = (B_P, 0, 0, 0)^\dagger \) is the source vector, with \( B_P \) the usual Planck function. Supposing \( \mathbf{K} \) independent of \( \tau \) – which implies the \( \tau \)-independence of the parameters \( B, \theta, \chi, \Delta \lambda_D \) (Doppler broadening), \( a \) (damping constant), and \( \eta_0 \) (line strength) –, and assuming a linear behavior of the Planck function as a function of \( \tau \)

\[
B_P = B_0 + B_1 \tau ,
\]  

(7)

one gets for the emerging Stokes parameters

\[
\mathbf{I} = B_0 \mathbf{U} + B_1 \mathbf{K}^{-1} \mathbf{U}
\]  

(8)

where \( \mathbf{U} = (1, 0, 0, 0)^\dagger \).

This formula, that can be generalized to the case of chromospheric lines by adding an exponential contribution to the \( \tau \)-behavior of the source function (see Lites et al., 1988 for further details), is generally referred to as the Unno-Rachkovsky equation. Being largely model-independent (the linear behavior of the Planck function mimics the basic structure of the solar atmosphere, while the other thermodynamic parameters – supposed constant – only affect the explicit expression of \( \mathbf{K} \), the Unno-Rachkovsky equation can be considered as a basic “recipe” for deducing the measurement of the magnetic field vector from observations. An inversion technique (based on a non-linear least-squares fit of the observed profiles to the theoretical profiles) is described in detail in Lites and Skumanich (1990) and yields very precise magnetic field measurements at least in sunspots.

A further “recipe” for the measurement of solar magnetic fields is the one that is at the basis of the so-called line-ratio technique (Stenflo, 1973). Referring to a pair of lines (line 1 and line 2) having the same thermodynamic behavior in the solar atmosphere (like the pair FeI \( \lambda 5250.21 \) and FeI \( \lambda 5247.05 \)), but different effective Landé factors (\( \tilde{g}_1 \) and \( \tilde{g}_2 \), with \( \tilde{g}_1 > \tilde{g}_2 \)), at equal wavelength distance \( \Delta \lambda \) from line center one has

\[
\frac{\tilde{g}_2 V_1(\Delta \lambda)}{\tilde{g}_1 V_2(\Delta \lambda)} = 1 + \mathcal{O} \left[ \left( \frac{\Delta \lambda_B}{\Delta \lambda_D} \right)^2 \right]
\]  

(9)

where the symbol \( \mathcal{O} \) means “order of”, and where \( \Delta \lambda_B \) is the Zeeman splitting. When the observed ratio appearing in the l.h.s. of Eq.(9) is smaller than 1, one can conclude that the magnetic field is “strong” (in the sense that the Zeeman splitting is comparable or larger than the Doppler broadening). This conclusion is totally model-independent and it is also independent of the filling factor \( \alpha \). However, the actual value deduced for the magnetic field is, in general, model dependent.
The final "recipe" that we want to mention here is the so-called "center of gravity method" (Semel, 1967, Rees and Semel, 1979). Defining the centers of gravity of the right and left circular polarization profiles, \( \lambda_r \) and \( \lambda_l \), through the equation

\[
\lambda_{r,l} = \frac{\int (I_c - I \mp V) \lambda \, d\lambda}{\int (I_c - I \mp V) \, d\lambda}
\]

the longitudinal component of the magnetic field can be recovered through the simple formula

\[
B \cos \theta = 4.28 \times 10^4 \frac{1}{g} \left( \frac{5000}{\lambda_0} \right)^2 (\lambda_r - \lambda_l)
\]

where all wavelengths are expressed in Å and the magnetic field in Gauss. This formula is indeed somewhat model-dependent. It is rigorously true only for a constant magnetic field directed along the line of sight (or in the opposite direction) and for Zeeman triplets, while, for anomalous patterns, it is valid only for those having symmetric \( \sigma \)-components. According to Rees and Semel (1979), this formula should however give the longitudinal component of the magnetic field with a maximum error of the order of 20% for any reasonable model atmosphere. A generalization of the same formula to two-component atmospheres has been suggested by Del Toro Iniesta et al. (1990).

3. A Recipe for Stochastic Magnetic Fields

Stochastic magnetic fields have received little attention in the literature. The only contribution in this field known to the author is a relatively old paper by Faulstich (1980). Simple formulae can be however obtained under a number of simplifying assumptions. Here we give the derivation of a formula that generalizes the Unno-Rachkovsky equation for stochastic fields.

We suppose that the magnetic field vector, as well as the other physical quantities affecting the propagation matrix \( K \) (like the line-of-sight velocity, the line strength, the Doppler broadening and the damping constant) are not deterministic, but have a stochastic distribution around mean values with possible correlation effects between different parameters. We also suppose that this stochastic distribution is independent of the spatial coordinate measured along the line of sight (or of optical depth).

Consider now the physical situation at a given time. We suppose that the line of sight crosses a sequence of independent "eddy,, the \( n \)-th eddy covering the interval \((\tau_{n-1}, \tau_n)\), with \( \tau_0 = 0 \). In that interval (or eddy), the physical parameters have a particular realization (drawn at random in the stochastic distribution), and the propagation matrix results in being constant and equal to \( K_n \). Obviously there is no correlation between the matrix \( K_i \) in the \( i \)-th interval and the matrix \( K_j \) in the \( j \)-th interval.

Supposing also that the Planck function has a linear behavior with \( \tau \) (see Eq.(7)), the transfer equation (6) can be solved analytically (with the help of some matrix algebra) to give for the emerging Stokes parameters

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\[ I = B_0 U + B_1 \sum_{n=1}^{\infty} \left\{ \exp(-\Delta \tau_1 K_1) \exp(-\Delta \tau_2 K_2) \cdots \exp(-\Delta \tau_{n-1} K_{n-1}) \right\} \times \left[ 1 - \exp(-\Delta \tau_n K_n) \right] K_n^{-1} \] U

where we have defined \( \Delta \tau_i = (\tau_i - \tau_{i-1}) \), and where the exponential of a matrix has its usual meaning given by the Taylor expansion
\[ \exp(-xK) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} x^n K^n. \]

We now average over all the possible partitions of the \( \tau \)-axis specified by the grid-points \( \tau_1, \tau_2, \ldots, \tau_n, \ldots \).Neglecting any possible correlation between the length of the turbulent eddies and the distribution of the physical parameters and assuming for the length of the eddies a Poisson distribution characterized by a mean value \( \tau_e \), the average is obtained by means of the integral
\[ \int_0^\infty \exp(-\Delta \tau_i / \tau_e) \frac{d\Delta \tau_i}{\tau_e} \int_0^\infty \exp(-\Delta \tau_2 / \tau_e) \frac{d\Delta \tau_2}{\tau_e} \cdots \int_0^\infty \exp(-\Delta \tau_n / \tau_e) \frac{d\Delta \tau_n}{\tau_e} \cdots \]
which brings to the following expression
\[ I = B_0 U + B_1 \sum_{n=1}^{\infty} \left\{ (1 + \tau_e K_1)^{-1} (1 + \tau_e K_2)^{-1} \cdots (1 + \tau_e K_{n-1})^{-1} \right\} \times \left[ 1 - (1 + \tau_e K_n)^{-1} \right] K_n^{-1} \] U.

Finally, we have to average over the distribution of the physical parameters. Taking into account our previous hypothesis over the absence of correlations between such parameters in different eddies, and indicating by the symbol \( \langle \cdots \rangle \) the average over such a distribution, we get
\[ I = B_0 U + B_1 \sum_{n=1}^{\infty} \langle (1 + \tau_e K)^{-1} \rangle^{n-1} \left\{ (K^{-1}) - \langle (1 + \tau_e K)^{-1} K^{-1} \rangle \right\} U. \]

On the other hand, the sum over \( n \) is nothing but the Taylor expansion of the matrix \( [1 - \langle (1 + \tau_e K)^{-1} \rangle]^{-1} \) so that we obtain the final equation
\[ I = B_0 U + B_1 \left[ 1 - \langle (1 + \tau_e K)^{-1} \rangle \right]^{-1} \left[ (K^{-1}) - \langle (1 + \tau_e K)^{-1} K^{-1} \rangle \right] U. \] (10)

It can be easily proved that this equation has the following limiting properties:
a) If the physical parameters are deterministic, the symbols \( \langle \cdot \cdot \cdot \rangle \) can be suppressed and the Unno-Rachkovsky equation (Eq.(8)) is recovered;
b) microturbulent limit
\[ \lim_{\tau_e \to 0} I = B_0 U + B_1 (K)^{-1} U; \]

c) macroturbulent limit
\[ \lim_{\tau_e \to \infty} I = B_0 U + B_1 (K^{-1}) U; \]

Eq. (10) can be considered as a simple generalization of the Unno-Rachkovsky equation to a stochastic medium. For the particular case where correlations are present between stochastic magnetic fields and stochastic velocities, it can be shown that this equation gives circular polarization profiles with skewed symmetry properties around line center and non-zero net circular polarization. Whether this fact is relevant to solar observations has still to be investigated.

Finally, we want to stress the fact that, although Eq. (10) has been obtained for a magnetized medium, it can also be applied to the so-called “scalar case” or, in other words, to a non-magnetic atmosphere. In this case the matrix \( K \) is proportional to unity (\( K = k I \)) and all the Stokes parameters except the intensity vanish. Eq. (10) now reduces to the simpler form
\[ I = B_0 + B_1 \left( \frac{1}{k} - \frac{1}{k(1 + \tau_e k)} \right) \left( 1 - \frac{1}{1 + \tau_e k} \right). \]

This equation can be applied to describe the intensity emerging from a non-magnetic atmosphere where the absorption coefficient \( k \) is a stochastic variable.

References
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