Time Variation of the Hard X-Ray Image during the Early Phase of Solar Impulsive Bursts

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(Received 1993 January 8; accepted 1993 May 7)

Abstract

The time variations of hard X-ray images of four impulsive bursts with simple source structures were investigated in a comparison with the magnetic structure. Two of them are limb bursts. Common variations during the early phase are as follows: i) The hard X-ray brightening seems to start at the top of a single coronal loop. ii) The X-ray source spreads during the increasing phase of the burst in both directions along the loop, and both ends become brighter, especially at higher energies with generally unequal brightness. The loop top is still bright, especially at lower energies, to show three peaks. The speed of the expansion of the X-ray source amounts to about $10^4$ km s$^{-1}$ in three cases. iii) At and after the peak of the X-ray flux, the source tends to be a single source at the loop top, especially at lower energies. iv) The effective temperature for quasi-thermal electrons and their number density during the early phase in the vicinity of the loop top are $(4-6) \times 10^7$ K and $(5-2) \times 10^9$ cm$^{-3}$, respectively, so that the electron mean free path is greater than three-times the local temperature scale height. These observations are consistent with the idea that anomalous resistivity, which triggers impulsive bursts, is caused by electron plasma waves generated in the process of heat conduction.

Key words: Sun: hard X-rays — Sun: impulsive phase of flares

1. Introduction

The time variation of hard X-ray images during the early phase of solar impulsive bursts is very important for studying the mechanism of solar flares. As the origin of hard X-ray bursts, thermal or nonthermal is still controversial (e.g., Dulk 1983; de Jager et al. 1986; Kane 1987). A self-heating ‘anomalous heat conduction’ has been proposed as a cause for the onset and early phase of impulsive hard X-ray bursts (Takakura 1992). This enhanced heating in the coronal magnetic filaments is caused by a Joule dissipation of field-aligned electric current by an anomalous resistivity. The anomalous resistivity is ascribed to electron plasma waves excited by a hump in the electron velocity distribution produced in an enhanced heat conduction initiated by a minor preheating near to the loop top (Takakura 1991).

Many hard X-ray images of solar bursts were obtained with the instruments aboard the SMM and the Hinotori spacecraft during the last solar cycle (e.g., Hoyng et al. 1981a,b; Harrison et al. 1983; Machado et al. 1983; Machado 1983; Takakura et al. 1983a,b, 1984a,b, 1985, 1987; Ohki et al. 1983; Tsuneta et al. 1984a,b, Wang et al. 1987; Kurokawa et al. 1988; Nakajima 1990; Nitta et al. 1990). However, no image during the very early phase of the bursts was obtained, due to insufficient sensitivity of the telescopes as well as a vibration of the Hinotori spacecraft caused by a speed up of a mechanical recorder at the onset of the flare mode. The aim of the present paper is to show the time variation of hard X-ray images of impulsive bursts observed with modulation collimators aboard the Yohkoh spacecraft.

A description of the instrumentation for the hard X-ray telescope, HXT, is given in a paper by Kosugi et al. (1991). There are four energy bands: $L = 13.9-22.7$ keV, $M_1 = 22.7-32.7$ keV, $M_2 = 32.7-52.7$ keV, and...
H = 52.7–92.8 keV. The spatial resolution is about 5", and highest time resolution is 0.5 s. The effective mean collecting area per each collimator is about 1 cm².

2. Observations

Since the start of the HXT observation in 1991 October, about 200 bursts have been observed over a period of 6 months (Kosugi et al. 1992); mappings of 46 impulsive bursts were made. In order to simplify the present study, any burst with a complex image and/or complex time profile are omitted. Furthermore, those bursts having sufficient counts for the mapping at least in two energy bands are selected. Although a major time variation of the X-ray sources, which is the main subject of the present paper, is found during the early phase of the flares, such an early phase is frequently recorded only in the lowest energy band. Thus, about 20 bursts were selected; four of them are shown in the present paper, including two limb flares (E and W limbs) presenting vertical structures above the limbs.

The contour maps of the hard X-rays presented in the following section were obtained by a maximum-entropy method. The pixel size is always set as 2"×2" square but the map size is set to 76°×6°–126° depending on the source size of the bursts. The contour steps are logarithmic √3-times step, and the minimum contour level is 0.1 times the peak brightness in each map. The vertical axis (upward) of the maps is directed to the solar north; however, a fluctuation of about 0°8 is not corrected.

The X-ray images are compared with the vector magnetogram (Makita et al. 1985) observed at the Okayama Observatory of the National Astronomical Observatory.

3. Bursts on the Solar Disk

3.1. 1991 December 7

This burst is classified as C6.4 on the GOES satellite scale. Time profiles are shown in figure 1a. X-ray images are shown in figures 1b and 1c, in which the photon energy and the time of the snapshot are indicated in each panel. The panel number of each X-ray map is identical to the number shown in the top panel of figure 1a indicating the time range of the snapshot of the X-ray map. The X-ray images in times (1) to (3) are available in only the L energy band; the later maps are for both the L and M1 bands. An image in the M2 band is also shown at time (8). In order to show the rapid time variation of the X-ray image at time (6), the snapshots in every one second in the L band are illustrated in figure 1d.

A vector magnetogram observed 1.3 d after the X-ray observation is shown in figure 1e, together with an X-ray map in the upper panel. The spatial scale of the X-ray map in the N-S (up-down) direction is set to be equal to that of the magnetic map. Furthermore, the center and orientation of the magnetic map are adjusted so that the time difference coincides with those of the X-ray map shown in the upper panel. The relative error in the position may be about 10", excluding the time variation of the magnetic structure during the time difference. If we compare the X-ray map with the magnetic structure, the X-ray source seems to be a single coronal loop, and two peaks in the X-ray map are legs or footpoints, which lie on the regions with opposite magnetic polarities within the position error. Thus, the single peak at times (2) to (5) in figure 1b may indicate the location of the loop top. The loop top is also bright in the M1 band at times (4) and (5).

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Fig. 1b. X-ray contour maps of the burst on 1991 December 7. The photon energy, the starting time, and the integration time of the map are given in each panel. The panel number is identical with the number shown in figure 1a, indicating the time of the snapshot. The minimum contour level is 0.1-times the peak brightness in each map and the contour step is $\sqrt{3}$-times the logarithmic step. The map size is 76'6" square. The map center is 15'07" east and 2'54" north from the solar center. The solar north is upward (y-axis) in the maps. In each panel, the X-ray count s$^{-1}$ and count/pixel in the brightest pixel are shown.
We can see that the bright region expands from the loop top towards the foot points at time (6) as shown in figure 1d. The expansion speed was estimated from the movement of a given contour which is 0.47 in brightness [c s^{-1}(pixel)^{-1}] in this case in both panels 2) and 3). It reduces to about $8 \times 10^5$ km s^{-1}. A similar variation can be seen at times (7) to (8). In the decay phase, the vicinity of the loop top becomes bright again, exceptionally so in the M1 band during this burst. The above-mentioned spatial variations of the X-ray source are common in the other bursts, as shown in the following sections. Note that the image sizes of the contour maps normalized in each map (as shown in figure 1b) are almost the same from times (1) to (4), but the source size at a given brightness expands with time, even during this period, though the speed is $2 \times 10^2$ km s^{-1} from times (3) to (4).

If nonthermal electrons predominate during the early phase, the foot points should be brighter than the loop top, even in the L band, since their propagation time is less than one second. Therefore, the electrons during the early phase may be quasi-thermal. An effective electron temperature, which represents the mean energy of the quasi-thermal electrons, and the electron number density about the loop top during the early phase can be estimated in the following way. At time (4), X-ray images are obtained in both the M1 and L bands. Therefore, the effective temperature can be computed based on the ratio between the brightness [count s^{-1}(pixel)^{-1}] in the M1 band and that in the L band in an assumption of thermal X-rays. The transmission efficiency of the filter for the collimators and the pulse-height distribution of the detectors are taken into account, as shown in Appendix. In this case, the ratio is multiplied by a correction factor $\alpha$ for the following reason. Because the total count $C_0$ in the M1 band is always smaller than that in the L band, the spurious image is generally greater in the M1 band. Consequently, we set

$$\alpha = \frac{(C_0/C_1)}{\text{in M1 band}/(C_0/C_1)} \text{ in L band},$$

(1)
Fig. 1d. X-ray maps every one sec in the L band at time (6).

where $C_1$ indicates the sum of counts in those pixels with a brightness greater than 0.1 times the maximum brightness. The corrected ratio between the average brightness on 3 x 3 pixels about the loop top gives $T_e \approx 5.8 \times 10^7$ K, which gives a number density of $n_e \approx 5.0 \times 10^9$ cm$^{-3}$ if the thickness of the source is equal to the width of the loop-like image without taking into account the filling factor.

Before time (4), the X-ray image in the M1 band is not available. However, if the time variation of the brightness about the loop top during the increase phase is attributed to a temperature variation alone, since $n_e$ variation is slow as it propagates at the proton thermal speed, we can estimate the temperature variation. We thus have

$$T_e \approx 4.2 \times 10^7 \text{ K, \quad at time (1)},$$

$$T_e \approx 4.1 \times 10^7 \text{ K, \quad at time (2)}.$$

It is remarkable that the effective electron temperature is already so high before a sharp increase in the X-ray flux, as will also be shown in the following samples.

The electron temperature and the number density can also be estimated from the soft X-ray images observed with the SXT(Tsuneta et al. 1991) aboard Yohkoh. In this case, however, the images are too small to make such a study (Hudson 1993, private communication).
Fig. 2a. Time profiles of the X-ray burst on 1992 February 2. See the caption to figure 1a.
3.2. 1992 February 2

This burst is classified as C5.5 in the GOES scale. The time profiles are shown in figure 2a. This burst is very spiky and has a very hard spectrum. The X-ray images are shown in figures 2b to 2d. In these figures, the X-ray maps in each column show the contour maps in each energy band at times (1) to (10) as indicated in figure 2a.

Although a reliable magnetogram for a comparison with the X-ray map is not available, the brightest point in the X-ray map of the L-band at time (1) in figure 2b may be a loop top. The loop top is also bright in the M1-band. The brightest peak shifts during the early phase from the loop top to the foot points. The speed is about $8 \times 10^3$ km s$^{-1}$ from times (2) to (3), if we assume a semi-circular loop. The loop top again becomes brightest in the decay phase (Times 5 to 7 and 10) in the L band.

Since the X-ray images are available in both the L and M1 bands at time (1), we can directly estimate the effective electron temperature and number density in the vicinity of the loop top at this moment:

$$T_e \simeq 5.6 \times 10^7 \text{ K},$$

$$n_e \simeq 5.4 \times 10^9 \text{ cm}^{-3}.$$

Although SXT was not observing this event, we can check the quasi-thermal assumption near to the loop top using the images in three energy bands, (M2, M1 and L). Though the images are not shown, those from $33^m$ to $22^m$ [i.e., time (2)+time (3)] give an effective temperature of $1.05 \times 10^8$ K from the M1 and L bands and $1.13 \times 10^8$ K from the M2 and L bands. This may support the quasi-thermal assumption concerning the loop top during the early phase. Note that the X-rays from the foot points are harden at time (4), and the contribution of a nonthermal component may be appreciable in the M2 and H energy bands.
Fig. 2c. Same as figure 2b, but at later times.
Fig. 2d. Same as figures 2b and 2c, but at later times.
Fig. 3a. Time profiles of the X-ray burst on 1992 January 13. See the caption to figure 1a.
are shown at times (1) to (8), as indicated in figure 3a. A vector magnetogram observed 2.6 d before the X-ray observation is shown in figure 3d, together with an X-ray map. In the magnetic map, the solar limb at the time of the X-ray burst is indicated by a thick solid curve. The spatial scale, center and orientation of the limb are adjusted so as to coincide with those of the X-ray map in the upper panel. Although the magnetic structure is somewhat complex, the X-ray source seems to be a single loop whose foot points lie at regions with opposite magnetic polarities.

The expansion of the source from the loop top to the foot points can be seen during the early phase of the burst from times (1) to (5). The lower estimate of the speed, due to the low time resolution of 16 s, is about $10^8$ km s$^{-1}$ from times (1) to (2).

The effective electron temperature and density at time (1) in the vicinity of the loop top are obtained in the same way as mentioned in section 3.1. Those at time (4) are

$$T_e \simeq 5.0 \times 10^7 \text{ K},$$

$$n_e \simeq 2.0 \times 10^9 \text{ cm}^{-3}.$$  \hfill (6)

At time (1) we have,

$$T_e \simeq 4.4 \times 10^7 \text{ K}.$$  \hfill (8)

Note that the electron temperature and the number density estimated roughly from the SXT at about time (5) are $10^7$ K and $5 \times 10^{10}$ cm$^{-3}$, respectively in the vicinity of the loop top. A more detailed comparison of such physical parameters in the loop, including foot points, is under way (Hudson 1993, private communication). However, because the coronal loop is inhomogeneous, the SXT, which is more sensitive to the lower temperature than the HXT, may give the parameters for regions with lower temperature.

4.2. 1992 April 1

This burst, classified as M2.3 according to the GOES scale, occurred at the east limb. The time profiles are given in figure 4a and the X-ray images are shown in figure 4b. At times (1) to (3), X-ray maps are available in only the L band. After time (4), the maps in each column of figure 4b show the contours in each energy band (L, M1, and M2). In order to show a rapid time variation of the X-ray image at time (4), the snapshots in every one second in the L band are illustrated in figure 4c. Note that although the X-ray maps after $10^{14}$ cm are not shown, the X-rays are always from a single source at the same location as the south source in figure 4b. A temporal change in the X-ray brightness from the center to two ends can be seen at times (1) to (4), as observed in the other examples. The apparent speed estimated from panels 1) and 2) in figure 4c reduces to $1.1 \times 10^4$ km s$^{-1}$.
Fig. 3c. Same as figure 3b, but at later times.
The effective temperature and number density at time (4) about the loop top are

\[ T_e \simeq 7.5 \times 10^7 \text{ K}, \]
\[ n_e \simeq 1.7 \times 10^9 \text{ cm}^{-3}. \]  

At time (1), we have

\[ T_e \simeq 6.1 \times 10^7 \text{ K}, \]  

in the same way as mentioned in subsection 3.1. Note that the SXT images available only after time (4) are saturated in the time range under consideration, and they do not show any impulsive foot points, which are probably behind the limb (Hudson 1993, private communication).

5. Discussion and Conclusion

The time variations of the hard X-ray images of four impulsive bursts with simple source structures are presented. The common characteristics during the early phase are as follows:

i) Hard X-ray brightening starts in the vicinity of the center of a single loop. At this time, the electron temperature for quasi-thermal electrons and their number density about the center are estimated to be \((4.2 - 6.1) \times 10^7\) and \((5.4 - 1.7) \times 10^9 \text{ cm}^{-3}\), respectively, as summarized in table 1.

ii) The source spreads in both directions along the loop, and both ends, which may be foot points, become brighter especially at higher photon energies. However, the two sources generally have unequal brightness. The loop center is still bright, especially at lower photon energies to show three peaks (the...
Table 1. Physical parameters of the loop top at the initial phase (time 1) and the expansion speed $v_p$ of the hot region from the top to the foot points.

<table>
<thead>
<tr>
<th>Date</th>
<th>$T_e$ $(10^7 \text{ K})$</th>
<th>$n_e$ $(10^9 \text{ cm}^{-3})$</th>
<th>$\lambda_e$ $(10^9 \text{ cm})$</th>
<th>$h_T$ $(10^9 \text{ cm})$</th>
<th>$\lambda_e/h_T$</th>
<th>$v_p$ $(10^3 \text{ km s}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 December 7</td>
<td>4.2</td>
<td>5.0</td>
<td>3.4</td>
<td>1.1</td>
<td>3.1</td>
<td>8</td>
</tr>
<tr>
<td>1992 February 2</td>
<td>5.6</td>
<td>5.4</td>
<td>5.9</td>
<td>1.0</td>
<td>6.0</td>
<td>8</td>
</tr>
<tr>
<td>1992 January 13</td>
<td>4.4</td>
<td>2.0</td>
<td>9.8</td>
<td>2.5</td>
<td>3.9</td>
<td>&gt;1.0</td>
</tr>
<tr>
<td>1992 April 1</td>
<td>6.1</td>
<td>1.7</td>
<td>22.</td>
<td>2.3</td>
<td>9.6</td>
<td>11</td>
</tr>
</tbody>
</table>

Fig. 4b. X-ray contour maps of the limb burst on 1992 April 1. The map size is 76″6 square. The map center is the east limb at 1°07 south. See the caption to figure 1b.
center and both ends). The speed of the expansion in the increasing phase is about $10^4$ km s$^{-1}$ in three cases, and more than $10^3$ km s$^{-1}$ in the other case mapped with a low time resolution.

iii) At and after the peak of the X-ray flux, the source tends to be a single source at the loop center, especially at lower energies. This characteristic was commonly observed during the last solar cycle with the SMM and the Hinotori spacecraft (e.g., Hoyng et al. 1981b; Harrison et al. 1983; Takakura et al. 1983b, 1984b, 1987; Wang et al. 1987), and it is ascribed to an increase in the gas density of the loop due to a flow of chromospheric gas from the heated foot points.

The number of bursts which clearly show the above-mentioned characteristics is 9 out of 20 bursts selected as mentioned in section 2. Furthermore, during the early phase of a big burst on 1991 December 3, a time variation similar to the present events was observed. This may suggest that if the sensitivity of the telescope is sufficiently high to be able to obtain images during the earlier phase of weaker bursts, a time variation like that in the present study could be detected in most bursts. Note that in this big burst, the effective electron temperature is about $4.2 \times 10^7$ K even at such an early phase that the X-ray flux is only 1% of peak flux, and that it is 1 minute before a steep increase in X-ray flux to reach the peak 1 minute later.

Characteristics ii) cannot be accounted for by only an energetic electron beam which is accelerated by a field-aligned electric field, since the expansion speed of the X-ray source is one order lower. At the time when a double-footpoint structure with a harder spectrum is formed, the contribution of a nonthermal component may be large in high photon energies at the foot points. This subject is not included in the present paper, and is left to a future study.

It has been shown by Takakura (1991, 1992) with numerical simulations that an anomalous resistivity may be created near to the loop top by electron plasma waves if this region is preheated above about $10^7.5$ K by a sufficiently small heat input, as compared with the total flare energy. The electron plasma waves are excited by a hump which appears in the electron velocity distribution for heat conduction under the locally enhanced temperature, if the mean free path $\lambda_e$ of the thermal electrons exceeds about 3-times the local temperature scale height $h_T$ along the loop (Takakura, 1990), where

$$\lambda_e \simeq 10^4 T_e^2 / n_e \quad \text{cm in cgs units},$$

$$h_T \equiv \left\{ \frac{|dT_e|}{dx} / T_e \right\}^{-1}.$$  

The $h_T$ is derived from the X-ray contour map, and the ratio $\lambda_e/h_T$ is given in table 1. The physical parameters given in this table, are those at time (1) in the vicinity of the loop top except for the last column. As shown in this table, the ratio is already above about 3 at the early phase before the steep increase in the X-ray flux. This result may support the above-mentioned idea that the trigger for the impulsive bursts can be attributed to the electron plasma waves caused by heat conduction in the locally preheated coronal loop.

If an anomalous resistivity appears near to the loop top, a field-aligned electric current which maintains the coronal magnetic filaments is dissipated so as to heat
this region more. The enhanced heat conduction causes anomalous resistivity over a wider region. Consequently, the anomalous heating continues and spreads in both directions in a self-heating way, even after the end of the minor preheating (Takakura 1992). This was named ‘anomalous heat conduction.’ The speed $v_p$ of the expansion of the hot region obtained by the simulation is of the order of the flux-limited heat conduct. It is about 0.3-times the thermal electron velocity, i.e.,

$$v_p \simeq 0.3 \, v_T = 1.65 \, T_1^{1/2} \, \text{km s}^{-1}. \quad (14)$$

The speeds estimated in three bursts are nearly equal to this value. On January 13, 1992, although it was not as high as that given in the last column of table 1, the lower speed could mainly be ascribed to the lower time resolution required for obtaining reliable maps.

Particle acceleration also occurs due to a high field-aligned electric field caused by anomalous resistivity. The electric potential increases with a spreading of the anomalous region, and it can be of the order of 10 Mvolts (Takakura 1992). If nonthermal X-ray component from both foot points is really predominant, the initial current must be bi-directional in a coronal loop.

The authors owe their deepest thanks to the staff of the Institute of Space and Astronautical Science and the staff of NASA Deep Space Network for their support in the operation of this mission. Further, thanks are due to all of the Yohkoh team members.

Appendix

The hard X-ray flux (photons cm$^{-2}$ s$^{-1}$) emitted from the electrons with an energy distribution $N(E)$ in thin plasma with density $n$ (cm$^{-3}$) is given at a distance $R$ (e.g., Brown 1971) as

$$I(K)dK = \frac{2\sqrt{2\pi} \, c \, n \, r_0^2}{3 \, R} \frac{dK}{K} \times \int_K^\infty \sqrt{mc^2/E} \log\left(\frac{1 + \sqrt{1 - K/E}}{1 - \sqrt{1 - K/E}}\right) N(E)dE, \quad (A1)$$

where $K$ and $E$ indicate the energies of the photon and electron, respectively, and $r_0 = e^2/mc^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius.

Equation (A1) reduces to

$$I(K)dK = \frac{7.33 \times 10^{-44} \, n \, dK}{K} \times \int_K^\infty \sqrt{E_0/E} \log\left(\frac{1 + \sqrt{1 - K/E}}{1 - \sqrt{1 - K/E}}\right) N(E)dE$$

(photons cm$^{-2}$ s$^{-1}$), \quad (A2)

where $E_0$ indicates the electron rest energy. In the case of thermal X-rays, we have

$$N(E)dE = \frac{2}{\sqrt{\pi}N \, E_T} \frac{dE}{E_T} \sqrt{E/E_T} \exp(-E/E_T), \quad (A3)$$

where $E_T = k_BT$ indicates the thermal energy and $V$ (cm$^3$) is the volume of the source. Equation (A2) reduces to

$$I(K)dK = 8.27 \times 10^{-44} n^2 V \sqrt{E_0/E_T} \frac{dK}{K} \times \int_K^\infty \log\left(\frac{1 + \sqrt{1 - K/E}}{1 - \sqrt{1 - K/E}}\right) \exp(-E/E_T) \frac{dE}{E_T}, \quad (A4)$$

Equations (A1) to (A4) are valid in any units of the energies ($K$, $E$, $E_0$, and $E_T$) if they are given by the same units erg, keV, etc., since $N(E)dE$ indicates a number of electrons, e.g., (A3), and all other terms for the energies are divided by some energy to be dimension-less.

The photon flux $F$ (counts cm$^{-2}$ s$^{-1}$) detected by the counter as photons with energies $k_1$ to $k_2$ is given by

$$F = \int_{k_1}^{k_2} \int_0^\infty I(K)\eta(K)G(K,k)[1 - f(K)]dKdk, \quad (A5)$$

where $\eta(K)$ indicates the transmission efficiency of the filter for the collimators (Kosugi et al. 1991), $G(K,k)$ indicates the pulse-height distribution of the detector given by a Gaussian with

$$FWHM \simeq 1.3\sqrt{K(\text{keV})}, \quad (A6)$$

and $f(K)$ indicates the K-escape given by

$$f(K) = 0.336 \left[1 - \frac{1}{\phi(K)} \log(1 + \phi(K))\right], \quad K(\text{keV}) > 33.2, \quad (A7)$$

$$f(K) = 0, \quad K(\text{keV}) < 33.2, \quad (A8)$$

where

$$\phi(K) \simeq 4.0 \left[40/K(\text{keV})\right]^{2.77}, \quad K(\text{keV}) < 100. \quad (A9)$$

The contribution by higher energy photons due to the K-escape is neglected, since the X-ray spectrum under consideration is soft.

If $k$ and $K$ are given in keV, (A5) reduces to

$$F = \int_0^\infty I(K)\eta(K)[1 - f(K)] \int_{k_1}^{k_2} G(K,k)dk \, dK, \quad (A10)$$

with

$$G(K,k)dk = \frac{0.723}{\sqrt{K}} \exp\{-1.64\frac{(K - k)^2}{K}\}dk. \quad (A11)$$
References


