On magnetic fields, stellar coronae and dynamo action in late-type dwarfs

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ABSTRACT
The chromospheric and coronal emission from late-type dwarf stars is thought to depend on the stellar magnetic fields. The observed relations between the average surface magnetic fields $B_z f_x$, the filling factors $f_x$ and the Rossby numbers $Ro$ are discussed and compared with the predictions of simple dynamo theory. The Ca ii excess flux density $\Delta F_{\text{HH}}$ as a fraction of the bolometric flux is found to correlate most closely with $Ro$, a conclusion similar to that drawn by Noyes et al. Coronal scaling laws, based on the minimum energy loss hypothesis of Hearn, are compared with observed coronal parameters, and on average can account for the observed trends. The total coronal heating flux required is found to scale approximately as $Ro^{-1}$. If magnetic reconnection provides this flux, a combination of the plasma parameters required to match the empirical heating flux can be found. The scalings of the implied coronal field and surface magnetic flux with $Ro$ cannot be reconciled with magnetic flux conservation, suggesting that not all the surface flux extends to the corona. The dependence of chromospheric and coronal parameters on $Ro$ is not yet understood. However, the correlation between the Ca ii excess flux density $\Delta F_{\text{HH}}$ and the X-ray flux (more strictly, the coronal emission measure) can be accounted for by relating these quantities through the electron pressure.

Key words: MHD – stars: activity – stars: chromospheres – stars: coronae – stars: late-type – stars: magnetic fields.

1 INTRODUCTION
Spatially resolved observations of the Sun show the morphology of the magnetic fields present. At the level of the photosphere direct measurements can be made; in higher layers the plasma emission is assumed to follow the field geometry. Although the mechanisms that heat the chromosphere and corona are not known in detail, the available evidence strongly suggests that the magnetic field controls coronal heating, whether by magneto-hydrodynamic (MHD) waves or reconnection processes, and that the heating of the chromosphere has both magnetic and non-magnetic components (see the reviews in Ulmschneider, Priest & Rosner 1990).

Spatially averaged magnetic fields can be measured in the photospheres of other cool main-sequence stars (see e.g. Saar 1990), and the fields and area filling factors can be found by modelling. Structures analogous to sunspots have been detected on some stars (e.g. Bopp & Stencel 1981; Hall 1976; Rodonó 1986), and the presence of activity cycles is well established through rotational modulation of emission in the Ca ii H and K lines (e.g. Baliunas & Vaughan 1985; Baliunas 1986; Dravins 1990; Saar & Baliunas 1992a). It is therefore reasonable to assume that the features observed in the solar chromosphere, transition region and corona have their analogues on other main-sequence stars.

The observed magnetic fields are thought to arise from dynamo action at the base of the subphotospheric convection zone. Reviews of the theory and recent progress can be found in Moss (1986) and Tuominen, Moss & Rüdiger (1991). The basic mechanisms of the dynamo action involve interaction between rotation, differential rotation and convection. After the field has been amplified in the deep regions of the convection zone it emerges through the photosphere in large-scale structures, such as active regions, or in the supergranulation network. The magnetic field structure above the photosphere has to be inferred from the plasma
emission. Images of the quiet Sun, obtained in ultraviolet emission lines, show the supergranulation up to about $6 \times 10^4$ K, but not at higher temperatures (Reeves 1976). Either the field lines diverge (see e.g. Gabriel 1976), or non-radial flux tubes overlap apparently to fill the supergranulation cell interiors.

Systematic studies of the fluxes in emission lines formed in the chromosphere and transition region and of broad-band X-ray fluxes, for a wide variety of stars, have shown the existence of scaling relations between such fluxes (Ayres, Marstad & Linsky 1981; Oranje 1986; Rutten & Schrijver 1987), which also hold for solar spatially resolved data (Capelli et al. 1989). Flux-flux correlations appear to arise from the energy balance within the atmosphere. For example, the tight correlation between transition region lines formed between temperatures of $\sim 2 \times 10^4$ and $10^5$ K can be understood in terms of a unique shape of the emission measure distribution with temperature, determined by the radiative power function, when radiation balances only a small non-thermal energy input (Jordan et al. 1987).

Because of the expected dependence of the heating process on the dynamo-generated magnetic fields, there have also been a number of studies of correlations between emission fluxes (or luminosities) and stellar parameters such as the rotation rate ($P_{rot}$) or the Rossby number ($R_o = P_{rot}/\tau_e$, where $\tau_e$ is the turnover time at the base of the convection zone) (e.g. Pallavicini et al. 1981; Mangeney & Praderie 1984; Noyes et al. 1984; Vilhu 1984; Simon, Herbig & Boesgaard 1985; Basri 1987; Simon & Fekel 1987; Maggio et al. 1987; Montesinos & Jordan 1988; Dobson & Radick 1989; Stepien 1988, 1989).

If the emission can be expressed in terms of the local plasma parameters, then such studies offer the hope of understanding the control of these parameters by the stellar dynamo parameters. Jordan & Montesinos (1991) showed that coronal parameters such as the temperature, $T_c$, the emission measure, $\text{Em}(T_c)$, and the implied magnetic field required to produce the heating, $B_c$, can all be expressed simply in terms of $R_o$, with a small gravity dependence.

For the limited number of stars with measured magnetic fields, the relations between the emission fluxes, $P_{rot}$, $R_o$, the surface fields $B_s$ and the filling factors $f_s$, can also be examined (Saar & Schrijver 1987; Saar 1990; Stepien 1991). Solar observations have also been used to find a relation between the Ca II H and K flux and $B_{f_s}$ (Skumanich, Smythe & Frazier 1975; Schrijver et al. 1989), which appears also to hold for stellar data.

In this paper we restrict our analysis to F, G and K dwarfs. We exclude M stars because their X-ray spectra cannot be fitted with single coronal temperatures, and because Ca II is not a major chromospheric radiation loss in these stars, as it is in the F-K dwarfs (Linsky et al. 1982). Stars in dwarf RS CVn systems are also excluded on the grounds that they are close interacting binaries (see also Rutten 1987a).

In Sections 2 and 3 we re-examine the relations between the surface magnetic fields, the filling factors, the Ca II H and K fluxes and $R_o$. The coronal scaling laws and the dependence of coronal parameters and the required heating flux on $R_o$ are discussed in Section 4. In Section 5 the scaling between the Ca II and X-ray fluxes is discussed, and the coronal fields are related to the surface fields and filling factors. Section 6 discusses the models used for the dynamo and the computation of the filling factors, and the observations are compared with these models in Section 7.

2 MAGNETIC FIELDS AND FILLING FACTORS

The sample of stars is given in Table 1. We have taken measurements of magnetic fields and filling factors from the compilation by Saar (1990). We have added two more measurements by Saar (1991a) for HD 17925 and V833 Tau, and one by Valenti (1991) for $\xi$ Boo A. The latter is a preliminary result, but the reported $B_{f_s}$ lies between the two previous values. For $\kappa$ Cet we also list the values of $B_{f_s}$ given by Saar & Baliunas (1992b), but they did not separate this product into $B_s$ and $f_s$. The value of $B_{f_s}$ for 61 Cyg A is not reliable since it exceeds the upper limit reported by Saar & Linsky (1985) and Saar (1987). We show this point in plots involving $B_{f_s}$, but do not include it when fitting correlations. Saar & Solanki (1992) gave an estimate of $\pm 20$ per cent for the errors in determining relative values of $B_{f_s}$ using the same technique, but systematic errors, mostly affecting $f_s$, may be as large as $\pm 50$ per cent.

For the Sun, Tarbell, Title & Schoolman (1979) found that about 0.83 per cent of the quiet Sun and about 8.5 per cent of a plage are filled with fields of 1200 G. Sheeley (1966) estimated that $\pm 20$ per cent of the Sun is covered by plage regions. Schrijver (1987) found similar results for active regions, i.e. that around 5–10 per cent of their area is covered by fields of 1000–2000 G. These numbers give a filling factor of $f_s = 2$ per cent. Saar & Schrijver (1987) used a smaller value for the area covered by active regions, but this is compensated by a larger network value at high activity, and their maximum value of $f_s$ is the same. Regarding $f_s = 1$ per cent as a typical low-activity value, we adopt a mean of 1.5 per cent in the plots discussed below.

We re-examine the relations between the magnetic parameters, the various Ca II flux parameters (see Section 3), $P_{rot}$ and $R_o$. We find correlations for the set of 15 stars for which all these quantities are known (see Tables 1 and, later, 2), and for all stars for which the relevant pair of parameters is known, but the restricted set gives more mutually self-consistent relations. All data points are shown, but stars not included in the correlations are indicated by a horizontal line on the corresponding symbol.

Of the possible forms of correlation (i.e. log $B_{f_s}$ with $P_{rot}$, $R_o$, log $P_{rot}$ or log $R_o$) we find that log $B_{f_s}$ is best fitted with a linear dependence on $R_o$, such that for the 15 stars

$$\log B_{f_s} = 3.27(\pm 0.08) - 0.97(\pm 0.08)R_o$$

although this form depends heavily on the solar values. In all the fits we have used a least-squares reduced major axis regression to find the coefficients and the uncertainties (Isobe et al. 1990). The same expression fits the 17 stars for which $B_{f_s}$ and $R_o$ are known.

The Rossby numbers listed in Table 1 have been calculated using the turnover time $\tau_e$ given by Noyes et al. (1984) as a function of $B - V$ for $\alpha = 2$ (the mixing length/pressure scaleheight). It is difficult to give a formal uncertainty in the values of $R_o$. Noyes et al. (1984) found least scatter in their relation between the Ca II flux ratio (see Section 3) and $R_o$ when $\alpha = 2$ was used. The remaining scatter was $\sim 0.10$ dex in $R_o$. The fit given by (2.1) is shown in Fig. 1(a).
The observed filling factors can be fitted with a similar form. For the 15 stars,
\[ \log f_s = -0.01(\pm 0.07) - 0.86(\pm 0.07)R_o, \]
and this is shown in Fig. 1(b). For the 17 stars,
\[ \log f_s = -0.06(\pm 0.06) - 0.82(\pm 0.06)R_o. \]
These results are consistent, to within the uncertainties, with that found by Stepien (1991) using the complete sample of Saar (1990).

Others (e.g. Saar, Linsky & Beckers 1986a) have suggested that the surface magnetic field is determined by equilibrium with the photospheric gas pressure, at a depth in the photosphere represented by \( \tau_{5000} = 1 \). Plots of \( B_s \) against \( P_{\text{rot}} \) or \( R_o \) show little or no correlation, and \( B_s f_s \) shows almost the same dependence on \( R_o \) as does \( f_s \) (Saar 1991a,b). We have used the photospheric models by Peytrellmann (1974) and Kurucz (1979) to calculate the values of \( B_s \). These can be fitted by the expression
\[ \log B_s = 3.46(\pm 0.01) - 3.1(\pm 0.1) \times 10^{-12} \alpha T_{\text{eff}}^4. \]
To compute the effective temperature we use throughout this paper the polynomial fit of \( 5040/T_{\text{eff}} \) with \( B - V \) given by Hauck (1985), which matches very well with the values of \( T_{\text{eff}} \) given by Böhm-Vitense (1981) in the interval \( 0.4 < B - V < 1.2 \).

On average these fields are only 0.10 dex larger than those observed. Assuming that the relative calculated fields are more accurate than those observed, one can use the observed product \( B_s f_s \), which should be more accurate than the separated quantities, to calculate \( f_s \). The fits are, for the 15 stars,
\[ \log f_s = -0.09(\pm 0.07) - 0.89(\pm 0.07)R_o, \]
and for the 17 stars,
\[ \log f_s = -0.10(\pm 0.06) - 0.88(\pm 0.06)R_o, \]
which are consistent with the observed relations (2.2) and (2.3).

The forms for \( B_s f_s \) and \( f_s \) show the saturation characteristics discussed by Vilhu (1984) and Saar & Linsky (1986), giving a filling factor that tends to approximately 1 for very small values of \( R_o \).

Although a dependence on \( P_{\text{rot}} \) would fit most of the stars, it does not provide such a good fit to the slow rotators. Further measurements of magnetic fields for such stars are needed.
3 Ca II ACTIVITY INDICATORS AND THEIR DEPENDENCE ON $Ro$

Skumanich, Smythe & Frazier (1975) found that in the solar atmosphere there is a good correlation between areas of enhanced Ca II H and K emission and regions of strong magnetic field; a topic that has been of continuing interest (Schrijver et al. 1989 and references therein).

Noyes et al. (1984) converted the observed Ca II $S_j$ index into $F'_{HK}$, the chromospheric flux, corrected for the photospheric contribution, and also found $R'_{HK} = F'_{HK}/aT_{eff}^4$. They examined the dependence of these quantities on both $P_{rot}$ and $Ro$ and concluded that, with $a \approx 2$, a plot of $R'_{HK}$ against $Ro$ gives the least scatter. Although they stressed that their sample of data was not sufficient to determine a clearly preferable functional relation between $R'_{HK}$ and $Ro$, they obtained a reasonable fit with

$$\log R'_{HK} = -4.2 - 0.4 Ro.$$  \hspace{1cm} (3.1)

On the other hand, Rutten (1987a) considered the quantity $\Delta F_{HK} = F_{HK} - F_{min}$, where $F_{HK}$ is the flux as defined by Rutten (1987a) and $F_{min}$ is a basal non-magnetic contribution, which exceeds the photospheric correction of Noyes et al. (1984), and plotted $\Delta F_{HK}$ against $P_{rot}$. Fig. 2(a) shows such a plot, using stars from the sample of Rutten (1987a). This plot includes stars that have their rotational periods determined through rotational modulation in the H and K lines (diagonal crosses), through broad-band modulation (vertical/horizontal crosses) and through their orbital periods (squares), and stars that are used in other correlations, but have only $v \sin i$ measurements (triangles). The stars that appear in both Tables 1 and 2 are indicated by circles. As Rutten (1987a) pointed out, there remains a colour dependence. Given the findings of Noyes et al. (1984), we also plot $\Delta R_{HK}/aT_{eff}^4$ against $Ro$, as shown in Fig. 2(b), and find a distinct reduction in the scatter, and a removal of the colour dependence.

![Figure 1](image1.png)

Figure 1. Observed (a) $\log R_{f_0}$ and (b) $\log f$, plotted against the Rossby number $\log Ro$ for stars in Table 1. The lines shown are the fits given by equations (2.1) and (2.2), respectively. The points crossed by a horizontal line are stars not included in the correlations. Vertical lines connect different measurements for the same star.

![Figure 2](image2.png)

Figure 2. (a) Ca II H and K flux density, $\log \Delta F_{HK}$, and (b) $\log \Delta R_{HK}/aT_{eff}^4$ plotted against the Rossby number $\log Ro$. Symbols indicate different methods of measuring the rotation periods: diagonal crosses, from Ca II modulation; vertical/horizontal crosses, from broad-band modulation; squares, from orbital periods; triangles, from $v \sin i$. Circles indicate stars in both Tables 1 and 2. The line shown in (b) is the fit given by equation (3.2).
Table 2. Can and Em(Tc) data.

<table>
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<th>Star</th>
<th>Log Teff</th>
<th>Log FHk</th>
<th>Log ΔRhk</th>
<th>Log Em(Tc)</th>
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FHk for HD 10476, 39587 and 155886 has been computed from the S1 index given by Noyes et al. (1984). For the remaining stars, FHk has been computed from the ΔFHk given by Rutten (1987a). †These stars are not included in the correlations; see text.

- Uncertainties were not given for these FHk measurements (Rutten 1987b). The emission measure for HD 165341 is for the unresolved A+B system.

For the 15 stars that appear in both Tables 1 and 2, the fit is

\[ \log \Delta R_{\text{hk}} = -3.94(\pm 0.03) - 0.50(\pm 0.03)R_0. \]  

(3.2)

Stepień (1989) reassessed the relation between \( R_0 \) and \( B - V \) using a larger set of stars from Rutten (1987a), including stars whose rotation rates were measured via \( v \sin i \). His relation differs from that of Noyes et al. (1984) for \( B - V \) greater than 0.8. He found a weaker dependence on \( T_{\text{eff}} \) in the relation between \( \Delta F_{\text{HK}} \), \( R_0 \) and \( T_{\text{eff}} \) given by

\[ \log \Delta F_{\text{HK}} = 0.68 + 1.60 \log T_{\text{eff}} - 0.43 R_0. \]  

(3.4)

For our sample, the scatter is less with our relation than with Stepień’s. In particular the F stars, for which the values of \( R_0 \) adopted are similar, show less scatter in our plot. Fig. 2(b) shows a remarkably tight correlation considering that the contribution from active regions to the total Ca II emission will vary with the activity cycle, and the stars are observed at random times in the cycles. HD 118100, a single, BY Dra star, deviates most from the mean relation. The effect of excluding this star from the fit has been investigated and the relation for the 14 remaining objects is

\[ \log \Delta R_{\text{hk}} = -3.95(\pm 0.03) - 0.49(\pm 0.02)R_0. \]  

(3.5)

which is closer to the fit (equation 3.2) for the larger sample, as one would expect.

From spatially resolved solar observations of active regions, Schrijver et al. (1989) found that

\[ \log \Delta F_{\text{HK}} = 0.60(\pm 0.10) \log B_\text{fA} + 4.8, \]  

(3.6)

where \( f_\text{A} \) is the filling factor for the active region. With the solar value of \( T_{\text{eff}} \), this converts to

\[ \log \Delta R_{\text{hk}} = 0.60(\pm 0.10) \log B_\text{fA} - 6.0. \]  

(3.7)

As Saar & Schrijver (1987) have noted, stellar observations also follow the form of relation (3.6). We have re-examined the stellar correlation using the same 15 stars used to find relations (2.1) and (2.2). The results are shown in Figs 3(a) and (b), and can be fitted by

\[ \log \Delta R_{\text{hk}} = 0.61(\pm 0.06) \log B_{\text{fA}} - 5.89(\pm 0.17) \]  

(3.8)

and

\[ \log \Delta R_{\text{hk}} = 0.69(\pm 0.08) \log f_\text{A} - 3.88(\pm 0.06), \]  

(3.9)

which are shown as full lines.

The stellar correlation (3.8) is consistent with (3.7), for solar active regions, to within the uncertainties. Combination of the correlations of \( \Delta R_{\text{hk}} \), \( B_{\text{fA}} \) and \( f_\text{A} \) with \( R_0 \) (equations 2.1 and 3.2, or 2.2 and 3.2) gives the same results to within the uncertainties. The relation (3.7) for the solar active regions is shown in Fig. 3(a) as a dashed line. The small differences between the solar and stellar relations (which are in fact within the uncertainties) are not surprising because (3.7) involves the active region filling factor, at 2.4-arcsec resolution, while (3.8) contains the total stellar filling factor. The stellar values of \( R_0 \) are almost constant, so the main stellar correlation is between \( \log \Delta R_{\text{hk}} \) and \( f_\text{A} \). For the Sun, however, where \( R_0 \) is fixed, the correlation is with \( B_{\text{fA}} \). Schrijver & Harvey (1989) have discussed the relation between solar spatially resolved and spatially integrated (i.e. over the whole star) fluxes. They argued that the distribution of magnetic flux densities changes smoothly over the solar...
cycle and should not be divided into two distributions, one for the quiet regions and another for active regions and the activity-enhanced network. For the relation between $\Delta F_{\text{HK}}$ and the magnetic flux they showed that a spatially resolved power law with gradient 0.6 results in a similar power law (gradient 0.56) for the integrated fluxes, but a constant smaller by a factor of 0.8. Their approach goes some way towards understanding why the stellar and resolved solar data follow a similar relation.

A larger sample of stellar magnetic field measurements and also simultaneous studies of variations in the observed fields and Ca ii fluxes would be of value. The variations found by Saar & Balunas (1992) in \kappa Cet do roughly fit the scalings found for the whole sample of stars. Also, modelling of the chromospheres of individual stars to calculate the radiation losses in Ca ii H and K is needed to check the $S_1$ index approach. So far this has been attempted only for the Sun (Noyes et al. 1984).

We conclude that the quantity $\Delta R_{\text{HK}}$ rather than $\Delta F_{\text{HK}}$ gives the tightest correlation with Ro. Since the excitation of the Ca ii H and K emission does not depend on the local filling factor, but will depend on the value of the local chromospheric magnetic field indirectly through the chromospheric temperature and height structure, the correlations with Ro, $B_{\text{HK}}$, and $f_c$ must arise through the systematic behaviour of the magnetic field and filling factor at the level at which the Ca ii emission is formed.

4 CORONAL PARAMETERS, SCALING LAWS AND HEATING

4.1 Coronal scaling laws

It is generally agreed that the heating of stellar coronae is controlled by the magnetic field, irrespective of the details of the unknown heating mechanism, which may be the dissipation of MHD waves or energy released in small-scale magnetic reconnection. The magnetic field in the corona must be determined by the surface fields and filling factors, but, as discussed below, it is likely that not all the surface flux extends to the corona. It is therefore of interest to express the observable parameters in terms of the coronal magnetic field.

The observed emission measure can be written in terms of the intrinsic emission measure $\int N_e^2 \, d\tau$ as

$$\text{Em}(T_e) = \frac{P_e^2 H_0^2}{2k^2 T_e^3},$$

(4.1)

where $P_e$ is the coronal electron pressure and $H/2$ is the isothermal pressure-squared scaleheight, given by

$$\frac{H}{2} = 7.1 \times 10^{-7} \frac{T_e^2}{g_*},$$

(4.2)

and $f_c$ is the unknown coronal filling factor $A_c/A_*$, where $A_c$ is normal to the radiation direction.

Observations with the Apollo Telescope Mount (ATM) on Skylab (Reeves 1976; Reeves, Vernazza & Withbroe 1976) show that at 5-arcsec resolution the supergranulation network continues through the transition region with approximately the same area filling factor (46 per cent) as the Ca ii network (40 per cent) (Skumanich, Smythe & Frazier 1975). However, images in the Fe xii and Fe xiii lines, formed at roughly $5 \times 10^8$ and $7 \times 10^8$ K, respectively, show less distinct structure, and by Mg x, formed at around $1.4 \times 10^9$ K (close to the temperature of the quiet corona), the structure has mostly disappeared, and is replaced by much more amorphous emission. If the field lines diverge to a uniform distribution (see Gabriel 1976), then $f_c = 1$. If small-scale loops are present, however, and on average are not radial, then the emission from along the loops could in projection effectively cover a large fraction of the supergranulation cell interior, giving the appearance of a filling factor close to 1. In the formulation below we set $f_c$ equal to 1, but discuss later the effect that $f_c$ and a non-radial field might have.

The electron pressure can be expressed in terms of the plasma $\beta$ and the coronal magnetic field $B_c$, so that

$$1.8 P_e = \frac{\beta B_c^2}{8\pi},$$

(4.3)

and equation (4.1) can then be rewritten as

$$B_c = 8.6 \times 10^{-10} \left( \frac{\text{Em}(T_e) T_e g_*}{\beta^{1/2}} \right)^{3/4} \text{G}.$$  

(4.4)
Jordan & Montesinos (1991) postulated that, for a fixed value of $\beta$, $B_\gamma$ might depend simply on $R_\gamma$. Using a sample of F, G and K main-sequence stars, for which a single-temperature fit to the coronal temperature is satisfactory (Schmitt et al. 1990), they found that a correlation does appear to be present. The sample of stars used did not extend to small values of $R_\gamma$, where saturation effects might be expected, and a log-linear fit was not justified. However, we have added one star to our sample, CC Eri, which has a very small value of $R_\gamma (6 \times 10^{-5})$, and we now want to compare with photospheric and chromospheric parameters, for which a log-linear fit seems to be more appropriate, so this form is now adopted. The other parameters for CC Eri are $\log E m(T_e) = 29.89$, log $T_e = 6.98$ and log $g_\star = 4.55$. We have also excluded $\alpha^2$ Cr B, which is a dwarf RS CVn system. In the absence of information concerning the relative contribution of the two stars to the X-ray flux, we have used the total emission measure. The numerical constants used below may need to be revised when data for a larger sample of stars become available from observations with ROSAT. Further comparisons of emission measures integrated over height, derived directly from the volume emission measures given by Schmitt et al. (1990), with values obtained from the tabulated X-ray fluxes using the calibration factors adopted by Schrijver, Meew & Walter (1984) suggest that Schmitt et al. (1990) do not allow for the fraction of the coronal flux that is emitted down towards the star ($-1/2$). To compensate for this the volume emission measures of Schmitt et al. (1990) should be divided by $2\pi R_\star^2$, not $4\pi R_\star^2$, and this change (which affects only the constants in the previous log-log correlations, not the powers) has now been made. For consistency we have re-derived other emission measures using the conversion factors given by Schrijver et al. (1984).

We previously adopted a solar emission measure $\log E m(T_e) = 27.50$ from Malinovsky & Heroux (1973). Their emission measure distribution is fact peaks at the temperature of Fe xv and Fe xvi, and is presumably affected by the presence of active regions. [Observations of rotational modulation in several stages of ionization of iron by Neupert (1965) show that the average coronal temperature is close to that where lines of Fe x and Fe xvi are formed, i.e. log $T_e = 6.2$. An earlier analysis of quiet Sun spectra (Jordan 1966) gives a lower emission measure of 27.08. We adopt the quiet Sun value in analytical fits. The revised values of $E m(T_e)$ for stars used in the correlation with $A R H K$ are given in Table 2. Other quantities are given in Table 1 of Jordan & Montesinos (1991). The quality of the spectral fits is given by Schmitt et al. (1990). The errors in log $T_e$ are given as $\pm 0.1$ dex.

The form of the fit is now

$$0.25 \log [E m(T_e) / g_\star] = a - b R_\gamma,$$  \hspace{1cm} (4.5)

where $a = 10.26 (\pm 1.04)$ and $b = 0.39 (\pm 0.04)$, giving

$$\log B_\gamma = 1.20 (\pm 0.04) - 0.39 (\pm 0.04) R_\gamma - 0.5 \log \beta.$$  \hspace{1cm} (4.6)

Further scalings between $E m(T_e)$, $T_e$, $P_e$ and $g_\star$ can be obtained if an assumption is made concerning the energy balance in the corona (see Section 4.2). Hearn (1975, 1977) proposed that coronae tend to a minimum energy loss configuration. He assumed that below some temperature, at the base of the corona, there is no external heating, with the radiation losses being balanced by thermal conduction from the corona. This allows the thermal conduction to be evaluated, and the total coronal losses, by conduction and radiation, can then be found. The total energy loss is then minimized, with respect to a varying temperature, at constant pressure. This provides a relation between $P_e$, $T_e$ and $g_\star$. Craig, McClymont & Underwood (1978) and Rosner, Tucker & Vaiana (1978) used a similar approach to evaluate the conductive flux, but also included a specific heating function and a closed loop geometry. When the loop length is replaced by the hydrostatic pressure-squared scaleheight, the same dimensional scaling law results, with a slightly different constant. In both methods the scaling law results from there being a fixed ratio between the coronal energy losses by thermal conduction ($F_c$) and radiation ($F_r$).

The precise powers in the scaling laws depend on the functional form adopted for the radiative power losses. For temperatures above $2 \times 10^5$ K one can put

$$P_{rad} = A T_e^{-\delta},$$  \hspace{1cm} (4.7)

where $\delta = 1/2$ (with $A = 1.3 \times 10^{-19}$ erg cm$^{-3}$ s$^{-1}$ K$^{1/2}$ (Jordan et al. 1987) or $\delta = 1$ (Craig et al. 1978) (with $A = 5.8 \times 10^{-17}$ erg cm$^{-3}$ s$^{-1}$ K$^{-1}$) have been used. With Hearn's method the general forms of the scaling laws are

$$P_e = 1.6 \times 10^{-27} \frac{g_\star (1 - \delta)}{A^{1/2} (\delta + 1)} \frac{T_e^{15 + 6/\delta}}{T_e}.$$  \hspace{1cm} (4.8)

and

$$E m(T_e) = 9.7 \times 10^{-15} \frac{g_\star (1 - \delta)}{A (\delta + 1)^2}.$$  \hspace{1cm} (4.9)

The data for the sample of dwarf stars studied here are fitted best by

$$\log [E m(T_e) / g_\star] = 1.411 (\pm 0.333) + 3.50 (\pm 0.51) \log T_e,$$  \hspace{1cm} (4.10)

suggesting $\delta = 1$, but allowing $\delta = 0.5$.

Equation (4.9) predicts, for $\delta = 1$,

$$\log [E m(T_e) / g_\star] = 1.32 + 3.5 \log T_e$$  \hspace{1cm} (4.11)

or, with $\delta = 1/2$,

$$\log [E m(T_e) / g_\star] = 4.52 + 3 \log T_e.$$  \hspace{1cm} (4.12)

Because there is a large scatter in the data about the mean relation given by (4.10), which may have a physical origin, one cannot formally conclude $\delta = 1/2$, although $\delta = 1$ apparently gives a better fit. There does seem to be a systematic behaviour in the departures of the observed values of $\log [E m(T_e) / g_\star]$ from (4.10), for a given $T_e$. The F dwarfs are observed to have larger values than the mean, by about a factor of 2.8, while the single giants in the sample discussed by Jordan & Montesinos (1991) have lower values than the mean. Schrijver et al. (1984) recognized the presence of a third parameter in the relation between $E m(T_e)$ and $T_e$, but suggested that this was the rotation period, rather than the gravity, which appears naturally in our formulation. However, the periods for the different types of star also show some systematic behaviour with colour. An alternative source of the deviations from the mean could be the factor $f_c$, and the effect of non-radial magnetic fields. If the coronal field is at an angle $\phi$ to the radial direction, the conductive flux is reduced by $\cos^2 \phi$ (see Hearn 1977). This leads to a factor of $\cos \phi$ on the right-hand side of (4.8) and a combined...
factor of \(f_c \cos^2 \phi\) on the right-hand side of (4.9). If the deviations were real they would indicate that \(f_c \cos^2 \phi\) would be closest to 1.0 for the F dwarfs, smaller for the G–K dwarfs and smallest for the single giants. One obvious factor is that the shallower convective zone of the F dwarfs may produce less widely spaced supergranulation structure, making it easier to fill the corona with more radial fields. (See also Giampapa & Rosner 1984.) Hearn (1977) used similar arguments to deduce that field lines in the quiet corona are on average more inclined than those in coronal holes. However, relation (4.11) (for \(\delta = 1\)) assumes that \(f_c\) and \(\cos^2 \phi\) are 1.0 and should give the largest possible values of \(\text{Em}(T_c)/g_s\) for a given \(T_c\), so the minimum energy loss approach is not entirely satisfactory. If one forces the fit to the observed data points to have a gradient of 3 (for \(\delta = 1/2\)), then the constant in (4.10) becomes 4.63(±0.10), so again the observed maximum values of \(\text{Em}(T_c)/g_s\) are larger than the minimum energy loss theory predicts. The theoretical constants are not significantly different if the approach of Rosner et al. (1978) is used instead.

If one writes equation (4.10) as

\[
\log[\text{Em}(T_c)/g_s] = c + d \log T_c, \tag{4.13}
\]

then the theoretical constants and dependences on \(Ro\) in the further scalings discussed by Jordan & Montesinos (1991) are determined by \(a\) and \(b\) in equation (4.5) and by \(c\) and \(d\) in equation (4.13). For \(\delta = 1/2\) or 1, respectively, these are

\[
\log(T_c g_s^{1/2}) = a - c/4 - bRo \tag{4.14}
\]

and

\[
\log[\text{Em}(T_c)/g_s^{1/2}] = 3a + c/4 - 3bRo, \tag{4.15}
\]

or

\[
\log(T_c g_s^{1/2}) = (8/9)a - 2/9c - (8/9)bRo \tag{4.16}
\]

and

\[
\log[\text{Em}(T_c)/g_s^{1/2}] = (28/9)a + (2/9)c - (28/9)bRo. \tag{4.17}
\]

Thus one can test the self-consistency of the observed relations. The results are summarized in Table 3. The first set of constants gives the observed fits. The second set gives the values using (4.5) and (4.10). The constant \(c = 4.63\) when \(\delta = 1/2\) is used. The third set uses (4.5) with the minimum energy loss relations, either (4.11) or (4.12).

For either \(\delta = 1/2\) or 1 the mean value of \(b\) giving the dependence on \(Ro\), is 0.40. The other constants are more self-consistent with \(\delta = 1\), although \(\delta = 1/2\) is not formally excluded. Thus the minimum energy loss theory, with \(\delta \sim 1\) to \(1/2\), gives on average a satisfactory prediction of the scalings of \(T_c\) and \(\text{Em}(T_c)/g_s\) individually with \(Ro\).

### 4.2 The total heating flux and the coronal magnetic field

We now express the energy losses in terms of the observable parameters and discuss these in relation to the minimum energy loss hypothesis. The empirical losses are also compared with a particular form of energy input.

The total non-thermal energy flux, \(F_n(T_c)\), is the sum of the losses by radiation, \(F_r(T_c)\), and conduction, \(F_c(T_c)\), where

\[
F_r(T_c) = 0.8 \text{Em}(T_c) A T_c^{-6.5} \tag{4.18}
\]

and

\[
F_c(T_c) = -\kappa T_c^{5/2} \frac{d T_c}{dr}. \tag{4.19}
\]

Since at some height \(F_n(T_c)\) tends to 0, an approximation must be made for the conductive flux back from the base of the corona. One approximation that can be used is that of Hearn (1975, 1977), in which the conductive flux is balanced against the radiation losses down to some transition region temperature, \(T_\alpha\). Then, for \(T_c\) greater than \(T_\alpha\), \(P_e = \text{constant}\), \(F_c(T_c) = 0\) and \(\delta = 1/2\),

\[
F_c(T_c)^2 = 2\kappa A 0.8 \left[\frac{P_e}{k}\right] T_c, \tag{4.20}
\]

which from (4.1) can be written in terms of the observable parameters as

\[
F_c(T_c) = \frac{1}{(7.1 \times 10^{14})^{1/2}} [1.6 \kappa A \text{Em}(T_c)/g_s]^{1/2} T_c. \tag{4.21}
\]

A second approximation is to assume that the conductive flux is constant (see e.g. Jordan et al. 1987). With \(P_e\) constant, this implies

\[
\text{Em}(T_c) = \text{Em}(T_c) \left[\frac{T_c^5}{T_\alpha^5}\right],
\]

### Table 3. Constants in equations (4.14)-(4.17) (i) derived directly from the observed fits, compared with those derived (ii) from equations (4.5) and (4.10) and (iii) from the minimum energy loss theory, equation (4.11) or (4.12).

<table>
<thead>
<tr>
<th>Equation</th>
<th>(i) Observed</th>
<th>(ii) Predicted using (4.5) and (4.10)</th>
<th>(iii) Predicted using (4.5) and (4.11) or (4.12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4.14)</td>
<td>9.09(±0.07)</td>
<td>9.11(±0.06)</td>
<td>9.13(±0.04)</td>
</tr>
<tr>
<td>(4.15)</td>
<td>0.39(±0.07)</td>
<td>0.39(±0.04)</td>
<td>0.39(±0.04)</td>
</tr>
<tr>
<td>(4.16)</td>
<td>32.02(±0.10)</td>
<td>31.94(±0.14)</td>
<td>31.92(±0.12)</td>
</tr>
<tr>
<td>(4.17)</td>
<td>8.44(±0.07)</td>
<td>8.81(±0.04)</td>
<td>0.39(±0.04)</td>
</tr>
<tr>
<td></td>
<td>0.42(±0.07)</td>
<td>0.39(±0.04)</td>
<td>0.39(±0.04)</td>
</tr>
<tr>
<td></td>
<td>32.27(±0.11)</td>
<td>32.24(±0.06)</td>
<td>32.22(±0.03)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.39(±0.04)</td>
<td>0.39(±0.04)</td>
</tr>
</tbody>
</table>

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leading to
\[ F_c(T_e) = \frac{1}{7.1 \times 10^7 \sqrt{2}} \kappa T_e^{5/2} g_\ast. \] (4.22)

Thus the total energy losses can be calculated in terms of the observable parameters, and their empirical dependence on \( R_0 \) can be found, assuming initially that no other parameters are important (but see below and Section 5). Using (4.18) and (4.21), one finds
\[ \log F_m = 7.89(\pm 0.08) - 1.00(\pm 0.10) R_0, \] (4.23)
and using (4.18) and (4.22) one finds the same relation, to within the uncertainties:
\[ \log F_m = 8.00(\pm 0.19) - 1.06(\pm 0.18) R_0. \] (4.24)

We can also compare relation (4.23) with the minimum energy loss solution. This fixes the ratio of the losses by radiation and conduction according to
\[ \frac{F_c(T_e)}{F_r(T_e)} = \frac{(\delta + 1)}{0.75 \pm \delta}. \] (4.25)

The total energy flux can then be expressed in terms of either \( T_e \) or \( \Em(T_e) \), using (4.8) or (4.1), giving, for \( \delta = 1/2 \),
\[ F_m(T_e) = 1.38 \times 10^{-14} g_\ast T_e^{5/2} \] (4.26)
or
\[ F_m(T_e) = 2.24 \times 10^{-18} g_\ast^{1/6} \Em(T_e)^{5/6}. \] (4.27)

The uncertainties in \( T_e \) are about \( \pm 0.1 \) dex and, if those in \( \Em(T_e) \) are similar, relation (4.26) is far more sensitive to the errors than is (4.27). This results in less scatter when (4.27) is used to compare with the total empirical losses. Apart from \( \nu^2 \) Col, which has an anomalously high temperature (as can be seen from fig. 3 of Jordan & Montesinos 1991), the main difference between the values from (4.27) and the total empirical losses is only 0.08 dex.

The relations (4.14) and (4.15) can be used (with the constants given in Table 3, column 3) to predict the dependence of \( F_m(T_e) \) on \( R_0 \), for the minimum energy loss solution. This is
\[ \log F_m(T_e) = 8.97(\pm 0.17) - 0.25 \log g_\ast - 0.98(\pm 0.10) R_0, \] (4.28)
in good agreement with the direct fits to the total fluxes calculated from (4.27):
\[ \log F_m(T_e) = 9.03(\pm 0.09) - 0.25 \log g_\ast - 1.05(\pm 0.09) R_0. \] (4.29)

We now allow for the small term in gravity when examining the dependence of the total empirical losses, from (4.18) and (4.21), on \( R_0 \); i.e., without assuming a minimum energy loss solution, the relation is
\[ \log F_m(T_e) = 8.99(\pm 0.09) - 0.25 \log g_\ast - 0.99(\pm 0.10) R_0, \] (4.30)
in good agreement with the minimum energy loss solution. Fig. 4 shows a plot of \( \log F_m(T_e) + 0.25 \log g_\ast \) against \( R_0 \), calculated from (4.18) and (4.21), with relation (4.30) shown as a full line, and the fit to the minimum energy loss values, given by (4.29), shown as a dotted line. As Jordan (1980) has discussed, the total energy loss is insensitive to the assumption of minimum loss. Departures of a factor of 5 from the minimum energy loss ratio of \( F_c(T_e)/F_r(T_e) = 3 \) lead to total losses that are only a factor of 1.8 larger than the minimum value. The hypothesis is therefore very powerful in predicting the total energy input required. As discussed in Section 4.1, the more sensitive test of plotting \( \Em(T_e)/g_\ast \) against \( T_e^3 \) does show departures from the minimum energy loss predictions, but, given the possibility of geometric factors and the observational uncertainties in \( T_e \), we cannot yet say whether or not these are significant.

The values of \( F_m(T_e) \) required by the observations can now be compared with theoretical forms of the heating flux. To illustrate this we use a heating flux of the form
\[ F_m(T_e) = \frac{\delta B^2}{8 \pi} \alpha \nu V_\alpha, \] (4.31)
where \( \delta B^2/8\pi \) is the energy available from a stressed magnetic field, \( \alpha \) is the fraction of the energy thermalized, \( V_\alpha \) is the Alfvén speed, and, if (4.31) represents heating by magnetic reconnection, \( \epsilon \) is determined by the details of the reconnection process, including its geometry (see Galeev et al. 1981). With \( \epsilon = 1 \) and a wave energy density of \( \delta B^2/4\pi \), the same form describes an Alfvén wave flux, which will not be discussed further here.

Equation (4.31) can be rewritten as
\[ F_m(T_e) = \frac{\delta B^2}{8 \pi} \frac{B^2}{8 \pi} \frac{\alpha \nu}{(4 \pi \rho)^{1/2}}. \] (4.32)
$B_e$ can be expressed in terms of $P_e$ and the plasma $\beta$ (4.3), and $P_e$ can be substituted from (4.1) (ignoring the geometric factors). Then

$$F_n(T_e) = 4.9 \times 10^{-16} \frac{1}{\beta^{3/2}} \frac{1}{B^2_e} \alpha e [\text{Em}(T_e) g_e]^{1/2} T_e \quad (4.33)$$

An equivalent expression can be given in terms of $g_e^{-1/4} B_e^{3/2}$, which, for dimensional reasons, leads to the same dependence of $F_n(T_e)$ on $g_e$ and $Ro$ as given by (4.30).

Thus the value of the unknown constants can be found by comparing the empirical values of $F_n(T_e)$ with those predicted by (4.33). The mean value of $(\delta B_e^2/\alpha e [B_e^2/\beta^{3/2}])$ is then 0.16.

Galeev et al. (1981) considered heating by non-linear tearing mode reconnection, albeit in a loop geometry. They equated $(\delta B_e^2)$ with $B_e^2$ and set $\alpha = 1/2$, so that if the energy in the gas is determined by the available energy density in the magnetic field, i.e.

$$1.8 P_e = \frac{2}{3} \alpha \frac{\delta B_e^2}{8 \pi}, \quad (4.34)$$

one finds $\beta = 1/3$. The observations then give $\epsilon = 6.2 \times 10^{-2}$, similar to the value of $5 \times 10^{-2}$ proposed by Galeev et al. (1981).

Alternatively, one can assume a maximum value of $(\delta B_e^2)/B_e^2 = 1/4$ [as in the non-linear damping of Alfvén waves by Holzer, Flà & Leer (1983), or similar in principle to the value of $B_e/B_e^2 = 1/2$ used by Golub et al. (1980) for the dissipation of twisted fields]. Then, if the energy density in the coronal gas is determined by the available energy in the magnetic field, which is eventually all thermalized, $\beta = 1/6$ and $\epsilon = 4.4 \times 10^{-2}$. Since the non-thermal density may be deposited at heights greater than those that dominate the emission measure, these numbers should be taken only as illustrative. In particular, in comparing the observed losses and theoretical heating we have had to assume that the pressures and temperatures are the same in both regions.

4.3 Predicted temperatures

In their discussion of temperatures derived from Einstein data, Schmitt et al. (1990) found no stars with a single-fit temperature between $\log T_e = 6.67$ and 6.88 (see their fig. 3). They concluded that this gap is due to the intrinsic properties of the stars and not solely to the spectral response characteristics of the IPC. Since it is difficult to identify a physical reason for such behaviour, we have applied our scaling law for the temperature to dwarfs for which Schmitt et al. (1990) found a two-temperature fit. The two values of $\delta$ give the same results to within the uncertainties in the observations. The form adopted is that for $\delta = 1/2$,

$$\log T_e = 9.09 - 0.38 \log Ro - 0.5 \log g_e. \quad (4.35)$$

There are nine stars for which the period has been measured and hence $Ro$ and $T_e$ can be found, and these are given in Table 4. All the temperatures fall in the range $\log T_e = 6.63$ to 6.76. For some of the stars, single-temperature fits have been made by Schrijver et al. (1984) or Giampapa et al. (1985), although errors are not quoted by the latter. Apart from $\xi$ Boo A, for which only a rough estimate was available, the agreement between these and our values is acceptable. We conclude that the gap found by Schmitt et al. (1990) is an artefact of the combination of the IPC filter response and the radiative power loss calculations used (Raymond & Smith 1977), and that coronae do exist with average temperatures in the above range, although some contribution from hotter active region material cannot be excluded. In an earlier study, Schmitt et al. (1987) found no significant difference in the temperatures when they used the calculations by Landini & Monsignori-Fossi (1970, 1984) instead of those by Raymond & Smith (1977). The results from the use of more recent radiative power loss calculations should be examined. Some of the stars in Table 4 have also been observed with EXOSAT, and their spectra have been fitted with two temperatures (Pallavicini et al. 1988). Our predicted temperatures are quite close to the low-temperature EXOSAT values. The conversion factor between the absolute Einstein fluxes and emission measures does not depend strongly on the temperature. The total observed fluxes lead to emission measures that are also consistent with the scaling laws above. Further observations of these stars with ROSAT would be useful, and we intend to carry out full emission measure distribution modelling on the targets we have observed.

Table 4. Predicted coronal temperatures.

<table>
<thead>
<tr>
<th>Star</th>
<th>$Ro$</th>
<th>log $g_e$</th>
<th>log $T_e$(K)</th>
<th>Other values of log $T_e$(K)†</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 17925</td>
<td>0.31</td>
<td>4.50</td>
<td>6.72</td>
<td>—</td>
</tr>
<tr>
<td>HD 22049</td>
<td>0.52</td>
<td>4.51</td>
<td>6.63</td>
<td>6.53 (a)</td>
</tr>
<tr>
<td>HD 39587</td>
<td>0.62</td>
<td>4.34</td>
<td>6.68</td>
<td>6.82 (6.75-6.88) (b)</td>
</tr>
<tr>
<td>HD 45088 A</td>
<td>0.33</td>
<td>4.52</td>
<td>6.71</td>
<td>—</td>
</tr>
<tr>
<td>HD 72905</td>
<td>0.46</td>
<td>4.40</td>
<td>6.72</td>
<td>6.88 (6.65-6.97) (b)</td>
</tr>
<tr>
<td>HD 118100</td>
<td>0.16</td>
<td>4.55</td>
<td>6.75</td>
<td>—</td>
</tr>
<tr>
<td>HD 131156 A</td>
<td>0.35</td>
<td>4.40</td>
<td>6.76</td>
<td>7.0 (a)</td>
</tr>
<tr>
<td>HD 206860</td>
<td>0.55</td>
<td>4.40</td>
<td>6.68</td>
<td>6.64 (c)</td>
</tr>
<tr>
<td>HD 234677</td>
<td>0.16</td>
<td>4.55</td>
<td>6.75</td>
<td>—</td>
</tr>
</tbody>
</table>

†Schmitt et al. (1990) found that two-temperature fits are necessary for at least one spectrum of these stars, although others have been fitted with one temperature.

(a) Walter et al. (1984), estimate only.
(b) Schrijver, Mewe & Walter (1984).
(c) Giampapa et al. (1985), no errors quoted.

5 COMPARISONS BETWEEN PHOTOSPHERIC, CHROMOSPHERIC AND CORONAL PARAMETERS

5.1 The chromospheric–coronal flux correlation

As discussed by Rutten et al. (1991) and the earlier authors to which they refer, there is a non-linear relation between X-ray fluxes and the Ca II H and K fluxes, $\Delta F_{HK}$, such that $F_X$ is proportional to $\Delta F_{HK}^{2.3}$. The X-ray fluxes measured with the Einstein Observatory are almost proportional to $\text{Em}(T_e)$. By expressing both $\text{Em}(T_e)$ and $\Delta F_{HK}$ in terms of the pressure, we can now see how this correlation arises.

Using $\delta = 1/2$, relation (4.8) shows that $T_e$ scales as $(P_e/g_e)^{1/2}$, so that, from (4.1) and (4.2), $\text{Em}(T_e) g_e^{1/2}$ scales as $P_e^{1/2}$. Models of the solar chromosphere have been used to find the relation between $\Delta F_{HK}$ and the electron pressure at the base of the transition region $P_e$. Harper (private communication) found that, for the solar values of $T_{\text{eff}}$ and $g_e$,

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\[ \Delta F_{\text{HK}} \text{ scales as } P_c^{0.77} \text{ over the pressure range relevant to most of our sample, and as } P_c^{0.90} \text{ at higher pressures. The latter is similar to that derived from the VAL A, C and F models (Vernazza, Avrett \& Loeser 1981). The scaling essentially arises through the dependence on } P_c \text{ of the temperature gradient and the temperature at which Ca II lines are formed. We will extend these calculations to other stellar parameters in future work. Normalization to the solar values } (P_o/k = 5.6 \times 10^{14} \text{ cm}^{-3} \text{ K, Jordan 1980) leads to} \]

\[ \log \Delta F_{\text{HK}} = X + n \log P_c, \]  

(5.1)

where \(0.77 < n < 0.90\) and \(6.59 < X < 6.73\). Previous modelling of dwarf stars, from the chromosphere to the corona, shows that \(P_c \) exceeds \(P_e\) by only about 10 per cent (Jordan et al. 1987). Thus, with \(P_o = P_e\), we can combine

\[ \log[\text{Em}(T_e)g_*^{1/2}] = 30.81 + 1.5 \log P_c \]  

(5.2)

with (5.1) to predict

\[ \log[\text{Em}(T_e)g_*^{1/2}] = 17.97 + 1.95 \log \Delta F_{\text{HK}} \]  

(5.3)

for \(n = 0.77\), or

\[ \log[\text{Em}(T_e)g_*^{1/2}] = 19.59 + 1.67 \log \Delta F_{\text{HK}} \]  

(5.4)

for \(n = 0.90\).

The direct plot of these quantities for our sample is shown in Fig. 5, and the best-fitting relation is

\[ \log[\text{Em}(T_e)g_*^{1/2}] = 18.05(\pm 0.92) + 2.00(\pm 0.14) \log \Delta F_{\text{HK}} \]  

(5.5)

in good agreement with (5.3). Although the term in \(g_*^{1/2}\) does not make much difference for the dwarf stars, its inclusion gives a better fit than \(\log F_X\) versus \(\log \Delta F_{\text{HK}}\) when the giants from the sample of Rutten et al. (1991) are also included. Thus the scaling \(F_X \propto \Delta F_{\text{HK}}^{1.5}\) found by Rutten et al. (1991) can be understood in terms of the way that the atmospheric structure responds to the pressure.

### 5.2 Relations between \(B_r, f_r, B_c\) and \(\Delta R_{\text{HK}}\)

Since we have independent relations between \(B_r, \Delta R_{\text{HK}}\) and \(B_c f_r\) or \(f_r\) and \(R_0\), we can now derive other combinations of parameters. Because there are only six stars for which both \(B_r\) and \(B_c f_r\) are known (see Table 1 of Jordan \& Montesinos 1991 and Table 1 of this paper), we cannot reliably find the relation between them directly. Instead, we combine (using \(\beta = 1/6\))

\[ \log B_c = 1.59 - 0.39 R_0 \]  

(5.6)

with (2.1) to give

\[ \log B_c = 0.28(\pm 0.29) + 0.40(\pm 0.07) \log B_c f_r \]  

(5.7)

and with (2.3) to give

\[ \log B_c = 1.62(\pm 0.06) + 0.47(\pm 0.09) \log f_r. \]  

(5.8)

We can also compare \(B_r\) and \(\Delta R_{\text{HK}}\) (or \(\Delta F_{\text{HK}}\)) directly, since there is sufficient overlap in the sample to do so (see Table 2). The combined relation is

\[ \log B_r = 4.64(\pm 0.54) + 0.77(\pm 0.12) \log \Delta R_{\text{HK}} \]  

(5.9)

using (3.2) and (5.6) and

\[ \log B_r = 4.22(\pm 0.33) + 0.68(\pm 0.08) \log \Delta F_{\text{HK}}. \]  

(5.10)

from a direct fit to the available data. These expressions agree to within the uncertainties.

So far we have assumed that \(B_c\) depends only on \(R_0\). In Section 5.1, however, we found that the correlation between \(\text{Em}(T_e)g_*^{1/2}\) and \(\Delta F_{\text{HK}}\) is consistent with a correlation between \(\Delta F_{\text{HK}}\) and \(P_c\). Since \(P_c\) is related to \(B_c^2\) through (4.3), use of (5.2) and (5.5) leads to the relation

\[ \log B_c = -3.04(\pm 0.31) + 0.67(\pm 0.15) \log \Delta F_{\text{HK}}. \]  

(5.11)

The direct fit to the observations is shown in Fig. 6 and is

\[ \log B_c = -2.61(\pm 0.35) + 0.60(\pm 0.05) \log \Delta F_{\text{HK}}, \]  

(5.12)

which is consistent with (5.11).

![Figure 5. log Em(T_e)g_*^{1/2} plotted against log \Delta F_{\text{HK}}. The line is the fit given by equation (5.5).](image)

![Figure 6. The coronal magnetic field log B_c from equation (4.4) with \beta = 1/6 plotted against log \Delta F_{\text{HK}}. The line shows the fit given by equation (5.12).](image)
Magnetic fields in late-type dwarfs

The relations for $B_e$ given by (5.10) and (5.12) give fits of similar quality. This is because the change from $\Delta R_{\text{HK}}$ to $\Delta F_{\text{HK}}$ affects mainly the scatter of individual points about the mean and not the power of the relations. If one converts $\log \Delta F_{\text{HK}}$ in (5.12) to $\log \Delta R_{\text{HK}}$, using the mean value of $\sigma T^4_{\text{eff}}$ for the sample (which is 10.82), then the relation becomes

$$\log B_e = 3.88 (\pm 0.89) + 0.60 (\pm 0.05) \log \Delta R_{\text{HK}},$$

consistent with (5.10).

If, however, the fundamental relation is between $B_e$ and $\Delta F_{\text{HK}}$, then this implies that the relation between $B_e$ and $R_o$ should include a term in $T_{\text{eff}}$, and that our constant involves the mean value of this term for our sample. Working from (5.11) and using (3.2), we obtain the expression

$$\log B_e = -5.67 (\pm 0.90) - 0.33 (\pm 0.09) R_o + 0.67 (\pm 0.15) \log (\sigma T^4_{\text{eff}}),$$

which is consistent with our previous expression if the mean value of $\sigma T^4_{\text{eff}}$ is used, i.e.

$$\log B_e = 1.55 - 0.33 (\pm 0.09) R_o.$$

Use of (5.14) does not reduce the scatter in the observed relation because, as pointed out earlier, there are larger (perhaps) systematic departures from the mean value of $\text{Em}(T_c)/g_8$, as a function of $T_c$, which could in principle arise from the geometric factors discussed above. A large sample of stars is needed to explore any systematic behaviour in the scatter, and for this reason we use only the average form for $B_e$, given by (5.6). Similarly, relations (5.7) and (5.8) are reproduced if the full expression and mean effective temperature are used.

Because of their different dependences on $R_o$ (which are not significantly changed by including the $T_{\text{eff}}$ term), it is clear that, with $f_\epsilon = 1$, the relation between $B_e$ and $B_e f_\epsilon$ (equation 5.7) cannot also satisfy the conservation of magnetic flux from the photosphere to the corona. If the unknown geometric factors are included, then equation (5.6) becomes

$$\log (B_e f_\epsilon \cos \phi) = 1.59 - 0.39 R_o + 0.75 \log f_\epsilon + \log \cos \phi,$$

so that to satisfy flux conservation one requires (using equation 2.1)

$$0.75 \log f_\epsilon + \log \cos \phi = 1.68 - 0.58 R_o.$$

Both $f_\epsilon$ and $\cos \phi$ must, however, be less than or equal to 1, so (5.17) can hold only if $R_o > 2.9$, which is larger than for any stars in our sample. Thus we conclude that only part of the surface magnetic flux extends to the corona. Stepenis (1988) came to a similar conclusion, but based on different arguments. Using theoretical estimates of the surface Alfvén wave flux and comparing these with the X-ray radiative losses, he concluded that only some fraction $f_\epsilon$ of the stellar surface participates in the generation of Alfvén waves that heat the corona. He found a correlation $f_\epsilon \propto 10^{-2 R_o}$, which is significantly different from our results below.

We define $f_\epsilon$ as the fraction of the stellar surface that contains magnetic fields that extend to the corona, so that

$$f_\epsilon = \frac{B_e f_\epsilon \cos \phi}{B_e}.$$

or

$$\log f_\epsilon = 1.59 - 0.39 R_o + 0.75 \log f_\epsilon + \log \cos \phi - \log B_e.$$

We do not know $f_\epsilon$ or $\log(\cos \phi)$, so setting these to 1 will give only an upper limit to $f_\epsilon$. We also require $f_\epsilon < f_\epsilon$. For the Sun, $f_\epsilon$ is $0.4$ of the quiet-Sun filling factor for strong fields (see Section 2). Since $B_e$ does not vary by a large factor over the spectral types in the sample, $f_\epsilon$ is determined mainly by $R_o$. The ratio $f_\epsilon f_\epsilon$ must clearly be $\ll 1$, since $B_e$ and $B_e$ are assumed to be related to the coronal and photospheric gas pressures as described above, and $P_e \ll P_{\phi e}$.

Table 5 illustrates values of $f_\epsilon (\text{max})$, which is predicted from (5.19) with a mean value of $\log B_e = 3.22$; $f_\epsilon$, which is predicted from (2.3); and $f_\epsilon (\text{max})/f_\epsilon$, which is given by

$$\log \frac{f_\epsilon (\text{max})}{f_\epsilon} = -1.57 + 0.43 R_o.$$

Although both $f_\epsilon$ and $f_\epsilon$ increase with decreasing $R_o$, the ratio $f_\epsilon/f_\epsilon$ decreases with decreasing $R_o$, so that $f_\epsilon$ is still $\ll 1$ even when $f_\epsilon$ approaches its saturated value of 1; i.e., for a given spectral type, a faster rotator has a larger surface magnetic flux and filling factor, and a larger coronal magnetic flux, but a smaller fraction of the surface flux extends to the corona. As $R_o$ increases, there is a limiting value $\approx 3.7$ at which $f_\epsilon = f_\epsilon$, so that, although the surface flux and filling factor are very small, all the flux extends to the corona.

6 THE DYNAMO MODEL

6.1 The convection zone magnetic field

The method for estimating the convection zone magnetic field was originally described by Durney & Robinson (1982) (hereafter DR), and was modified by Montesinos, Fernández-Figueroa & de Castro (1987) and applied to a sample of active stars. We assume that the field has a toroidal configuration of flux tubes at the base of the convection zone. DR showed that, by keeping only the generation terms of the dynamo equation and using the equations given by Parker (1979) relating several convection zone quantities, it is possible to estimate the Alfvén speed $V_A$ and thus $B_{\text{ez}}$ through the expression $B_{\text{ez}} = V_A (4\pi \rho)^{1/2}$. The equation for $V_A$ is then

$$V_A = \left( \frac{3}{8} \frac{R_o}{L} \right)^{1/2} \frac{1}{uv} \left( 1.31 - \ln \frac{3\pi R_o}{vL} \right)^{1/2}.$$

Table 5. Values of $f_\epsilon (\text{max})$ (with $\log B_e = 3.22$); $f_\epsilon$ and $f_\epsilon f_\epsilon$.

<table>
<thead>
<tr>
<th>Ro</th>
<th>$\log f_\epsilon$</th>
<th>$\log f_\epsilon$ (max)</th>
<th>$\log f_\epsilon/f_\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-1.67</td>
<td>-1.64</td>
<td>-1.53</td>
</tr>
<tr>
<td>0.3</td>
<td>-1.75</td>
<td>-0.31</td>
<td>-1.44</td>
</tr>
<tr>
<td>1.0</td>
<td>-2.03</td>
<td>-0.88</td>
<td>-1.14</td>
</tr>
<tr>
<td>2.0</td>
<td>-2.43</td>
<td>-1.70</td>
<td>-0.71</td>
</tr>
<tr>
<td>3.0</td>
<td>-2.83</td>
<td>-2.52</td>
<td>-0.28</td>
</tr>
<tr>
<td>3.65</td>
<td>-3.05</td>
<td>-3.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
where $R_0$ is the radius of the flux tube, $L$ is the pressure scaleheight, $u$ is the rate of rise of a flux tube through the generation region and $v$ is the convective velocity. The dynamo is assumed to operate at the layer located one pressure scaleheight above the base of the convection zone. This is a matter of debate, since the key point in determining where the dynamo should be placed is that the amplification time should match the typical interval between two activity cycles. If the dynamo is placed well above the base of the convection zone, buoyancy mechanisms act much too quickly and the field is not sufficiently amplified, whereas if the dynamo is placed in the stable layer underneath the convection zone the fields can be maintained for times much longer than the duration of a typical cycle ($\sim 10 \text{ yr}$ [see, for example, the discussions by Parker (1975) and Choudhuri & Gilman (1987), and references therein].

The convective velocity $v$ has been computed using the expression $v = L/\tau_c$, where $\tau_c$ is the convective turnover time for a convective cell. This characteristic time has been computed following the work by Noyes et al. (1984) as in Section 2.1.

All the other parameters, apart from $u$, have been extracted by interpolating in a grid of model stellar interiors (Maeder, private communication) with chemical composition $X = 0.699$, $Y = 0.282$ and $Z = 0.019$ (solar metallicity). The radius of the tube has been set to $R_0 = L/2$ so that the physical variables do not change significantly over a tube cross-section.

The rate of rise for a tube, $u$, has been computed under the assumption that the amplification time for the magnetic field, $t_\alpha$, equals the rise time of the tube through the generation zone, $t_\alpha = L/u$. The expression for $t_\alpha$ is

$$t_\alpha = \left( \frac{R_0^2 \omega}{R_g} \right)^{1/3},$$

(see DR and Montesinos et al. 1987), where $\omega$ is the angular velocity at the generation zone. The constant $c_0$ is found by introducing in (6.2) the appropriate parameters either for the Sun or for another star with a well-monitored magnetic cycle, and matching $t_\alpha$ with the time elapsed from a minimum to a maximum of activity, which can be considered as a good approximation for the amplification time. The value of $c_0$ depends on the form of $\omega(r)$; two approaches have been chosen – see below for details – for which the values of $c_0$ are $7.3 \times 10^{-4}$ and $4.0 \times 10^{-3}$ respectively. $R_g$ is the radius of the magnetic field generation zone, i.e. $R_g = R_g + L$, where $R_g$ is the distance from the centre of the star to the base of the convection zone.

One of the major problems in applying dynamo models to stars other than the Sun is the choice of the dependence of the angular velocity on depth and on the two angular coordinates. Both $\omega$ and its first derivative with respect to the radial coordinate, $d\omega/dr$, appear in our computations. A sensible way to proceed, given the lack of data for other stars, would be to scale laws that have been derived for the Sun from helioseismological measurements. We have chosen two extreme approaches. The first one, which will be referred to as [1], is based on work by Durney (1985), who found the following dependence for the solar angular velocity from data obtained by Duval et al. (1984):

$$\omega(r, \theta) = \frac{\omega(R_0, \pi/2)}{c_1} \left[ \omega_0(r) + \omega_2(r) P_2(\cos \theta) \right],$$

(6.3)

where $r$ is the distance to the centre of the Sun, $\theta$ is the co-latitude and

$$\omega_0 = 1 + \frac{(R_\odot - r)}{R_\odot},$$

and

$$\omega_2 = -0.189 \left[ 1 + \left( 1 - \frac{r}{R_\odot} \right) \left( 1 + \frac{L}{2R_\odot} \right) \right],$$

where $P_2(\cos \theta)$ is the second-order Legendre polynomial and $L$, $L'$ are typical horizontal and vertical dimensions of a convective eddy near the solar surface. We set the ratio $L/L' = 1$ and $\theta = \pi/2$ for our computations. The constant $c_1$ that appears in (6.3) has a value of 1.10, in order that for $r = R_\odot$ and $\theta = \pi/2$ the right-hand side of the equation equals the equatorial solar angular velocity $\omega(R_\odot, \pi/2)$.

We assume that a similar dependence of $\omega$ on $r$ occurs in other stars. The general expression we use has the same form as (6.3) but with $\omega(R_\star, \pi/2)$ and $R_\star$ replacing $\omega(R_\odot, \pi/2)$ and $R_\odot$ in the right-hand side of the equation. In this approach the shear in angular velocity, $d\omega/dr$, is proportional to the surface angular velocity of the star.

The second approach, which will be referred to as [2], assumes that the shear in angular velocity does not depend on the stellar angular velocity itself. We have taken as a guideline the results of the analyses of solar oscillation data by Brown et al. (1989). A linear fit to one of the curves shown in fig. 14 of their paper – the one obtained using a technique referred to as ‘asymptotic analysis’ – yields the following dependence of the solar angular velocity on depth:

$$\omega(r) = \omega(R_\odot, \pi/2) - 6.75 \times 10^{-7} \left( 1 - \frac{r}{R_\odot} \right).$$

(6.4)

This fit is appropriate over the range $0.6 \leq r/R_\odot \leq 1.0$ and for latitudes near the equator. Other techniques applied by Brown et al. (1989) give slightly different laws for the angular velocity, but all of them seem to have the same slope near the bottom of the convection zone.

To scale equation (6.4) for a given star we have changed $\omega(R_\star, \pi/2)$ and $R_\star$ to $\omega(R_\odot, \pi/2)$ and $R_\odot$, but have kept the same constant multiplying the parenthesis in the second term of the equation. $d\omega/dr$ is thus made proportional to $1/R_\star$, i.e. there is a dependence on spectral type through the stellar radius, but there is no dependence on rotation.

Both assumptions [1] and [2] are probably extreme and oversimplified, but, since we do not have information from other stars to test our approaches, we assume that they provide some insight into how the global determining $B$, and $f_\alpha$, might behave.

### 6.2 Estimates of the filling factor

In what follows, the ‘filling factor’ ($f_\alpha$) is defined as the fraction of the stellar surface area covered by fields of the order of $10^3 \text{ G}$ or larger. In this subsection we present a procedure for computing the filling factor which aims to match the values of $f_\alpha$ and $B_\alpha$ obtained from observations.
The magnetic structures in the solar photosphere can be divided into large-scale polarity patterns and small-scale structures. The most conspicuous examples of the former are the active regions, which represent the largest concentrations of magnetic flux. These regions emerge from the solar convection zone as compact, bipolar regions, which contain sunspots in some cases, and decay over periods of weeks to months. On a smaller scale there is a background field which is not smoothly distributed but occurs in a pattern forming a network. The lifetime of these cells is of the order of 1 d. It has been shown that more than 90 per cent of the magnetic flux reaching the solar photosphere, outside sunspots, is in the form of kG flux elements whose typical dimensions are of the order of 100 km, i.e. below the resolution limits of the currently available telescopes (Stenflo 1989), hence one can consider the supergranulation network to be formed by bundles of such magnetic elements.

It is clear that a theoretical prediction of the filling factor must take into account both kinds of structure. On the other hand, the filling factors obtained from the observations take into account fields in active regions and in the supergranulation network but not in spots, owing to the fact that these dark regions contribute little to the line profiles used to find \( f_s \) and \( B_s f_s \). Furthermore, we wish to estimate the contributions to the filling factor from both types of region. In this case the total filling factor can be split into two parts:

\[
f_s = \frac{A_{AR} + A_{SN}}{A_s},
\]

where \( A_{AR} \) and \( A_{SN} \) are the total areas on the photosphere covered by strong fields in active regions and in the supergranulation network, respectively.

We now derive expressions for the filling factor assuming that all magnetic structures appear either as active regions (Section 6.2.1) or as supergranulation network (Section 6.2.2). A weighting is then introduced in Section 6.2.3 in order to combine the contributions from the two types of structure.

### 6.2.1 Estimation of the active-region filling factor \( f_{AR} \)

Consider a toroidal configuration of \( N_e \) flux tubes at the generation zone, where each tube is a bent structure with \( N_e \) emerging sections. The tube, after the amplification process, leaves the generation zone and is pushed up by buoyancy. We assume that the tube rises without destroying its initial configuration, i.e. keeping the same number of bends during the whole rising phase. This hypothesis, obviously, cannot be tested by observations, but computations of the evolution of loop structures in flux rings within the solar convection zone (Choudhuri 1989) show that the overall structure of the ring is conserved as the tube rises through the convection zone, although the amplitude and shape of the bends may change.

We must point out at this stage that we are bypassing an important problem that appears in studying the behaviour of magnetic tubes rising through the convection zone. If this zone is regarded as passive, the rising tubes from the original toroidal configuration at both sides of the equator are deflected by the Coriolis forces to emerge at rather high latitudes, polewards of typical sunspot locations (Choudhuri & Gilman 1987; Choudhuri 1989). The mechanisms by which the tubes are redirected towards the 'sunspot belt' are still unknown, but some studies are being made in this field (Choudhuri & D'Silva 1990; D'Silva & Choudhuri 1991; Moreno-Insertis 1992).

As discussed above, active regions emerge as compact bipolar structures, so it is reasonable to assume that the magnetic flux tube from which they originate does not undergo important diffusion and fragmentation during the buoyancy process. In this case, the filling factor would be the fractional area of the emerging sections of the buoyant flux tube intersecting the photosphere. The filling factor can be written as

\[
f_{AR} = \frac{1}{4\pi R_{ph}^2} N_e \pi R_{ph}^2
\]

where \( R_{ph} \) is the radius of the magnetic flux tube at the photosphere and can be estimated, according to our hypothesis above, assuming magnetic flux conservation, i.e. \( B_s (L/2)^2 = B_e R_{ph}^2 \). The factor of 2 accompanying \( N_e \) comes from the fact that each eruption contributes two intersections. The photospheric field \( B_e \) can be calculated by assuming that the equipartition principle is valid, i.e. that \( (B_e^2/8\pi) P_e = 1 \), where \( P_e \) is the photospheric gas pressure outside the tube. This approach has been suggested by Parker (1978), and Saar (1990) showed that there are not large differences between the measured fields and the equipartition fields. For each star, we have taken values of \( P_e \) from the grid of models by Peytremann (1974) and Kurucz (1979) at an optical depth \( \tau_{2000} = 1 \). For stars with \( T_{eff} \) between 5000 and 4400 K we have extrapolated \( P_e \) guided by the models of Carbon & Gingerich (1969) (see also Gray 1992). For the Sun this gives \( B_e = 1850 \) G.

Although this way of looking at active regions as if they were flux tubes with cylindrical cross-sections intersecting the photosphere is an obvious oversimplification, the approximation can be tested for the solar case. The assumption of magnetic flux conservation, plus the results for \( B_{az} \) and \( L \) from the formalism described in the previous section and the equipartition field \( B_e \), leads to \( R_{ph} = 11,000 \) km for the Sun. This value is smaller than the radius of the leading or following part of a typical active region. However, the magnetic flux across the tube, \( \Phi = \pi R_{ph}^2 B_e \), is \( \Phi = 7 \times 10^{21} \) Mx, which is typical of the magnetic flux observed in a large-to-medium size active region (e.g. Schrijver 1987). Thus our approximation gives a good estimate of \( \Phi \) for active regions.

We need to estimate \( N_e \) since we do not know the angular width in altitude of the generation zone about the equator. It is well known that in the Sun the area in which spots and active regions appear lies between latitudes \( \sim -\pi/5 \) and \( \sim \pi/5 \). If one starts from a toroidal configuration of tubes, concentrated between latitudes \( \sim \phi/2 \) and \( \phi/2 \), then, since the diameter of a tube in the generation zone has been set to \( L \), the number of tubes, or more precisely an upper limit to the actual number of tubes present, can be expressed as

\[
N_e \approx \frac{R}{L}
\]

Introducing (6.7) in (6.6), one obtains the following expression for the filling factor:
There are still some unknowns in the expression for $f_{AR}$, namely $\varphi$ and $N_r$. A reasonable value for $\varphi$ would be $2\pi/5$, i.e. the solar one, and then either the solar filling factor, or the filling factor of another chosen reference star, could be used to fix a value for $N_r$ to be used for the remaining stars.

Introducing typical values for the Sun in (6.8), namely $R_a = 5.7 \times 10^{10}$ cm, $L = 4.2 \times 10^9$ cm, $B_{CA} = 500$ G and $B_{ph} = 1850$ G, one obtains $f_{AR} = 0.002 N_r$, which implies $N_r \approx 7$ eruptive sections per tube to match a solar filling factor of 0.015.

### 6.2.2 Estimation of the supergranulation filling factor $f_{SN}$

Let us consider the same configuration as in Section 6.2.1 for the toroidal magnetic field and assume that each tube from this configuration is subdivided into a bundle of tubes of smaller radius $R_{tb}$. We introduce this 'ad hoc' hypothesis because this sort of 'fibrillar' topology is more appropriate for understanding how the turbulent motions near the photosphere would be able to split the tubes in small elements to form the supergranulation network. Let us assume that these bundles of flux tubes rise through the generation zone at a speed $u$. We assume, as before, that each tube is a curved structure of $N'_0$ emerging section, where $N'_0$ is different from $N_r$ owing to the smaller scale of the supergranulation network. Let $F_{SN}$ be the total magnetic flux corresponding to the tubes that intersect the spherical surface of radius $R_p$.

The rate at which the rising tubes contribute to this flux is given by the expression

$$\frac{d F_{SN}}{d t} = \frac{N'_0 \pi R_{tb}^2 B_{ex} u}{L},$$

(see also DR). If we consider the field emerging at the photosphere, the total (unsigned) magnetic flux is given by $4\pi R_p^2 B_{SN}$.

In order to model the filling factor while assuming that all the magnetic flux emerges as a small-scale magnetic pattern, which we identify with the supergranulation network, some diffusion effects on the buoyant tubes must be included. Whereas the lifetime for large-scale regions is of the order of weeks or months, the smaller a structure, the shorter its lifetime, and although the flux is not itself destroyed the flux elements have a limited lifetime, comparable to that of the turbulent eddies, due to the reshuffling of magnetic lines (Stenflo 1989).

Following the ideas of DR, we assume that the mechanism responsible for the destruction of individual small-scale magnetic elements is the turbulent diffusivity. If $t_0$ is the characteristic diffusion time at the photosphere, then the expression (6.9) must be equivalent to $4\pi R_p^2 f_{SN} B_{Sn}/t_0$, where the subscript 'SN' stands for 'supergranulation network'. DR proposed that $t_0$ is given by

$$t_0 = \frac{L_D}{\eta_{ph}},$$

(6.10)

where $L_D$ is a typical length-scale for this small-scale photospheric turbulence and $\eta_{ph}$ is the magnetic diffusivity at the photosphere, which can be estimated using the expression $\eta_{ph} \propto L_D u_{ph}$, $v_{ph}$ being the convective velocity at the photosphere. The constant of proportionality in the estimate of $\eta_{ph}$ varies from 1/3 (adopted by DR) to $\sim 1/10$ (Moss 1986). The latter value has been used in our calculations.

The value of $L_D$, the diffusion length, has been approximated by the pressure scaleheight, $L_{ph}$, and has been computed for each star by interpolating the appropriate quantities in the grid of models by Peytremann (1974) and Kurucz (1979) at an optical depth $\tau_{5000} = 1$. Between 5000 and 4400 K we have scaled $L_D$ according to $T_{eff}/T_0$ since, as Gray (1992) pointed out, $T_{eff} = T_{eff}/T_0$ is a good approximation to models by Kurucz (1979) and Carbon & Gingerich (1969).

Stepien's (1988) approximation has been used to calculate the convective velocities, $v_{ph}$, at the photosphere. For the Sun this gives $\sim 1.9$ km s$^{-1}$, which, when combined with $L_{ph} \approx 150$ km, leads to a typical diffusion time of $\sim 13$ min. The value of the solar photospheric scaleheight matches quite well the estimate of $\sim 100$ km for the size of the small-scale magnetic elements given by Stenflo (1989), so we assume that this is also a good approximation for stars other than the Sun.

In this approximation the resulting magnetic field configuration is a set of thin structures within the supergranulation, these being the final stage of bundles of tubes that rise through the convection zone and are split up into smaller tubes as they reach the solar surface. The entire network is not covered by kG fields because during the process of fragmentation the flux elements become more and more separated by almost field-free areas (Stenflo 1989).

A similar picture was proposed by Golub et al. (1981) to explain the origin of the small-scale fields in X-ray bright points (XBP) and the turbulent magnetic fields. They suggested that, whereas the dynamo process is likely to operate deep in the convective zone, where the active-region fields originate, both the fields in XBP and the turbulent fields cannot originate directly from the same location as do active regions, but must undergo some processing at a higher level.

Using the above arguments, the filling factor can be expressed in the following manner:

$$f_{SN} = \frac{1}{4\pi R_p^2} \frac{N'_0 \pi R_{tb}^2 B_{ex} u}{B_{SN} v_{ph}} = \frac{u 10 L_{ph}}{L}.$$  

(6.11)

The number of tubes $N'_0$ can be estimated as follows. As we pointed out above, we assume that each of the $N_r$ tubes from the initial configuration with radius $R_{tb} = L/2$ is split into a bundle of slender tubes with radius $R_{tb}'$. It is straightforward to see that

$$N'_0 = N_r \pi (L/2)^2 / \pi R_{tb}^2.$$  

(6.12)

Using expression (6.7) one obtains

$$N'_0 \approx \frac{\varphi R L}{4 R_{tb}},$$  

(6.13)

and substituting expression (6.13) in (6.11) one obtains

$$f_{SN} \approx \frac{5N'_0 \varphi R L}{4 R_p} \frac{B_{ex}}{B_{SN} v_{ph} L_{ph}}.$$  

(6.14)
Note that $R'_s$ cancels out, so no estimate of this is required. Introducing typical solar values into (6.14), we obtain $f_{SN} = 9.3 \times 10^{-9} N'_i$, which implies $N'_i = 1.6 \times 10^6$, to match $f_0 = 0.015$. Given the lack of knowledge about this parameter, the same value of $N'_i$ has been adopted for all stars.

Note that the velocity $u$ appears in the expression (6.14) for the filling factor. As we saw in the previous subsection, this velocity is computed by using the expression $u = L/L_i = L_0/L_i$, where $t_0$, the amplification time, is given by equation (6.2). This expression contains $\omega/\partial r$ explicitly. Use of the two different relations for $\omega/\partial r$ described by (6.3) and (6.4) causes a distinct difference in the dependence of $B_0 f_0$ on rotation. DR made the approximation $\omega/\partial r = \Delta \omega/R_0$, $\Delta \omega$ being a typical longitudinal shear in the angular velocity at the surface. Rather than adopting this approach for estimating both the magnetic field at the bottom of the convection zone and the filling factor, we have introduced explicit expressions for $\omega$ and its first derivative, as pointed out in the previous section. The results are discussed below.

In their original approach, DR applied the hypothesis of flux conservation to obtain the length-scale for the photospheric magnetic structures and then applied arguments about the diffusion process to these elements, but no distinction was made between large- and small-scale structures. Montesinos et al. (1987) applied DR’s method to estimate filling factors for a sample of active stars. The values given by them were scaled to the value of the solar filling factor chosen by DR, namely $f_0 = 0.002$, which actually corresponds to the filling factor for sunspots. This scaling led to lower filling factors than those now computed.

6.2.3 The total filling factor

Using the expressions for the filling factor for active regions (6.8) and for the supergranulation network (6.14), the total filling factor is

$$ f_i = \xi f_{FR} + (1 - \xi) f_{SN}, $$

(6.15)

where the weight $\xi$ is a number between 0 and 1 which indicates the proportional contribution of active regions to the total filling factor. Note that in the derivation of $f_{FR}$ and $f_{SN}$ we assumed that all the tubes from the original toroidal configuration give rise either to active regions or to small-scale elements; a weighting is therefore necessary to account for both contributions. A value $\xi = 1$ would imply that all the magnetic structures appear as active regions, whereas $\xi = 0$ would represent a field with a purely fibrillar structure. The only star for which $\xi$ is known is the Sun, where the separate contributions from active regions and the supergranulation network can be estimated. From the values given in Section 2.1, with a network area of 40 per cent, one obtains $\xi = 0.5$.

7 OBSERVATIONS AND THEORETICAL PREDICTIONS

In this section we compare the theoretical calculations with the observational results. In order to find the actual dependence of $B_0 f_0$ on $R_0$ one would need to establish correlations for groups of stars with the same spectral type and different rotation rates, so one would be able to separate the effects of stellar and structural parameters on the correlations. Unfortunately, the sample of stars with reliable detections of $B_0$ and $f_0$ is not large enough to allow a subdivision into such groups, so the correlations below refer to the same set of 14 stars as in Section 2.1. In the graphs we show the corresponding theoretical predictions and all data points. The relations between $B_0 f_0$ or $f_0$ and $R_0$ shown below therefore contain an implicit dependence on stellar parameters. A sample containing a different distribution of spectral types would give slightly different dependences on $R_0$, but our interest here is in comparing theory and observations for the given set of stars.

Four sets of calculations have been carried out: for each of the assumptions [1] and [2] for the dependence of the angular velocity on depth, two extreme approaches have been adopted, i.e. it is assumed that all magnetic fields appear either (a) as active regions or (b) in the form of a supergranulation network. The calculations have been performed matching a value of 0.015 for the solar filling factor.

The results of the correlations are, with $\omega/\partial r$ as in assumption [1],

$$\begin{align*}
\log (B_0 f) & = 2.55 (\pm 0.08) - 0.69 (\pm 0.11) R_0, \\
\log (B_0 f) & = 3.21 (\pm 0.10) - 0.99 (\pm 0.12) R_0, \\
\log (B_0 f) & = -0.83 (\pm 0.07) - 0.60 (\pm 0.10) R_0, \\
\log (B_0 f) & = -0.15 (\pm 0.08) - 0.93 (\pm 0.10) R_0,
\end{align*}$$

(7.1)

with $\omega/\partial r$ as in assumption [2],

$$\begin{align*}
\log (B_0 f) & = 2.32 (\pm 0.05) - 0.56 (\pm 0.09) R_0, \\
\log (B_0 f) & = 2.82 (\pm 0.08) - 0.82 (\pm 0.12) R_0, \\
\log (B_0 f) & = -1.07 (\pm 0.04) - 0.46 (\pm 0.07) R_0, \\
\log (B_0 f) & = -0.56 (\pm 0.07) - 0.74 (\pm 0.10) R_0.
\end{align*}$$

(7.2)

Comparing relations (7.1) with (2.1) and (2.2), it can be seen that the assumption that the field emerges mostly in the supergranulation network gives a closer fit to the observations, and the expressions in fact agree to within the deviations. The same conclusion can be drawn from comparing the set of correlations given in (7.2) with the observations. The agreement is, however, less good, although still formally within the deviations.

Whereas using (7.1) the theory and the observations can be brought into even closer agreement by involving a small contribution from active regions which reduces the dependence on $R_0$, this is not the case using (7.2). Use of assumption [1] and the value of $\xi = 0.5$ found above, for all the stars, leads to $B_0 f < 10^{-0.88 (\pm 0.12) R_0}$ and $f < 10^{-0.81 (\pm 0.10) R_0}$ in agreement with (2.1) and (2.2) to within the deviations. With assumption [1] the best fit to the observations is found for a combination of active regions and supergranulation with a weight factor $\xi = 0.25$, although $\xi$ may well be a function of both $R_0$ and the spectral type. The results of these calculations are shown in Fig. 7. Empty circles represent the observational results and solid circles are the theoretical predictions. The regression fits to the theoretical predictions give

$$\begin{align*}
\log (B_0 f) & = 3.11 (\pm 0.10) - 0.94 (\pm 0.12) R_0, \\
\log f & = -0.26 (\pm 0.08) - 0.88 (\pm 0.10) R_0.
\end{align*}$$

(7.3)
For V833 Tau, the theoretical filling factor exceeds the physical upper limit of 1, but this point is not one that is included in the correlations. The point shown is the upper limit with $f_s = 1$.

Although the whole sample can be fitted with a log-linear form for the dependence of $B_{f_1}$ and $f_s$ on $R_o$, this form does not naturally appear in the theory as applied to just one spectral type. The theory gives relations closer to a log-log form. However, not only the observations of $B_{f_1}$ and $f_s$, but also those of $\Delta R_{\text{HK}}$ appear to show a ‘saturation’ at small values of $R_o$ (Noyes et al. 1984; Vilhu 1984). Stauffer & Hartmann (1987) reported observations of rapidly and slowly rotating Pleiades and $\alpha$ Per stars and found modest differences in chromospheric emission for stars ranging in $v \sin i$ from 10 to 100 km s$^{-1}$. Also, to explain successfully the distribution in rotational velocities observed in the Pleiades, these authors suggested the existence of a different braking mechanism, based on angular momentum loss independent of the rotational velocity for stars with $v_{\text{eq}} \geq 10$ km s$^{-1}$. This difference must also be reflected in the dependence of $\omega$ on $R_o$. We have therefore calculated the locus of the values of $B_{f_1}$ for stars of a fixed spectral type (G2 V), first using assumption [1] (the shear in $\omega$ depends on $\omega$) for stars with $P_{\text{rot}} \geq 5$ d, which corresponds to $R_o \geq 0.4 \ (v_{\text{eq}} < 10$ km s$^{-1}$) and gives a steep gradient, and then keeping a constant shear in $\omega$ for stars with $R_o < 0.4 \ (v_{\text{eq}} > 10$ km s$^{-1}$), which gives a less steep gradient. The locus is shown in Fig. 7 as a solid line. (The locus for G0 V stars lies slightly higher, and the loci for types later than G2 V lie increasingly higher). Thus the observed log-linear form of the whole sample of stars, and the tendency to saturate at small values of $R_o$, could arise naturally from a change in the dependence of $d\omega/d\ln r$ on $\omega(r)$ at $R_o = 0.4$.

Given the simplicity of our model it is worth pointing out some of its shortcomings to see where the theoretical predictions may not be valid.

The dynamo model is a ‘kinematic’ one, i.e. the velocity field in the convection zone is an input of the model and is not changed. This approach can be considered as valid if the kinetic energy density of the convective elements, $\rho \omega^2/2$, is larger than the magnetic energy density, $B_0^2/4\pi$, stored in the field. In our case, for a range of spectral types G0 V to K5 V this assumption breaks down for Rossby numbers less than $\sim 0.1$, where the two energy densities are similar. Thus, for the fast rotators (say, $R_o \leq 0.3$), some other factors in addition to those suggested above could account for the saturation seen in the relations between $B_{f_1}$ and $\Delta R_{\text{HK}}$ versus $R_o$.

Some of the approximations and constants are based on what is known for the Sun. This is an obvious oversimplification. Even for stars with spectral types similar to that of the Sun, but with different rotation periods, the similarities might be less close than one would expect. For example, Donahue & Baliunas (1992) have shown that the cycle of the activity cycle of HD 114710 ($\beta$ Com, G0 V, $P_{\text{rot}} = 12.4$ d) is opposite to that of the Sun, with the rotation period increasing as the activity declined during the last observed cycle. This star apparently shows a non-solar behaviour either in the surface acceleration pattern or in the progression of active regions during its cycle. If these differences are confirmed with further observations they would imply that different patterns of internal angular velocity are present depending on the rotation period, even for stars of similar spectral type. Clearly, further systematic studies of this type would be of great value.

The present simple approach is, however, able to account quite well for the observed values of $B_{f_1}$ and $f_s$ and their dependence on $R_o$.

8 CONCLUSIONS

The observed surface magnetic fields and filling factors can be fitted with a dependence on the Rossby number, $R_o$. To compare with the simple dynamo theory one ideally needs a sample of stars of the same spectral type but with different rotation periods. The agreement between the theory and the present sample of observations is, however, encouraging given the simplifying assumptions that we have to make in the theory. The reduced dependence of $B_{f_1}$ and $f_s$ on $R_o$ at small values of $R_o$ may in fact arise from a difference in the dependence of $\omega(r)$ on $r$ for stars with large and small values of $R_o$.

We confirm the tight correlation of the $\Delta R_{\text{HK}}$ index, rather than $\Delta F_{\text{HK}}$, with $R_o$, but this is not yet understood, since the magnetic fields and filling factors at the height at which the Ca II emission is formed are not known. In the type of stars
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Chromospheric models, for a fixed $T_{\text{eff}}$, give a preliminary scaling between the Ca ii flux and the transition region pressure, $P_{\text{tr}}$, through the chromospheric temperature and temperature gradient. The chromospheric models do not assume a particular form of heating, but simply show how the structure changes to match the observed fluxes, as the pressure $P_{\text{tr}}$ changes. The coronal emission measure (and the X-ray flux) can be expressed in terms of the coronal pressure, and, since $P_{\text{tr}} \approx P_{\text{c}}$, the relation between the Ca ii flux and the X-ray flux can be predicted, and is in satisfactory agreement with that observed. Even if one understood the dependence of the coronal flux on $R_0$, and could thus predict the dependence of the chromospheric fluxes on $R_0$, the relation between the chromospheric heating and structure would remain to be understood.

The main energy loss hypothesis of Hearn (1975, 1977) gives a good prediction of the total energy losses. There are, however, some systematic differences between the observed values of $E_{\text{tot}}(T_e)/g_s$ and those predicted from $T_e$, which may indicate that the coronae are not exactly in a minimum energy loss configuration, although geometric factors could also be involved. As discussed by Jordan & Montesinos (1991), the coronal parameters appear to depend mainly on $R_0$, although a term in $T_{\text{eff}}$ does appear. Similarly, using an average $T_{\text{eff}}$, the absolute coronal heating flux required scales basically as $R_0^{-1}$. For dimensional reasons this is also the case whether the heating is by some magnetic field reconnection process or by MHD waves.

Comparisons of the dependences of the implied coronal field $B_c$ and the surface magnetic flux $B_{f,S}$ on $R_0$ show that only part of the surface flux extends to the corona. For the same spectral type, a faster rotator has a larger surface flux and filling factor and a larger coronal field, but a smaller fraction of the surface flux extends to the corona, and vice-versa.

The sample of stars available for deriving the correlations is still small, but results from ROSAT should improve the situation regarding coronal observations. Further measurements of surface magnetic fields and filling factors are also required.

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