NORMAL INCIDENCE X-RAY TELESCOPE POWER SPECTRA OF X-RAY EMISSION FROM SOLAR ACTIVE REGIONS. II. THEORY

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ABSTRACT

In a previous paper, we used the very high resolution images of coronal active regions obtained by the Normal Incidence X-Ray Telescope to study the size distribution of X-ray-emitting structures. A Fourier analysis of these images showed a broad-band, isotropic, power-law spectrum for the spatial distribution of soft X-ray intensities. The presence of a broad-band spectrum indicates that magnetic structures of all sizes are present, at least down to the resolution limit of the instrument, which is about $\frac{1}{3}''$.

In the present paper, we present a model that relates the basic features of coronal magnetic fluctuations to the subphotospheric hydrodynamic turbulence that generates them. The main result of this paper is that from this model we obtain a theoretical power spectrum for the X-ray intensity, which falls off with increasing wavenumber as $k^{-3}$, fitting remarkably well the observed spectra that we obtained from a sample of topologically different active regions.

We speculate that the nonlinear interactions of these externally driven fluctuations will contribute to establish a magnetohydrodynamic turbulent regime in the corona. We suggest that the bulk of the energy delivered to the corona from footpoint motions directly cascades down to very microscopic length scales, where it efficiently dissipates and heats the plasma. However, since the wavenumber range associated with the cascade and dissipation regions are still beyond present-day spatial resolution limits, the presence of a turbulent regime cannot be observationally confirmed.

Subject headings: MHD — Sun: activity — turbulence

I. INTRODUCTION

The existence of highly inhomogeneous and fibrillated structures of the coronal plasma as well as their close relationship with magnetic fields certainly is the most important observational finding since the very discovery of the solar corona. Very high resolution images recently obtained by the Normal Incidence X-Ray Telescope (NIXT) have shown internal structure for coronal loops down to the resolution limit of the instrument, suggesting that structuring on still finer scales will be seen as the spatial resolution is improved (Gómez, Martens, & Golub 1992).

Using X-ray images obtained by NIXT during its 1989 September campaign, Gómez et al. 1992 (hereafter Paper I) analyzed the size distribution of the internal X-ray-emitting structure of a number of active regions. For each active region, they computed its two-dimensional power-spectrum, and its azimuthally integrated version, or so-called omnidirectional spectrum. The main results from that analysis were that (1) the two-dimensional power spectra displayed a high degree of isotropy in the wavenumber plane $(k_x, k_y)$ and (2) a common $k^{-3}$ power-law distribution for the omnidirectional spectra of all active regions was obtained, regardless of their different topology. The existence of a power-law distribution is indicative of the absence of "preferred" length scales for coronal X-ray structures, while the constancy of the power index strongly suggests that the same process generates the X-ray emission of all active regions.

The very active role of the magnetic energy contained in these structures in heating the corona has been repeatedly pointed out in the literature (see Narain & Ulmschneider 1990; see also Gómez 1990 for recent reviews). The magnetic field lines are deeply rooted in the convective region, below the photosphere, and evolve attached to the fluid because of the very high electric conductivity there. The turbulent subphotospheric fluid motions stochastically stress the magnetic field lines and pump magnetic energy into coronal active regions. By this mechanism, the convective region replenishes the coronal magnetic energy which is continually being dissipated. This scenario is quite appealing, because it is a theoretically plausible way for these magnetic structures to obtain enough energy to compensate for the conductive and radiative losses of the coronal plasma (Parker 1983). However, since the Joule relaxation of the above-mentioned magnetic stresses is extremely slow, the main challenge for current coronal heating theories is to find a mechanism capable of enhancing Joule dissipation.

The broadband power spectra for coronal X-ray emission reported in Paper I are a strong observational indication of the aforementioned connection between coronal magnetic structures and the stochastic subphotospheric motions. Recent observational studies based on Doppler imaging of photospheric motions (Zahn 1987; Chou et al. 1991; Hathaway et al. 1991; Tarbell 1992) strongly indicate the existence of a broadband, power-law kinetic energy spectrum, as expected for a turbulent medium like the...
subphotospheric convective region. A similar analysis performed on magnetograms (Knobloch & Rosner 1981) has also shown a power-law spectrum for the photospheric magnetic field. In this context, if the closed magnetic structures that confine active regions are being externally driven by a broad-band spectrum of footpoint motions, it seems natural to expect that the size distribution of their internal X-ray-emitting features also displays a broad-band power spectrum.

In the present paper we explore this connection further. In § 2 we present a theoretical derivation from first principles for the power-law spectra for the spatial distribution of X-ray emission. We first express the X-ray intensity in terms of the fluctuating velocity and magnetic field in the corona. Then we relate the fluctuating coronal fields to the photospheric velocity and magnetic fields. Performing a statistical averaging procedure over these relationships, we find the power spectrum for the observed X-ray intensity, which turns out to be independent of the power spectra for the photospheric kinetic and magnetic energy.

We believe that the presence of a broad-band spectrum of magnetic fluctuations, combined with the large Reynolds numbers for the coronal plasma, sets favorable conditions for the development of a magnetohydrodynamic (MHD) turbulent regime in solar active regions. If this is the case, the associated energy cascade to microscopic length scales will efficiently enhance Joule dissipation until the heating rate matches the conductive and radiative losses of the plasma (van Ballegooijen 1986; Gómez & Ferro Fontán 1988). In § 3 we speculate about the implications of a turbulent regime in active regions. However, our main result, the relation between the photospheric and coronal power spectra, holds regardless of the existence of a turbulent regime, so this section can be skipped without harm. In § 4 we discuss the correspondence between the observational results reported in Paper I and the theoretical results reported in this paper.

2. THEORY OF X-RAY POWER SPECTRA

In the following we will assume that the X-ray emission observed in each NIXT pixel is proportional to the Poynting flux entering that pixel, either from the photosphere or from the sides (see Fig. 1). This is a simple consequence of the apparent stationarity (on length scales equal to or larger than the pixel size) of the active regions observed (energy input must equal energy output), and the approximate equipartition between conductive and radiative losses in coronal loops. This result cannot be strictly true, since there will also be heat conducted from one pixel to the next along the field lines, but we make the assumption that the average of this sideways transport is zero (we discuss this approximation further in § 2.2).

If the Poynting flux per pixel is proportional to the X-ray emission per pixel, then of course their power spectra over an ensemble of pixels must have the same form. This then relates two distributions that, in principle, can be measured: we have already determined the distribution of the X-ray emission, and that of the Poynting flux entering through the photosphere can be derived from observations of the photospheric footpoint motions and magnetic field. In the following we derive a theoretical expression linking the slope of the power spectrum for the coronal X-ray emission to those for the photospheric magnetic field and velocity field.

We have no photospheric observations simultaneous with the NIXT data available to test this relationship. However, we can resort to a simplifying hypothesis: we assume that the photospheric magnetic field is passively advected by the convective fluid.

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**Fig. 1.**—Elementary box containing the coronal column mass that emits the photons to be collected by one single pixel.
motions. This assumption is not strictly true everywhere on the photospheric surface (for instance, it is well known that the magnetic field inside sunspots strongly suppresses photospheric motions), but we use it as a zeroth-order approximation (see § 2.3). We hope that simultaneous observations of magnetograms and photospheric motions will soon allow us to understand this relationship better.

Inserting the expression for a passively advected magnetic field in our expression for the slope of the X-ray power law, we surprisingly will find $S(k) \propto k^{-3}$, in perfect agreement with the observations (see Gómez et al. 1992 for a detailed description of the observations). This is the main result of our analysis.

Note that in the above scenario nothing has been said about the actual coronal heating mechanism, and it appears that the above results should be valid for any heating mechanism. However, in adopting the expression for passive field advection, we have tacitly assumed that these motions are of the slow type, since they advect the magnetic field (rapid motions would generate MHD waves and not change the average position of the field lines). Therefore, our main theoretical result only applies to DC heating mechanisms. That does not mean that for heating mechanisms of AC (wave) type equally good agreement with the X-ray power spectrum might not be achieved, but that remains to be shown. Given these results, we are inclined to favor DC mechanisms, such as Joule heating at the end of a cascade generating very small length scales (Gómez & Ferro Fontán 1988) or nanoflares resulting from dissipation at tangential discontinuities (Parker 1991).

In § 2.1 we discuss which are the leading terms contributing to the Poynting flux in coronal loops. The energy balance assumed for each NIXT pixel is detailed in § 2.2. The connection between the coronal magnetic field and the longitudinal component measured at the photosphere is discussed in § 2.3. Finally, in § 2.4 we relate the power spectrum for the coronal Poynting flux (which, as we said, is assumed to be proportional to the soft X-ray intensity), to the power spectra corresponding to magnetograms and photospheric motions.

### 2.1. Magnetohydrodynamic Equations

The full set of MHD equations for an incompressible plasma is

\[
\partial_t B = \nabla \times (u \times B) + \eta \nabla^2 B, \tag{1}
\]

\[
\partial_t u = - (u \cdot \nabla) u - \frac{1}{\rho} \nabla p + \frac{(\nabla \times B) \times B}{\rho} + \nu \nabla^2 u, \tag{2}
\]

\[
\nabla \cdot u = 0 = \nabla \cdot B, \tag{3}
\]

where $u$ and $B$ are the velocity and magnetic field expressed in velocity units. The dissipation coefficients are represented by $\eta$ (resistivity) and $\nu$ (viscosity), $p$ is the gas pressure, and $\rho$ is the (constant) mass density. Using these equations, it is straightforward to derive the evolution equation for the energy density $W_{\text{MHD}} = \frac{1}{2}(B^2 + u^2)$,

\[
\partial_t W_{\text{MHD}} = -\nabla \cdot F_{\text{MHD}} - E_{\text{MHD}}, \tag{4}
\]

\[
F_{\text{MHD}} = B^2 u_\perp + \left( \frac{p}{\rho} + \frac{u^2}{2} \right) u + \eta (\boldsymbol{J} \times \boldsymbol{B}) + \nu (\nabla \times u), \tag{5}
\]

\[
E_{\text{MHD}} = \eta J^2 + \nu \Omega^2, \tag{6}
\]

where $\boldsymbol{J} = \nabla \times \boldsymbol{B}$ is the electric current, $\boldsymbol{\Omega} = \nabla \times u$ is the vorticity, and $u_\perp$ is the component of the velocity perpendicular to the magnetic field.

Assuming that a stationary equilibrium is achieved,

\[
E_{\text{MHD}} = -\nabla \cdot F_{\text{MHD}}. \tag{7}
\]

This equation indicates that, for any closed volume of fluid, the net energy flux entering through the walls balances the resistive and viscous dissipation occurring inside that volume. It has been shown (Choudhuri 1986) that subphotospheric motions produce a net energy input on coronal loops [i.e., the integral over the loop of $(\nabla \cdot F_{\text{MHD}})$ remains positive].

We now turn to the expression for the energy flux in equation (5). Note that since the Reynolds numbers in the corona ($R_u = u_c / \nu$: Reynolds number; $R_B = B_c / \eta$: magnetic Reynolds number) are much larger than unity, the two last terms in the right-hand side of equation (5) are negligibly small. Since the motions involved are sufficiently slow ($u^2 \ll p/\rho$), we also neglect the kinetic energy flux. Moreover, the fact that the gas pressure is much smaller than the magnetic pressure allows us to drop the term $(p/\rho)u$ in equation (5). Therefore, the energy flux reduces to the MHD Poynting flux

\[
F_{\text{MHD}} \approx B^2 u_\perp. \tag{8}
\]

The very high value for the magnetic Reynolds number ($R_B \approx 10^{12}$) does also allow us to assume that the fluid moves glued to the magnetic field lines (frozen-in condition). Thus,

\[
u = \frac{B \times \dot{A}}{B^2}, \tag{9}
\]

where $\dot{A}$ is the vector potential ($B = \nabla \times A$) and the dot indicates a derivative with respect to time. It follows that the Poynting flux can simply be written as

\[
F_{\text{MHD}} \approx B \times \dot{A}. \tag{10}
\]
2.2. The Energy Balance

The mechanism outlined above explains how the energy contained in fluctuations of the magnetic and velocity fields can become available as a source of thermal energy for the plasma. However, we note that no particular process to enhance magnetic dissipation has been specified. Hereafter, we shall simply assume the presence of an efficient mechanism, such as the topological dissipation proposed by Parker (1972) or the development of a direct energy cascade (van Ballegooijen 1986; Gómez & Ferro Fontán 1988, 1992). In coronal active regions, the thermal energy source or heating rate reaches an equilibrium against radiative and conductive losses (see Withbroe 1981 for a review). The role of thermal conductivity is very important for the stability of this equilibrium, but integrated along a field line, from one footpoint of a loop to the other, its net contribution vanishes (conduction merely redistributes energy along the loop; see, for instance, Kuin & Martens 1982). Therefore, assuming that the heating rate is \( E_{\text{MHD}} \), and provided that \( E_{\text{MHD}} = -\nabla \cdot \mathbf{F}_{\text{MHD}} \) (see eq. [7]),

\[
-\nabla \cdot \mathbf{F}_{\text{MHD}} \approx L_{\text{RAD}},
\]

where \( L_{\text{RAD}} \) are the radiative losses (in ergs cm\(^{-3}\)s\(^{-1}\)). We note that since the X-ray emission is observed to be distributed rather evenly along the loop, equation (11) implies that the heating should also be distributed smoothly along the loop. However, if this were not the case, the net effect of conductivity would be to share out the energy release over the loop, and the operational result would be the same as if the heating rate were evenly distributed. For example, suppose a heating spike takes place in a small region of diameter \( a \). In Fourier space this would generate a signal at approximately \( k = (1/a, 1/a) \). Conductivity will spread out the X-ray emission over a loop of length \( L \), and, hence, if we choose the \( x \)-axis along the loop, a signal at \( k = (1/L, 1/a) \) will be generated. In the omnidirectional spectrum, this will shift the measured signal from \( k \approx 2^{1/2}a \) to \( k \approx 1/a \) (since \( L \gg a \)), which is rather insignificant compared with the large range in \( k \) we are considering.

Our goal in this section is to compare the outcome of these calculations with the observed power spectra described in \$2. Therefore, we discretize equation (11) by performing spatial averages over boxes which are infinitely high along the line of sight and whose square bases are 1 pixel in size (see Fig. 1). We obtain

\[
-\frac{C}{D^2} \oint_{\text{box}} dS \cdot \mathbf{F}_{\text{MHD}} = D^2 I_{i,j},
\]

where \( D \) is the linear size of one pixel, and

\[
I_{i,j} = \frac{C}{D^2} \oint_{\text{box}} d^3x L_{\text{RAD}}
\]

is the X-ray intensity of pixel \((i, j)\) (in ergs cm\(^{-2}\)s\(^{-1}\)). The dimensionless constant \( C \) accounts for the fraction of radiated photons effectively detected by the instrument, mainly determined by its narrow-band sensitivity and its solid-angle aperture. The surface integral \( \oint_{\text{box}} \) in equation (12) can be separated into two parts. One part corresponds to the photospheric base of the box and reduces to \( D^2 \int_{\text{box}} d^3x \mathbf{F}_{\text{box}}(z = 0) \). The second part corresponds to the side walls of the box and can be expressed as the integral over \( z \) of a divergence term. Therefore

\[
C^{-1} I_{i,j} = \int_0^z \nabla_\perp \cdot \mathbf{F}_{i,j},
\]

where \( \nabla_\perp \) is the finite element expression (because it operates over discretized functions) of the divergence operating on the \((x, y)\)-plane. In the following it is important to keep in mind the finite-element character of functions and operators, and the fact that many physical processes responsible for the heating of the plasma do actually occur on scales well below the linear pixel size (~450 km).

According to equations (10) and (14), the soft X-ray emission results

\[
C^{-1} I = (\nabla \times \mathbf{A})_{(z = 0)} - \int_0^z dz \nabla_\perp \cdot (\mathbf{B} \times \mathbf{A}),
\]

where, for brevity, the \((i, j)\) subscripts have been dropped. Equation (15) expresses the relationship between the X-ray emission and magnetic structure of coronal active regions. Note the importance of a dynamic behavior of the magnetic structure \((\mathbf{A} \neq 0)\) in order to explain its brightness in X-rays.

Unfortunately, the coronal magnetic field cannot be observed directly. However, the magnetic structure can be extrapolated from magnetograms, which display the spatial distribution of the vertical component of the photospheric magnetic field. The simplest case is to assume a current-free configuration and perform a potential \((\mathbf{B} = \nabla \phi)\) extrapolation for the corona, using magnetograms as boundary conditions. We follow this approach in \$2.3 as a zeroth-order approximation. In \$4 we discuss how our results are affected when a small force-free (i.e., \( \mathbf{J} \times \mathbf{B} = 0 \)) contribution is considered.

2.3. Magnetic Field Extrapolations

Hereafter we assume that a potential (current-free) field extrapolation from the photospheric magnetic field provides a good zeroth-order description of the coronal field. This hypothesis holds best for small-wavenumber modes (like the ones we are interested in), since electric currents are expected to be spatially distributed in a small-scale filamentary, and nonuniform, fashion.

We define a potential function \( \phi \), such that \( \mathbf{B} = \nabla \phi \). This scalar function satisfies a Laplace equation \((\nabla^2 \phi = 0)\), subject to the boundary condition

\[
\frac{\partial \phi}{dz}(z = 0) = b,
\]

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where $b_x(t)$ is the $z$-component of the photospheric magnetic field, which is displayed in magnetograms. After straightforward calculations,

$$B_k = \left( z - i \frac{k}{k} \right) e^{-\frac{k^2}{k^2}} b_d(t), \quad (17)$$

$$A_k = i \left( - \frac{k \times z}{k^2} \right) e^{-\frac{k^2}{k^2}} b_d(t), \quad (18)$$

For the particular case of a two-dimensional photospheric velocity field, the normal magnetic field component $b = z \cdot B$ is advected by the fluid just like a scalar field (Knobloch & Rosner 1981). In other words,

$$\partial_t b = -(U \cdot \nabla) b, \quad (19)$$

where $U = U_{i,j}(t)$ is the photospheric velocity field. We Fourier-transform this equation and formally write down the solution as

$$b_k(t) = b_k(0) + \sum_{k_1+k_2=k} i \int_0^t dt' k' \cdot U_{k'}(t') b_{k'}(t') , \quad (20)$$

where the sum also involves the symmetrization with respect to permutations of $k', k''$. We can now Fourier-transform our equation (15) and use equations (17) and (18) to express $I_d(t)$ in terms of the photospheric fields

$$C^{-1} I_d(t) = \sum_{k_1+k_2=k} \left( \frac{1}{k_1} + \frac{1}{k_1+k_2} \right) \frac{k_1 \cdot k_2}{k_2^2} b_{k_1}(t) \delta_{k_2}(t), \quad (21)$$

where $b_{k_1}(t)$ and $\delta_{k_2}(t)$ can be obtained from equation (20).

### 2.4. Power Spectra

Because of the stochastic nature of turbulent regimes, their theoretical description is usually statistical, all relevant quantities being described through random variables. Hereafter, we assume that the subphotospheric velocity field evolves in a turbulent fashion. Therefore, the subphotospheric velocity and magnetic field components will be considered as random variables. Equation (21) implies that $I_{d,m}(t)$ should be considered as a stochastic variable too. Therefore, the calculation of statistical averages like $\mathcal{S}(k) = \langle D^2 k / 2 \pi \rangle \langle |I_d|^2 \rangle$ is of interest.

Formally, the angular brackets represent an average over an infinite set of independent realizations of the dynamic evolution of the active region under study. This operation can be approximated in practice either by performing averages over different active regions (spatial averages) or by integrating the variables in time (temporal average), provided that the dynamic evolution of these systems is sufficiently ergodic.

We assume this stochastic process to be statistically isotropic and stationary. "Statistically isotropic" means that statistical averages only depend on $k = |k|$, while "statistically stationary" implies that these averages are independent of time. Therefore,

$$\mathcal{S}(k) = \frac{D^2 k}{2 \pi} \langle |I_d(t)|^2 \rangle = \frac{D^2 k}{2 \pi} \langle |I_d(0)|^2 \rangle , \quad (22)$$

where the angular brackets include an integration over the polar angle in Fourier space.

From equations (20) and (21), we derive our key result:

$$\mathcal{S}(k) = \frac{D^2 C^2 k}{2 \pi} \left| \sum_{k_1+k_2=k} \left( \frac{1}{k_1} + \frac{1}{k_1+k_2} \right) \frac{k_1 \cdot k_2}{k_2^2} b_{k_1}(0) \sum_{k' + k'' = k_2} ik' \cdot U_{k'}(0)b_{k''}(0) \right|^2 . \quad (23)$$

This expression allows us to compute the power spectrum for X-ray emission $[\mathcal{S}(k)]$ from simultaneous detections of photospheric velocity and magnetic fields. If, as indeed is the case, we only know very general features of these turbulent fields, like their power spectra, we can apply scaling techniques on equation (23). Scaling techniques were first used by Kolmogorov (1941) to describe the statistically stationary regime of hydrodynamic turbulence. In what follows, we use Kolmogorov’s approach to find a relationship between the power spectrum $\mathcal{S}(k)$ and the corresponding power spectra for photospheric velocity and magnetic fields.

Let us assume that the nonlinear interaction of Fourier modes in the photospheric turbulent motions is essentially local (that is, all $k$’s in the sums of eq. [23] are of the same order of magnitude). This locality assumption, which basically means that the strongest nonlinear interactions occur between modes whose wavenumber magnitudes are similar, is supported by theoretical arguments (Kraichnan 1971) and numerical simulations (Kraichnan & Montgomery 1980) in a number of turbulent systems. It is usual in this kind of Kolmogorov-like theories to define "tilded" quantities related to the relevant random variables of the problem and having the same units. For instance, we define $I^T_l$ in such a way that $I^T_l$ represents the total power of the variable $I$ contained in a ring in (Fourier space) of mean radius $k$ and width $k$, so that $\mathcal{S}(k) = \frac{I^T_l}{k}$ (see, for instance, Hasegawa 1985). The usefulness of these "tilded" variables lies in the fact that they are not random variables but statistical averages of them.

Let us also assume that all relevant quantities display power-law spectra. In Paper I we have shown observational evidence in favor of this assumption for the X-ray intensity. There is also observational evidence supporting this hypothesis for the photospheric...
fields (Knobloch & Rosner 1981; Chou et al. 1991; Hathaway et al. 1991; Tarbell 1992). For power-law spectra,

\[
\frac{\bar{U}_k^2}{k} = \frac{D^2 k}{2\pi} \langle |U_k|^2 \rangle \approx k^{-\alpha}, \tag{24}
\]

\[
\frac{\bar{b}_k^2}{k} = \frac{D^2 k}{2\pi} \langle |b_k|^2 \rangle \approx k^{-\alpha_s}, \tag{25}
\]

\[
\frac{\bar{I}_k^2}{k} = \frac{D^2 k}{2\pi} \langle |I_k|^2 \rangle \approx k^{-\alpha_v}, \tag{26}
\]

From equations (20) and (21) we find that the scaling between the relevant variables goes as \( \bar{I}_k \approx \bar{b}_k^2 \bar{U}_k \), and therefore we obtain the following equation relating the indices of the different power spectra:

\[
\alpha_s = 2\alpha_v + \alpha - 2. \tag{27}
\]

Since the field \( b_{ij} \) is advected by the photospheric velocity field like a passive scalar, the indices \( \alpha_v \) and \( \alpha \) are not independent. For the inertial region of a passive scalar (region of Fourier space where the scalar is not being externally pumped or dissipated), their relationship is (Tennekes & Lumley 1972),

\[
\alpha_v = \frac{5 - \alpha}{2}. \tag{28}
\]

Inserting this expression in equation (27), we finally find

\[
\alpha_s = 3. \tag{29}
\]

This theoretical derivation of the spectral index of the active region X-ray emission fits the observed values remarkably well (see Fig. 2). Also, its independence from the spectral structure of the photospheric velocity and magnetic fields is to be noted.

3. MHD TURBULENCE SCENARIO

The relative importance of nonlinearities compared with dissipative terms in the MHD equations (eqs. [1]–[3]), can be measured by two Reynolds numbers (\( R_u = u\ell/v \): Reynolds number; \( R_B = B\ell/\eta \): magnetic Reynolds number). Typical coronal values for these numbers range between \( 10^{10} \) and \( 10^{12} \), indicating that the dynamical evolution of this plasma will be subject to nonlinear effects. Therefore, these magnetically dominated fluctuations pumped into the system by footpoint motions (as discussed in § 2) are likely to couple nonlinearly to one another to produce a redistribution of energy (and other ideal invariants) in Fourier space. Since the magnetic field lines are supposed to follow the so-called line-tying condition, this additional component to the fluctuating fields, generated inside the corona by nonlinear interactions of the externally driven fluctuations, must have vanishing velocity components at the footpoints. Let us emphasize that we are not in a position to prove the occurrence of these interactions; we just affirm that, given the physical conditions of the coronal plasma, they are likely to take place, as has also been assumed by Heyvaerts &
Priest (1992). In the remainder of this section we simply assume their existence, describe the corresponding physical scenario, and discuss its potential relevance as an enhancing mechanism for Joule dissipation.

The effect of nonlinear terms is basically to redistribute excitations or fluctuations from one wavenumber to another in a virtually stochastic fashion. Only those excitations at sufficiently large wavenumbers decay as a consequence of dissipative effects. Therefore, a net flow of excitations in Fourier space is established, toward those regions which are deficient with respect to the values calculated from an ideal (without dissipation) model (Montgomery 1983). For instance, this nonlinear redistribution continuously replenishes the excitations being drained at the large-wavenumber region. An increase in Reynolds number only raises the value of the typical wavenumber at which dissipation begins to dominate, but it does not inhibit the excitation flow in Fourier space.

According to this scenario, three regions in Fourier space (wavenumber space) can be identified, each of them displaying a different turbulent behavior.

a) The energy-containing region.—This region comprises those modes that are being excited directly by the external driver, and is usually located toward the low-wavenumber zone. However, it does not necessarily include the very lowest wavenumbers, which are of the order of unity over the size of the system.

b) Dissipation region.—This region corresponds to those modes where fluctuations are being efficiently quenched by dissipative (viscous or ohmic) effects. The region is normally located at the largest wavenumbers, where the linear dissipative terms become comparable to the nonlinear terms.

c) Energy inertial region.—In this region both external forces and dissipation are negligible. Only nonlinearities play a role, transferring fluctuations from one mode to another, while keeping the total energy constant. This region normally bridges the gap between the low-wavenumber energy-containing region and the large-wavenumber dissipative zone.

Kolmogorov (1941) following heuristic arguments, has shown that when a three-dimensional, incompressible fluid is submitted to external forcing with a narrow spectrum, a direct energy cascade is generated and a stationary energy spectrum is achieved, displaying the well-known $k^{-5/3}$ distribution in the energy inertial region. Kolmogorov's ideas, mainly based on scaling properties of the ideal (dissipative) equations and on the existence of a net energy flow through the corresponding inertial range, are usually known as cascade theory (Montgomery 1983; Hasegawa 1985) and have been applied to a number of turbulent systems, including two- and three-dimensional MHD turbulence. The power spectra predicted by cascade theory for the energy inertial range have in many cases been confirmed by experiments (Grant, Stewart, & Molliet 1962; Matthaeus & Goldstein 1982; Sommier 1986) and numerical simulations (Lilly 1969; Herring & Kraichnan 1972; Fyfe, Montgomery, & Joyce 1977; Meneguzzi, Frisch, & Pouquet 1981; Matthaeus & Lamkin 1986; Biskamp & Welter 1989; Politano, Pouquet, & Sulem 1989).

Gómez & Ferro Fontán (1988) have shown that Joule dissipation of stationary MHD turbulence is a plausible mechanism for coronal heating. They assume that the subphotospheric forcing has a narrow spectrum whose wavelength is around the length scale of granular convection ($10^3$ km). Van Ballegooijen (1986) has calculated the evolution of a force-field driven by the turbulent subphotospheric velocity field and demonstrates the development of a direct energy cascade to the large-wavenumber spectral region. This scenario of energy cascades in coronal loops has recently been extended to broadband photospheric power spectra (Gómez & Ferro Fontán 1992). Unfortunately, we are still unable to test these theoretical predictions observationally. Typical length scales for structures associated with the energy cascade region ($<10^3$ km) are still beyond the present-day capabilities of spatial resolution.

In terms of the aforementioned classification of regions in Fourier space for a turbulent medium, the range of length scales covered by NIXT images seems to correspond to the energy-containing region rather than to the inertial range. The energy spectrum in the energy-containing region does not display the universal features of the spectrum in the inertial range and is in general strongly dependent on the characteristics of the external driving force. Therefore, in our theoretical analysis (see §2) we avoided applying arguments of cascade theory, which are only appropriate for the inertial range.

In what follows we describe an alternative derivation of the result $\mathcal{S}(k) \approx k^{-3}$, based on scaling properties of the magnetohydrodynamic equations. The ideal MHD equations (eqs. [1]–[3] with $\eta = 0 = \nu$) are invariant if we simultaneously scale the distance by $\lambda$, the field $\mathbf{u}$ by $\lambda^4$, the field $\mathbf{B}$ by $\lambda^6$ and time by $\lambda^{-1}$. In the force-free approximation, the scaling exponents $h$ and $h_b$ can be arbitrarily different, while in the general case the scaling holds for $h = h_b$. These equations describe not only the evolution of the coronal part of the loops (in their limit $|u| \ll |B|$) but also the footpoint motions (in the opposite limit, $|u| \gg |B|$). In fact, equation (20), which describes the passive advection of the normal magnetic field component at the photospheric level, is nothing but a particular expression of the more general equation (1).

The first theory of scaling in turbulence was proposed by Kolmogorov (1941) for the Navier-Stokes equations and rests on the following assumptions (see Frish & Orszag 1990 for a comprehensive discussion). First, the scale invariance holds for average ("tilded") quantities, whereas it need not hold for detailed structures. Second, a finite flux of energy cascades from large scales to small ones, where it dissipates. Third and last, the energy flux at a particular wavenumber is assumed to depend on quantities evaluated in a neighborhood of that wavenumber (the energy flux is a local, diffusion-like process in Fourier space). The energy flux in Fourier space goes as $\mathcal{F} \propto k^2 \propto \lambda^3$. Since this energy flux must be scale-independent, the scaling exponent must be $h = \frac{1}{2}$, which in turn leads to the well-known Kolmogorov power law $h(k) \approx k^{-5/3}$. For a scalar field $\phi(x, t)$ being passively advected by the velocity field, the corresponding flux in Fourier space is $\mathcal{F}_\phi = k^2 \overline{\dot{\phi}} \propto \lambda^{2k+4h-1}$. Assuming that this flux of passive scalar is scale-invariant (i.e., we are in the inertial range of the passive scalar), we obtain $h = (1 - h)/2$.

The power spectrum for X-ray intensity behaves as $\mathcal{S}(k) = T^2/k = (\langle B^2 \rangle/k) \propto \lambda^{2k+4h-1}$. Let us make the following assumptions: (1) the scaling properties of $B$ and $h$ are the same, and therefore $h = h_b$ (this is true for potential extrapolations, and may also be valid for more general configurations); (2) $b$ is passively advected by the photospheric velocity field; and (3) the range of scales that we are looking at ($10^3$–$10^5$ km) falls into the inertial range of $b$ and thus $h = 2h_b = 1$. Under these assumptions, we again obtain $\mathcal{S}(k) \propto k^{-3}$ as in equation (29). Note that we did not assume that we were working on either the photospheric kinetic energy or the
coronal magnetic energy inertial ranges. However, our assumption 3 is dubious, because it means that no new magnetic structures (of sizes between $10^3$ and $10^5$ km) emerge to the photospheric level. But the observed relationship between the photospheric kinetic and magnetic energy does not depart much from the case of the inertial range of a passively advected magnetic field (Tarbell 1992). The availability of simultaneous observations of photospheric velocity and magnetic fields would also allow a better computation of $\mathcal{K}(k)$ by using equation (23).

4. DISCUSSION

In Paper I, following a Fourier analysis on a sample of images from active regions obtained by NIXT, we found a broad-band, isotropic, power-law spectrum $k^{-3}$ for the spatial distribution of soft X-ray intensities, regardless of the different topological structures of the active regions.

In the present paper, we explore theoretically the connection between coronal magnetic features and their driving agent, the photospheric convective motions, and present a theoretical derivation of the power-law spectra for the spatial distribution of X-ray emission. We first express the X-ray intensity in terms of the fluctuating velocity and magnetic field in the corona, assuming that the X-ray emission per pixel is proportional to the MHD Poynting flux entering that pixel. Then we relate the fluctuating coronal fields to the photospheric velocity and magnetic fields. Performing a statistical averaging procedure over these relationships, and assuming that the longitudinal component of the photospheric magnetic field is passively advected by footpoint motions, we find a power spectrum $\mathcal{S}(k) \approx k^{-3}$, which is in perfect agreement with the spectra of the active regions we studied.

We emphasize that the theory presented in § 2.4 is statistical and does not make predictions about individual events. For instance, our assumption of stationarity by no means excludes the possibility of time dependence for individual realizations (observations of time variations in a particular active region during a particular period of time). The variable $I_s(i)$ is probably a function of time, but $\mathcal{S}(k) = (D^n k/2n)(|I_s|^2)$ is not.

We perform our theoretical analysis without specifying any particular coronal heating mechanism. However, our assumption that footpoint motions occur on time scales much longer than the Alfvén time of coronal loops tacitly implies the action of a DC heating mechanism. The features corresponding to dissipative structures are expected to become apparent at length scales well beyond present-day resolution capabilities. Thus it may be impossible to decide, on the basis of the observations analyzed here, which DC coronal heating mechanism applies.

The scenario for MHD turbulence that we explore further in § 3 is only one of the many possible heating mechanisms discussed in the literature. It predicts a natural cascade of energy from large magnetic structures (which receive this energy from the convective motions) to highly microscopic structures. This cascade then provides the energy released at very small length scales. The role of MHD turbulence in heating of the solar corona has been discussed in a number of papers (van Ballegooijen 1986; Similon & Sudam 1989; Gómez & Ferro Fontán 1988, 1992; Heyvaerts & Priest 1992; see also Gómez 1990 for a recent review). Most of the theoretical models for coronal heating proposed in the literature require the formation of confined and strongly sheared magnetic zones where the associated electric currents resistively dissipate the magnetic energy. Examples of these strongly sheared regions throughout the literature are the tangential discontinuities assumed by models based on MHD wave dissipation (Heyvaerts & Priest 1983; Davila 1987) or a stochastic distribution of them to enhance dissipation of Alfvén wave packets (Similon & Sudam 1989), the current sheets thought to be responsible for impulsive events like solar flares (Heyvaerts, Priest, & Rust 1977), the spontaneously generated tangential discontinuities in the topological dissipation model (Parker 1972, 1983), or the resonant regions of models based on an analogy between coronal loops and LRC circuits (Ionson 1978, 1982). We speculate that this direct cascade of magnetic energy associated with an MHD turbulent regime can be regarded as a source of the aforementioned highly sheared zones.

In our opinion, magneto-hydrodynamic turbulence provides an appropriate theoretical starting point for detailed models of coronal heating.

The power spectra we have derived are not expected to hold for the largest wavenumber modes, which define the macroscopic shape of the active region. Loosely speaking, the mean magnetic field is oriented along the axis of the loop and is large compared with the turbulent fluctuations of the magnetic and velocity fields, thus causing the total distribution of fluctuations to be highly anisotropic. While the perpendicular motions may develop small dissipative scales giving rise to turbulent dissipation, spatial variations along the mean field generally remain smooth, their dynamics being determined by weak interaction of Alfvén waves (Biskamp & Welter 1989). In Paper I we investigated the division between the anisotropically distributed low modes and the more isotropic distribution of high-wavenumber modes by removing high-wavenumber modes from the observed images, and found that even if we remove up to 95% of the highest wavenumber modes, the filtered image looks very similar to the original one. This simple experiment verifies that only the very lowest wavenumber modes are responsible for the observed anisotropic shape of these coronal active regions. On the other hand, those modes contributing to the isotropic power-law spectrum in the intermediate-wavenumber range reflect the stochastic nature of the footpoint motions.

The question remains, of course, of what determines the distribution of the large coronal structures. We suggest that it is the underlying photospheric field, since potential extrapolations from magnetograms provide a good zeroth-order description of the coronal magnetic structure as evidenced by comparison with X-ray images (Poletto et al. 1975), although departures from the potential structure have also been reported (Krieger, de Feiter, & Vaiana 1976; Vaiana & Rosner 1978). A force-free extrapolation from vector magnetograms (which map the three components of the photospheric magnetic field) probably provides a better description. With respect to our approximation of a potential magnetic field, let us remark that we have only used this assumption on the truncated set of Fourier modes corresponding to scales between $10^3$ and $10^4$ km; we do allow for the existence of electric currents in modes whose wavenumbers are larger than our upper cutoff. These currents are indeed essential to provide for the dissipation of magnetic energy.

To get some insight into the extent to which small, almost spatially uniform electric currents would influence our results, we extend our calculations to the case of linear force-free field extrapolations, $\mathbf{V} \times \mathbf{B} = \beta \mathbf{B}$ in the limit $\beta \ll k$. In this case, the magnetic
field and its vector potential shown in equations (17) and (18) acquire an extra (force-free) term,

$$\delta B_k = i \beta \frac{k \times z}{k^2} e^{-k z} b_k(t),$$

(30)

$$\delta A_k = \beta \frac{z}{k^2} e^{-k z} b_k(t).$$

(31)

Keeping terms up to first order in $\beta$, we find that $\delta \mathcal{A}(k) \approx \mathcal{A}(k) \frac{|| \delta B_k ||}{|| B_k ||}$, and therefore

$$\mathcal{A}(k) \approx k^{-3} \left( 1 + \frac{\beta}{k} \right).$$

(32)

Hence, for $\beta \ll k$, equation (32) implies no difference between the omnidirectional spectrum of a potential and a linear force-free field.

In summary, we developed a simple theoretical model to link the fluctuating fields in the corona with stochastic motions in the convective region. This model leads to the prediction of a power spectrum $\mathcal{A}(k) \approx k^{-3}$ that fits perfectly well with the ones reported in Paper I for a number of structurally different coronal active regions.

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