NORMAL INCIDENCE X-RAY TELESCOPE POWER SPECTRA OF X-RAY EMISSION FROM SOLAR ACTIVE REGIONS. 1. OBSERVATIONS

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ABSTRACT

We use the very high resolution images of coronal active regions obtained by the Normal Incidence X-Ray Telescope to search for features of magnetohydrodynamic (MHD) fluctuations. By Fourier analyzing these images we find a broad-band, isotropic, power-law spectrum for the spatial distribution of soft X-ray intensities. The presence of a broad-band spectrum indicates that magnetic structures of all sizes are present, at least down to the resolution limit of the instrument, which is ~0.75.

From a sample of topologically different active regions, we obtain power spectra for their X-ray intensities which falls off with increasing wavenumber as $k^{-3}$.

Subject headings: MHD — Sun: activity — turbulence

1. INTRODUCTION

Ever since on-disk coronal imaging became available in the late 1960s, high spatial resolution soft X-ray observations have consistently indicated the presence of inhomogeneous magnetic structures in the solar corona (Vaiana & Rosner 1978). The spherical and spatially uniform image of the corona has been replaced by a hierarchy of coronal loops of different sizes and shapes as a consequence of the presence of magnetic fields confining the coronal plasma and governing its dynamics. The existence of these structures of the coronal plasma as well as their close relationship with magnetic fields certainly is the most important observational finding since the very discovery of the solar corona. Very high resolution images recently obtained by the Normal Incidence X-Ray Telescope (NIXT) have shown internal structure for coronal loops down to the resolution limit of the instrument, suggesting that structuring on still finer scales will be seen as the spatial resolution is improved (Golub et al. 1990).

The very active role of the magnetic energy contained in these structures in heating the corona is being intensively studied (for recent reviews see Narain & Ullschneider 1990; Gómez 1990). The magnetic field lines are deeply rooted into the convective region, below the photosphere, and evolve attached to the fluid because of the very high electric conductivity there. The turbulent subphotospheric fluid motions stochastically stress the magnetic field lines and pump magnetic energy into coronal active regions. By this mechanism, the convective region replenishes the coronal magnetic energy which is continually being dissipated. This energy input is sufficient to compensate for the conductive and radiative losses of the coronal plasma (Parker 1983). However, since the Joule relaxation of the above-mentioned magnetic stresses is extremely slow, the main challenge for current coronal heating theories is to find a mechanism able to speed up Joule dissipation.

According to this scenario and regardless of the dissipative process involved, the spatial distribution of coronal magnetic structures is expected to reflect the complexity of the subphotospheric driving motions. Recent observational studies based on Doppler imaging of photospheric motions (Zahn 1987; Chou et al. 1991; Hathaway et al. 1991; Tarbell 1992) strongly indicate the existence of a broad-band, power-law kinetic energy spectrum, as expected for a turbulent medium like the subphotospheric convective region. A similar analysis performed on magnetograms (Knobloch & Rosner 1981) has also shown a power-law spectrum for the photospheric magnetic field. Using NIXT images, magnetic structures from 10⁵ km (typical size for an active region) to about 600 km have been resolved. We performed Fourier transforms for several topologically different active regions and computed their corresponding power spectra. They all display an isotropic power spectrum on the wavenumbers plane $(k_x, k_y)$. Their observed topological differences only show up in the lowest wavenumber Fourier modes. The corresponding omnidirectional power spectra are power-law, and all of them can be fitted by $k^{-3}$.

In § 2 we briefly describe the NIXT characteristics and detail the Fourier technique we used. In § 3 we summarize the procedures we followed to subtract a number of spurious observational effects from our data, such as the MTF of the film and the digitization procedure, aliasing due to pixelization of the image, or noise effects due to low photon statistics. In § 4 we list and briefly discuss our results. In a subsequent paper (Gómez, Martens, & Golub 1993), we present a theoretical model that relates the power spectrum of coronal X-ray intensity to the power spectrum for photospheric motions. As a result of this model, a power spectrum of $k^{-2}$ for the spatial distribution of X-ray emission is predicted, in perfect agreement with the observed spectra reported in the present paper.

2. X-RAY POWER SPECTRA FROM NIXT IMAGES

The NIXT payload for the flight of 1989 September 11 included a multilayer-coated telescope, configured as a 25 cm
Therefore, for a square array of $N \times N$ pixels,

$$I_k = \sum_{n=1}^{N} \sum_{m=1}^{N} I_{n,m} \exp \left( \frac{2\pi i}{N} (nn' + mm') \right),$$

(1)

where

$$k = \frac{2\pi}{ND} (n', m'), \quad n', m' = 0, 1, \ldots, (N-1),$$

(2)

and $D$ is the linear pixel size ($D \approx 450$ km on the solar surface).

In Figure 2 we show the two-dimensional power spectrum of region C, which is simply defined as

$$\mathcal{F}_{2D}(k_x, k_y) = \frac{D^2}{4\pi^2} |I_k|^2,$$

(3)

so that the total power $\mathcal{F} = \sum_{k_x, k_y} I_{k_x, k_y}^2$ can also be computed as $\mathcal{F} = \int \mathcal{F}_{2D} \, d^2k$. Besides the obvious symmetry $|I_{-k}^2| = |I_k|^2$ (which holds for every Fourier transform of a real function, because $I_{-k} = I_k^*$), Figure 2 shows a high degree of isotropy in the distribution of Fourier modes, that is, the power spectrum is essentially a function of $|k|$.

Figure 2 also shows some features of aliasing in the largest Fourier modes (see § 3.3). The nonzero pixel size $D$, automatically introduces an upper cutoff,

$$k_{\text{max}} = \frac{\pi}{D},$$

(4)

Fig. 2.—Two-dimensional power spectrum for the X-ray intensity of region C. Note the high degree of isotropy displayed at intermediate wavenumbers.
Fig. 1.—X-ray images of five coronal active regions (labeled A–E) selected for the present study. The bar in the upper right of each image corresponds to 15 Mm. The dark region was used for comparison. These images have been obtained by NIXT during its flight of 1989 September 11.

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which is known as the Nyquist wavenumber. The presence of aliasing is consistent with the idea that the structuring of coronal magnetic fields goes beyond the present spatial resolution capabilities.

The basic features shown in Figure 2 are common to the power spectra of all the active regions that we have studied (regions A–E), even though their topological structures are quite different (see Fig. 1). Their different topology only shows up in the Fourier distribution of the highly emitting structures (those modes concentrated around the peak in Fig. 2), but the middle and low emitting regions of Fourier space look amazingly similar, displaying the aforementioned isotropic distribution. In order to focus on the radial (in Fourier space) dependence of these two-dimensional power spectra, we computed the so-called omnidirectional spectra, which are the integrals over the polar angle as defined by

$$\mathcal{S}(k) = k \int_0^{2\pi} d\theta \mathcal{S}_{2D}(k \cos \theta, k \sin \theta).$$

(5)

The omnidirectional power spectra are shown in Figure 3. All the active regions display a power-law spectrum close to

$$\mathcal{S}(k) \propto k^{-3}.$$  

(6)

We estimated the relative error for X-ray intensity at about 15% (i.e., ΔI/I ≈ 0.15). Following the rules for error propagation, we end up with an error bar for the power spectra of Δ[log (|S(k)|)] = 2(ΔI/I) ≈ 0.3, as shown in Figure 3. We also corrected for the film MTF (modulation transfer function) and estimated the noise level due to photon statistics (see § 3). In Figure 4 we show the noise curve for active region A and for the dark region. The power spectra are reliable only in the region of Fourier space where they are safely above the noise curve. Figure 4 shows that for region A the noise curve reaches the spectrum at wavenumbers where the power spectrum departs from power law, which is also the case for the other four active regions (B–E). We note that our estimate for photon noise effects was a conservative one. Therefore we expect this spectral region (corresponding to spatial features between 900 and 2000 km) to be somewhat influenced by noise but not to show serious blurring effects, which in fact can be confirmed from a direct inspection of the images. We believe that the high-wavenumber flattening of our spectra is due to the combined effect of aliasing and photon noise, as discussed at the end of § 3.

For isotropic power spectra displaying a power-law dependence, it is straightforward to show that one-dimensional power spectra

$$\mathcal{S}_{1D}(k_x) = \int dk_y \mathcal{S}_{2D}(k_x, k_y) \approx k_0 \sum_{k_y} \mathcal{S}_{2D}(k_x, k_y),$$  

(7)

$$\mathcal{S}_{1D}(k_y) = \int dk_x \mathcal{S}_{2D}(k_x, k_y) \approx k_0 \sum_{k_x} \mathcal{S}_{2D}(k_x, k_y)$$

(8)

are also power-law, and have the same spectral index. The constant $k_0 = 2\pi/ND$ is the smallest wavenumber of the system, related to the size of the image. One interesting property of these one-dimensional distributions (which is useful for computational purposes) is that they are equivalent to com-

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**Fig. 3.**—Omnidirectional power spectra corresponding to the active regions A–E. We also show the power spectrum of the dark region, for reference.

**Fig. 4.**—The full traces correspond to the power spectra of active region A and to the dark region (also shown in Fig. 3). The dotted lines are the corresponding noise levels due to photon statistics.

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puting the power spectrum for each row (column) of the square array of data and averaging over the columns (rows). We computed these marginal distributions and verified the validity of the power-law behavior $I(k) \propto k^{-3}$. Perhaps more important is the fact that the sums involved in equations (7)-(8) (or the integral corresponding to the omnidirectional spectrum in eq. [5]) constitute, to some extent, a statistical averaging procedure. This statistical averaging capability is particularly relevant here, since we are planning to compare our observational results with theoretical predictions from intrinsically statistical models (Gómez et al. 1993).

In Figure 3 we also show the power spectrum for the dark region displayed in Figure 1. It appears as a uniformly dark region, but it has, in fact, an emission intensity level higher than the threshold value for the photographic emulsion that was used. In Figure 4 we show that except for the largest wavenumber modes, the power spectrum lies above the noise level. In Figure 3 we can see that the power spectrum corresponding to this dark region cannot be fitted by a $k^{-3}$ power law. Moreover, it can hardly be fitted by any power law. In this respect, the dark region behaves qualitatively different from the active regions we have studied, which share a common power-law distribution.

If coronal active regions were formed by a bundle of magnetic flux tubes of given sizes and with no internal structure, their power spectra should display a few peaks (corresponding to the sizes of the tubes) emerging from a noisy background. Instead, a power-law distribution is indicative of the presence of a whole hierarchy of magnetic structures of all sizes, as expected for a system driven by turbulent motions.

3. DECONVOLUTION ANALYSIS

3.1. The Derivation of Corrected Spectra

Image formation in a two-dimensional invariant linear system is a process that is well understood, and for which a complete mathematical formalism is available. The term “invariant linear” refers here to a point-spread function (PSF, i.e., the image formed from a point-source object) that does not vary over the telescope's field of view. For a brief and clear description see, for example, Goodman (1968). We will assume in the following that the NIXT's PSF is linear invariant.

The key point in two-dimensional invariant linear systems is that since the image $I(x, y)$ is formed through a convolution of the PSF with the object $O(x, y)$, that is,

$$I(x, y) = \int O(x', y')PSF(x - x', y - y')dx'dy',$$

(9)

the “convolution theorem” guarantees that the Fourier transform of the image is simply the product of the Fourier transform of the object times the Fourier transform of the PSF, called the optical-transfer function (OTF), which is in general a complex function. Hence,

$$I(k_x, k_y) = O(k_x, k_y)OTF(k_x, k_y),$$

(10)

and in principle, from calculating or measuring the PSF of an optical system, one can reconstruct the Fourier transform of the object for any image, and therefrom the object itself. However, inevitably the OTF of an optical system will decrease to zero with increasing wavenumber ($k$), and the inversion of equation (10) becomes impossible. In theory the upper limit $(k)$ is the diffraction limit, but very few telescopes even get close to that. Therefore every image has a resolution limit (highest $k$).

The absolute value of the OTF is the modulation-transfer function (MTF). In many cases the PSF is symmetric around its peak and OTF $\equiv$ MTF up to the first zero. The two-dimensional power spectrum of an image has been defined in equation (3) as proportional to the square of the absolute Fourier transform of an image, and thus it follows that the power spectrum of the object (which is what we are after) is obtained by applying a correction factor $1/(MTF)^2$.

3.2. The MTF of the Digitized NIXT Pictures

By repetitively applying the “convolution theorem” it is easy to show that the MTF of a complete optical system is the product of the MTFs of each element in the image formation. For example, the MTF of a telescope with a CCD camera for image recording is the product of the telescope MTF and that of the CCD. For NIXT the total MTF is the product of the MTF of the telescope, that of the film, and that of the digitization. The NIXT mirrors are of exceptionally high quality, and hence the telescope MTF is very broad. We will assume in the following that it is effectively unity below the Nyquist frequency for the digitization.

The MTF for the Kodak Technical Pan film 2415 was kindly provided to us by Kodak from unpublished data. It is measured in the standard way (e.g., Kodak 1973) in visible light (400–700 nm). The data were obtained after development in D-19 for 4 minutes (the NIXT film was developed in D-19 for 7.5 minutes). We found a good fit to the data with the expression

$$MTF_{film} = 1.06 \exp\left(-0.0166k\right),$$

(11)

where $k$ is the absolute value of the wavenumber vector, expressed in cycles mm$^{-1}$. It is likely that the actual film MTF for the NIXT pictures is better than the result above, since less scattering in the emulsion would be expected for X-rays than for visible light. However, Kodak had no data taken in X-rays, and no estimates for the MTF wavelength dependence. Therefore we will use the expression above, keeping in mind that it represents an upper boundary for the MTF correction.

The digitization was achieved as follows. The original film was enlarged 4 times, leading to a plate scale of 40 $\mu$m arcsec$^{-1}$ (we will ignore the loss of resolution because of that procedure). The enlarged film was scanned with a focused laser beam with a FWHM of 33 $\mu$m, in steps of 25 $\mu$m. Thus the pixel size ($\Delta$) is 0.625, and we assume the distribution for the intensity in the laser focus to be Gaussian. The latter leads to an MTF from the scanning of

$$MTF_{scanning} = \exp[-\pi^2(0.79k)^2],$$

(12)

where the wavenumber $k$ is expressed in units of $1/\Delta$. (Note: to find the film MTF in units of $1/\Delta$ one has to use 1/159 cycles mm$^{-1}$.) The effect of scanning with a finite step length $\Delta$ is a sampled spectrum, with upper and lower limits of plus or minus, respectively, the Nyquist frequency $k_N = 1/(2\Delta) = 79$ cycles mm$^{-1}$ (see Goodman 1968, pp. 21–25). In addition, when the spectrum to be sampled is not effectively zero outside this range, aliasing will occur. We will estimate that effect separately below.

The curves in Figures 3 and 4 were obtained by applying the correction factor $1/(MTF)^2$ to the spectra of the images, with MTF the product of the film and digitization MTFs (eqs. [11] and [12]). We note that this correction leads to the flattening
of the omnidirectional spectra as the Nyquist frequency is approached. However, we will demonstrate below that this flattening cannot be considered physically meaningful because of the errors introduced by photo statistics.

3.3. Aliasing

The sampled spectrum resulting from scanning with finite steplength $\Delta$ is easily derived from the Fourier spectrum of the image sampled (see Goodman 1968, p. 21–22). The result is

$$I(k_x, k_y)_{\text{sampled}} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} I(k_x - \frac{n}{\Delta}, k_y - \frac{m}{\Delta}).$$  \hspace{1cm} (13)

Aliasing occurs when there are significant contributions to $I(k_x, k_y)_{\text{sampled}}$ from other terms than $n = 0, m = 0$. Obviously, to derive exactly the amount of aliasing in a spectrum one needs to know the original spectrum $I(k_x, k_y)$, which is not known. However, irrespective of the form of the spectrum, the sampled Fourier spectrum will be enhanced by up to a factor 2 near the Nyquist frequency because at, for example, $I(k_x = k_n, k_y)$ there will be a $n = 1$ contribution from $I(k_x = -k_n, k_y)$, which is of the same order of magnitude. In the power spectrum, which goes as the Fourier spectrum squared, this factor would be about 4, while contributions from other values of $n$ and $m$ would even further enhance it.

We performed the following simple numerical experiment to determine the maximum extent to which aliasing can “spoil” a power spectrum. Assume an image with a $k^{-3}$ omnidirectional power spectrum. Then the two-dimensional spectrum for the absolute value of the Fourier transform would be $k^{-2}$. We cannot know the phase of the elements in the Fourier spectrum without knowing the original image. However, the largest amount of aliasing is obtained when all the elements in the summations of equation (13) have the same phase, so let us assume that this is the case. By numerically performing the summation of equation (13) we found a ratio of about 13 between the aliased and unaliased power spectrum near the Nyquist frequency decreasing to a factor of about 2 for log (k) half a unit smaller (cf. Figs. 3 and 4). Hence it appears that aliasing may account for a significant part of the upturn at high $k$ of the power spectra of the solar images we studied. Furthermore, the slope of the spectra seems to suffer little influence from aliasing below the last 0.5 stretch in log (k).

3.4. Photon Noise

From the Kodak-provided characteristic curve for Technical Pan 2415, and knowledge of the exposure time, we were able to calculate the intensity of the incident X-rays from the opacity of the NIXT negatives, and therefrom the photon counts, since the energy per photon is nearly constant (195 eV) due to NIXT’s narrow wavelength response.

As is well known, the uncertainty in photon counts is simply the square root of the count, and the propagation of these errors through the linear Fourier transforms is straightforward. Since each pixel represents an independent measurement, the sum-of-squares rule applies, and the error in the absolute value of the Fourier transform $|\delta I_k|_{\text{pixel}}$ turns out to be approximately constant as a function of $k$, and therefore equal to the error in the $k = 0$ component $|\delta I_k|_{\text{pixel}}$. The $k = 0$ component of the Fourier transform is a simple sum of the photon counts over the whole image, and hence, after again applying the sum-of-squares rule, the error in the $k$-components is found to be the square root of the sum of photon counts.

Assuming that the error in the Fourier components is small compared to the spectrum itself, one then finds for the uncertainty in the two-dimensional power spectrum

$$\delta S_{2D}(k_x, k_y) = 2 |I(k_x, k_y)| \delta I(0, 0).$$  \hspace{1cm} (14)

The calculation of the omnidirectional power spectrum $S(k)$ (eq. [5]) involves averaging $S_{2D}(k)$ over a ring bounded by $|k - dk, k + dk|$. Since the elements of $S_{2D}(k)$ represent independent measurements, the uncertainty in $S(k)$ goes like $\delta S(k) = 2 \pi k \delta S_{2D}(k)/N_k^{-2}$, where $N_k$ is the number of elements in the ring $[k - dk, k + dk]$, and $S_{2D}(k)$ denotes the average over the ring. Applying this expression to the power spectra of Figure 3 yields uncertainties far smaller than the spectra for every value of $k$. However, we have chosen to use the average error rather than the error-in-the-average in our figures, the average error being

$$\delta S(k) = 2 \pi k \delta S_{2D}(k).$$  \hspace{1cm} (15)

The reason for this choice is that when the error becomes comparable to the values of $S_{2D}(k)$ itself, the physical meaning of the average over these positive definite quantities is rather doubtful. Once the uncertainty in $S_{2D}(k)$ is comparable to the spectrum itself, the noise limit is reached, and the spectra will flatten out. Hence we believe that the (much larger) average error is a better indicator for the accuracy of the spectra.

In Figure 4 the dotted curves represent the average error in the power spectra due to photon statistics (for active region A and the dark region). We note that the uncertainty becomes an appreciable fraction of the spectrum only near the Nyquist frequency, where the upturn of the spectrum is also noticeable. Thus we suggest that the observed upturn is partly due to the statistical error introduced by low photon counts and partly due to aliasing effects as discussed above. Further we are confident that the obtained power spectra are quite reliable up to the last half-unit in log (k).

4. DISCUSSION

We selected a number of active regions from NIXT’s 1989 September campaign to analyze the size distribution of their internal X-ray emitting structure. To this end, we performed a Four analysis on the corresponding X-ray images. For each active region, we computed its two-dimensional power spectrum, which is simply the square of the absolute value of the Fourier transform of the image matrix. The azimuthally integrated, or so-called omnidirectional spectra, are obtained from the digitized spectra by overlaying the two-dimensional spectrum with rings of equal cross section, and increasing radius, and adding the elements in each ring (see Gomez et al. 1991 for a preliminary version of this study). These spectra have been properly deconvolved, correcting for MTF effects both in the film and in the scanning procedure. We also estimated the potential influence of other effects, such as aliasing caused by undersampling or noise due to poor photon statistics. We found that all these spurious effects only become significant at the largest wavenumbers.

The main conclusion of this paper is that, for a number of topologically different active regions, NIXT has observed (1) an isotropic two-dimensional power spectrum for their spatial distribution of X-ray emission, and (2) a common $k^{-3}$ power-law distribution for the corresponding omnidirectional spectra, regardless of their different topology. The existence of a power-law distribution demonstrates further that there is no
The heating of coronal plasma is known to be closely related to the presence of magnetic fields. The subphotospheric fluid motions permanently stress the coronal magnetic field lines, thus replenishing the magnetic energy which is continually being dissipated (Parker 1972). We believe that the existence of broad-band power spectra for the X-ray emission of active regions is therefore a consequence of this connection between coronal magnetic structures and the subphotospheric turbulent motions. In a separate paper (Gómez et al. 1993) we explore this connection from a theoretical point of view. Assuming that the longitudinal component of magnetic field is passively advected by photospheric motions, we obtain a theoretical power spectrum of $k^{-3}$ for the spatial distribution of X-ray emission, which agrees perfectly well with the observed spectra reported in this paper.

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NORMAL INCIDENCE X-RAY TELESCOPE POWER SPECTRA OF X-RAY EMISSION FROM
SOLAR ACTIVE REGIONS. II. THEORY

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ABSTRACT

In a previous paper, we used the very high resolution images of coronal active regions obtained by the Normal Incidence X-Ray Telescope to study the size distribution of X-ray-emitting structures. A Fourier analysis of these images showed a broad-band, isotropic, power-law spectrum for the spatial distribution of soft X-ray intensities. The presence of a broad-band spectrum indicates that magnetic structures of all sizes are present, at least down to the resolution limit of the instrument, which is about $\frac{1}{2}^\circ$. In the present paper, we present a model that relates the basic features of coronal magnetic fluctuations to the subphotospheric hydrodynamic turbulence that generates them. The main result of this paper is that from this model we obtain a theoretical power spectrum for the X-ray intensity, which falls off with increasing wavenumber as $k^{-3}$, fitting remarkably well the observed spectra that we obtained from a sample of topologically different active regions.

We speculate that the nonlinear interactions of these externally driven fluctuations will contribute to establish a magnetohydrodynamic turbulent regime in the corona. We suggest that the bulk of the energy delivered to the corona from footpoint motions directly cascades down to very microscopic length scales, where it efficiently dissipates and heats the plasma. However, since the wavenumber range associated with the cascade and dissipation regions are still beyond present-day spatial resolution limits, the presence of a turbulent regime cannot be observationally confirmed.

Subject headings: MHD — Sun: activity — turbulence

1. INTRODUCTION

The existence of highly inhomogeneous and fibrillated structures of the coronal plasma as well as their close relationship with magnetic fields certainly is the most important observational finding since the very discovery of the solar corona. Very high resolution images recently obtained by the Normal Incidence X-Ray Telescope (NIXT) have shown internal structure for coronal loops down to the resolution limit of the instrument, suggesting that structuring on still finer scales will be seen as the spatial resolution is improved (Gómez, Martens, & Golub 1992). Using X-ray images obtained by NIXT during its 1989 September campaign, Gómez et al. 1992 (hereafter Paper I) analyzed the size distribution of the internal X-ray-emitting structure of a number of active regions. For each active region, they computed its two-dimensional power-spectrum, and its azimuthally integrated version, or so-called omnidirectional spectrum. The main results from that analysis were that (1) the two-dimensional power spectra displayed a high degree of isotropy in the wavenumber plane ($k_x$, $k_y$) and (2) a common $k^{-3}$ power-law distribution for the omnidirectional spectra of all active regions was obtained, regardless of their different topology. The existence of a power-law distribution is indicative of the absence of "preferred" length scales for coronal X-ray structures, while the constancy of the power index strongly suggests that the same process generates the X-ray emission of all active regions.

The very active role of the magnetic energy contained in these structures in heating the corona has been repeatedly pointed out in the literature (see Narain & Ulmschneider 1990; see also Gómez 1990 for recent reviews). The magnetic field lines are deeply rooted in the convective region, below the photosphere, and evolve attached to the fluid because of the very high electric conductivity there. The turbulent subphotospheric fluid motions stochastically stress the magnetic field lines and pump magnetic energy into coronal active regions. By this mechanism, the convective region replenishes the coronal magnetic energy which is continually being dissipated. This scenario is quite appealing, because it is a theoretically plausible way for these magnetic structures to obtain enough energy to compensate for the conductive and radiative losses of the coronal plasma (Parker 1983). However, since the Joule relaxation of the above-mentioned magnetic stresses is extremely slow, the main challenge for current coronal heating theories is to find a mechanism capable of enhancing Joule dissipation.

The broad-band power spectra for coronal X-ray emission reported in Paper I are a strong observational indication of the aforementioned connection between coronal magnetic structures and the stochastic subphotospheric motions. Recent observational studies based on Doppler imaging of photospheric motions (Zahn 1987; Chou et al. 1991; Hathaway et al. 1991; Tarbell 1992) strongly indicate the existence of a broad-band, power-law kinetic energy spectrum, as expected for a turbulent medium like the...
subphotospheric convective region. A similar analysis performed on magnetograms (Knobloch & Rosner 1981) has also shown a power-law spectrum for the photospheric magnetic field. In this context, if the closed magnetic structures that confine active regions are being externally driven by a broad-band spectrum of footpoint motions, it seems natural to expect that the size distribution of their internal X-ray-emitting features also displays a broad-band power spectrum.

In the present paper we explore this connection further. In § 2 we present a theoretical derivation from first principles for the power-law spectra for the spatial distribution of X-ray emission. We first express the X-ray intensity in terms of the fluctuating velocity and magnetic field in the corona. Then we relate the fluctuating coronal fields to the photospheric velocity and magnetic fields. Performing a statistical averaging procedure over these relationships, we find the power spectrum for the observed X-ray intensity, which turns out to be independent of the power spectra for the photospheric kinetic and magnetic energy!

We believe that the presence of a broad-band spectrum of magnetic fluctuations, combined with the large Reynolds numbers for the coronal plasma, sets favorable conditions for the development of a magnetohydrodynamic (MHD) turbulent regime in solar active regions. If this is the case, the associated energy cascade to microscopic length scales will efficiently enhance Joule dissipation until the heating rate matches the conductive and radiative losses of the plasma (van Ballegooijen 1986; Gómez & Ferro Fontán 1988). In § 3 we speculate about the implications of a turbulent regime in active regions. However, our main result, the relation between the photospheric and coronal power spectra, holds regardless of the existence of a turbulent regime, so this section can be skipped without harm. In § 4 we discuss the correspondence between the observational results reported in Paper I and the theoretical results reported in this paper.

2. THEORY OF X-RAY POWER SPECTRA

In the following we will assume that the X-ray emission observed in each NIXT pixel is proportional to the Poynting flux entering that pixel, either from the photosphere or from the sides (see Fig. 1). This is a simple consequence of the apparent stationarity (on length scales equal to or larger than the pixel size) of the active regions observed (energy input must equal energy output), and the approximate equipartition between conductive and radiative losses in coronal loops. This result cannot be strictly true, since there will also be heat conducted from one pixel to the next along the field lines, but we make the assumption that the average of this sideways transport is zero (we discuss this approximation further in § 2.2).

If the Poynting flux per pixel is proportional to the X-ray emission per pixel, then of course their power spectra over an ensemble of pixels must have the same form. This then relates two distributions that, in principle, can be measured: we have already determined the distribution of the X-ray emission, and that of the Poynting flux entering through the photosphere can be derived from observations of the photospheric footpoint motions and magnetic field. In the following we derive a theoretical expression linking the slope of the power spectrum for the coronal X-ray emission to those for the photospheric magnetic field and velocity field.

We have no photospheric observations simultaneous with the NIXT data available to test this relationship. However, we can resort to a simplifying hypothesis: we assume that the photospheric magnetic field is passively advected by the convective fluid.

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Fig. 1.—Elementary box containing the coronal column mass that emits the photons to be collected by one single pixel
motions. This assumption is not strictly true everywhere on the photospheric surface (for instance, it is well known that the magnetic field inside sunspots strongly suppresses photospheric motions), but we use it as a zeroth-order approximation (see § 2.3). We hope that simultaneous observations of magnetograms and photospheric motions will soon allow us to understand this relationship better.

Inserting the expression for a passively advected magnetic field in our expression for the slope of the X-ray power law, we surprisingly will find $S(k) \propto k^{-3}$, in perfect agreement with the observations (see Goméz et al. 1992 for a detailed description of the observations). This is the main result of our analysis.

Note that in the above scenario nothing has been said about the actual coronal heating mechanism, and it appears that the above results should be valid for any heating mechanism. However, in adopting the expression for passive field advection, we have tacitly assumed that these motions are of the slow type, since they advect the magnetic field (rapid motions would generate MHD waves and not change the average position of the field lines). Therefore, our main theoretical result only applies to DC heating mechanisms. That does not mean that for heating mechanisms of AC (wave) type equally good agreement with the X-ray power spectrum might not be achieved, but that remains to be shown. Given these results, we are inclined to favor DC mechanisms, such as Joule heating at the end of a cascade generating very small length scales (Goméz & Ferro Fontán 1988) or nanoflares resulting from dissipation at tangential discontinuities (Parker 1991).

In § 2.1 we discuss which are the leading terms contributing to the Poynting flux in coronal loops. The energy balance assumed for each NIXT pixel is detailed in § 2.2. The connection between the coronal magnetic field and the longitudinal component measured at the photosphere is discussed in § 2.3. Finally, in § 2.4 we relate the power spectrum for the coronal Poynting flux (which, as we said, is assumed to be proportional to the soft X-ray intensity), to the power spectra corresponding to magnetograms and photospheric motions.

### 2.1. Magnetohydrodynamic Equations

The full set of MHD equations for an incompressible plasma is

$$\partial_t B = \nabla \times (u \times B) + \eta \nabla^2 B, \quad (1)$$

$$\partial_t u = -\left( u \cdot \nabla \right) u - \frac{1}{\rho} \nabla p + (\nabla \times B) \times B + \nu \nabla^2 u, \quad (2)$$

$$\nabla \cdot u = 0 = \nabla \cdot B, \quad (3)$$

where $u$ and $B$ are the velocity and magnetic field expressed in velocity units. The dissipation coefficients are represented by $\eta$ (resistivity) and $\nu$ (viscosity), $p$ is the gas pressure, and $\rho$ is the (constant) mass density. Using these equations, it is straightforward to derive the evolution equation for the energy density $W_{\text{MHD}} = \frac{1}{2}(B^2 + u^2)$,

$$\partial_t W_{\text{MHD}} = -\nabla \cdot F_{\text{MHD}} - E_{\text{MHD}}, \quad (4)$$

$$F_{\text{MHD}} = B^2 u_\perp + \left( \frac{p}{\rho} + \frac{u^2}{2} \right) u + \eta (J \times B) + \nu (\Omega \times u), \quad (5)$$

$$E_{\text{MHD}} = \eta J^2 + \nu \Omega^2, \quad (6)$$

where $J = \nabla \times B$ is the electric current, $\Omega = \nabla \times u$ is the vorticity, and $u_\perp$ is the component of the velocity perpendicular to the magnetic field.

Assuming that a stationary equilibrium is achieved,

$$E_{\text{MHD}} = -\nabla \cdot F_{\text{MHD}}. \quad (7)$$

This equation indicates that, for any closed volume of fluid, the net energy flux entering through the walls balances the resistive and viscous dissipation occurring inside that volume. It has been shown (Choudhuri 1986) that subphotospheric motions produce a net energy input on coronal loops [i.e., the integral over the loop of $(-\nabla \cdot F_{\text{MHD}})$ remains positive].

We now turn to the expression for the energy flux in equation (5). Note that since the Reynolds numbers in the corona ($R_u = u / \nu$: Reynolds number; $R_B = B / \eta$: magnetic Reynolds number) are much larger than unity, the two last terms in the right-hand side of equation (5) are negligibly small. Since the motions involved are sufficiently slow ($u^2 \ll p/\rho$), we also neglect the kinetic energy flux. Moreover, the fact that the gas pressure is much smaller than the magnetic pressure allows us to drop the term $(p/\rho)u$ in equation (5). Therefore, the energy flux reduces to the MHD Poynting flux

$$F_{\text{MHD}} \simeq B^2 u_\perp. \quad (8)$$

The very high value for the magnetic Reynolds number ($R_B \simeq 10^{12}$) does also allow us to assume that the fluid moves glued to the magnetic field lines (frozen-in condition). Thus,

$$u_\perp = \frac{B \times \mathbf{A}}{B^2}, \quad (9)$$

where $\mathbf{A}$ is the vector potential ($B = \nabla \times \mathbf{A}$) and the dot indicates a derivative with respect to time. It follows that the Poynting flux can simply be written as

$$F_{\text{MHD}} \simeq B \times \dot{A}. \quad (10)$$

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2.2. The Energy Balance

The mechanism outlined above explains how the energy contained in fluctuations of the magnetic and velocity fields can become available as a source of thermal energy for the plasma. However, we note that no particular process to enhance magnetic dissipation has been specified. Hereafter, we shall simply assume the presence of an efficient mechanism, such as the topological dissipation proposed by Parker (1972) or the development of a direct energy cascade (van Ballegooijen 1986; Gómez & Ferro Fontán 1988, 1992). In coronal active regions, the thermal energy source or heating rate reaches an equilibrium against radiative and conductive losses (see Withbroe 1981 for a review). The role of thermal conductivity is very important for the stability of this equilibrium, but integrated along a field line, from one footpoint of a loop to the other, its net contribution vanishes (conduction merely redistributes energy along the loop; see, for instance, Kuin & Martens 1982). Therefore, assuming that the heating rate is $E_{\text{MHD}}$, and provided that $E_{\text{MHD}} = -\nabla \cdot \mathbf{F}_{\text{MHD}}$ (see eq. [7]),

$$-\nabla \cdot \mathbf{F}_{\text{MHD}} \approx L_{\text{RAD}},$$  \hspace{1cm} (11)

where $L_{\text{RAD}}$ are the radiative losses (in ergs cm$^{-3}$ s$^{-1}$). We note that since the X-ray emission is observed to be distributed rather evenly along the loop, equation (11) implies that the heating should also be distributed smoothly along the loop. However, if this were not the case, the net effect of conductivity would be to share out the energy release over the loop, and the operational result would be the same as if the heating rate were evenly distributed. For example, suppose a heating spike takes place in a small region of diameter $a$. In Fourier space this would generate a signal at approximately $k = (1/a, 1/a)$. Conductivity will spread out the X-ray emission over a loop of length $L$, and, hence, if we choose the x-axis along the loop, a signal at $k = (1/L, 1/a)$ will be generated. In the omnidirectional spectrum, this will shift the measured signal from $k \approx 2^{1/2}a$ to $k \approx 1/a$ (since $L \gg a$), which is rather insignificant compared with the large range in $k$ we are considering.

Our goal in this section is to compare the outcome of these calculations with the observed power spectra described in § 2. Therefore, we discretize equation (11) by performing spatial averages over boxes which are infinitely high along the line of sight and whose square bases are 1 pixel in size (see Fig. 1). We obtain

$$-C \oint_{\Box} \mathbf{dS} \cdot \mathbf{F}_{\text{MHD}} = D^2 I_{i,j},$$  \hspace{1cm} (12)

where $D$ is the linear size of one pixel, and

$$I_{i,j} = \frac{C}{D^2} \oint_{\Box} \mathbf{dS} \cdot \mathbf{L}_{\text{RAD}},$$  \hspace{1cm} (13)

is the X-ray intensity of pixel $(i, j)$ (in ergs cm$^{-2}$ s$^{-1}$). The dimensionless constant $C$ accounts for the fraction of radiated photons effectively detected by the instrument, mainly determined by its narrow-band sensitivity and its solid-angle aperture. The surface integral in equation (12) can be separated into two parts. One part corresponds to the photospheric base of the box and reduces to $D^2 \cdot \mathbf{F}_{i,j}(z = 0)$. The second part corresponds to the side walls of the box and can be expressed as the integral over $z$ of a divergence term. Therefore

$$C^{-1}I_{i,j} = z \cdot \mathbf{F}_{i,j}(z = 0) - \int_0^\infty \mathbf{d}(z) \cdot \mathbf{V}_{\perp} \cdot \mathbf{F}_{i,j},$$  \hspace{1cm} (14)

where $\mathbf{V}_{\perp}$ is the finite element expression (because it operates over discretized functions) of the divergence operating on the $(x, y)$-plane. In the following it is important to keep in mind the finite-element character of functions and operators, and the fact that many physical processes responsible for the heating of the plasma do actually occur on scales well below the linear pixel size ($\sim 450$ km).

According to equations (10) and (14), the soft X-ray emission results

$$C^{-1}I = (z \cdot \mathbf{B} \times \mathbf{A})|_{z = 0} - \int_0^\infty d(\mathbf{z}) \cdot \mathbf{V}_{\perp} \cdot (\mathbf{B} \times \mathbf{A}),$$  \hspace{1cm} (15)

where, for brevity, the $(i, j)$ subscripts have been dropped. Equation (15) expresses the relationship between the X-ray emission and magnetic structure of coronal active regions. Note the importance of a dynamic behavior of the magnetic structure ($\mathbf{A} \neq 0$) in order to explain its brightness in X-rays.

Unfortunately, the coronal magnetic field cannot be observed directly. However, the magnetic structure can be extrapolated from magnetograms, which display the spatial distribution of the vertical component of the photospheric magnetic field. The simplest case is to assume a current-free configuration and perform a potential ($\mathbf{B} = \mathbf{V}\phi$) extrapolation for the corona, using magnetograms as boundary conditions. We follow this approach in § 2.3 as a zeroth-order approximation. In § 4 we discuss how our results are affected when a small force-free (i.e., $\mathbf{J} \times \mathbf{B} = 0$) contribution is considered.

2.3. Magnetic Field Extrapolations

Hereafter we assume that a potential (current-free) field extrapolation from the photospheric magnetic field provides a good zeroth-order description of the coronal field. This hypothesis holds best for small-wavenumber modes (like the ones we are interested in), since electric currents are expected to be spatially distributed in a small-scale filamentary, and nonuniform, fashion.

We define a potential function $\phi$, such that $\mathbf{B} = \mathbf{V}\phi$. This scalar function satisfies a Laplace equation ($\mathbf{V}^2 \phi = 0$), subject to the boundary condition

$$\partial_t \phi|_{z = 0} = b,$$  \hspace{1cm} (16)
where $b_{z}(t)$ is the $z$-component of the photospheric magnetic field, which is displayed in magnetograms. After straightforward calculations,

$$B_{k} = \left( z - i \frac{k}{k} \right) e^{-i k \cdot r} b_{z}(t),$$  \hspace{1cm} (17)

$$A_{k} = \left( \frac{k \times z}{k^2} \right) e^{-i k \cdot r} b_{z}(t),$$  \hspace{1cm} (18)

For the particular case of a two-dimensional photospheric velocity field, the normal magnetic field component $b = z \cdot B$ is advected by the fluid just like a scalar field (Knobloch & Rosner 1981). In other words,

$$\vec{\partial} t b = - (U \cdot \nabla) b,$$  \hspace{1cm} (19)

where $U = U_{x}(t)$ is the photospheric velocity field. We Fourier-transform this equation and formally write down the solution as

$$b_{k}(t) = b_{k}(0) + \sum_{k' + k'' = k} \int_{0}^{t} dt' k' \cdot U_{k'}(t') b_{k'}(t')$$  \hspace{1cm} (20)

where the sum also involves the symmetrization with respect to permutations of $k', k''$. We can now Fourier-transform our equation (15) and use equations (17) and (18) to express $I_{k}(t)$ in terms of the photospheric fields

$$C^{-1} I_{k}(t) = \sum_{k_{1} + k_{2} = k} \left( \frac{1}{k_{1}} + \frac{1}{k_{1} + k_{2}} \right) \frac{k_{1} \cdot k_{2}}{k_{2}} b_{k_{1}}(0) b_{k_{2}}(t),$$  \hspace{1cm} (21)

where $b_{k}(t)$ and $b_{k}(t)$ can be obtained from equation (20).

2.4. Power Spectra

Because of the stochastic nature of turbulent regimes, their theoretical description is usually statistical, all relevant quantities being described through random variables. Hereafter, we assume that the subphotospheric velocity field evolves in a turbulent fashion. Therefore, the subphotospheric velocity and magnetic field components will be considered as random variables. Equation (21) implies that $I_{k_{1}k_{2}}(t)$ should be considered as a stochastic variable too. Therefore, the calculation of statistical averages like $\langle \mathcal{S}(k) \rangle = \langle D^{2} k / 2 \pi \rangle \langle |I_{k}|^{2} \rangle$ is of interest.

Formally, the angular brackets represent an average over an infinite set of independent realizations of the dynamic evolution of the active region under study. This operation can be approximated in practice either by performing averages over different active regions (spatial averages) or by integrating the variables in time (temporal average), provided that the dynamic evolution of these systems is sufficiently ergodic.

We assume this stochastic process to be statistically isotropic and stationary. “Statistically isotropic” means that statistical averages only depend on $k = |k|$, while “statistically stationary” implies that these averages are independent of time. Therefore,

$$\mathcal{S}(k) = \frac{D^{2} k}{2 \pi} \frac{|I_{k}|^{2}}{2 \pi} = \frac{D^{2} k}{2 \pi} \langle |I_{k}(0)|^{2} \rangle$$  \hspace{1cm} (22)

where the angular brackets include an integration over the polar angle in Fourier space.

From equations (20) and (21), we derive our key result:

$$\mathcal{S}(k) = \frac{D^{2} C^{2} k}{2 \pi} \left| \sum_{k_{1} + k_{2} = k} \left( \frac{1}{k_{1}} + \frac{1}{k_{1} + k_{2}} \right) \frac{k_{1} \cdot k_{2}}{k_{2}} b_{k_{1}}(0) b_{k_{2}}(t) \right|^{2}$$  \hspace{1cm} (23)

This expression allows us to compute the power spectrum for X-ray emission [$\mathcal{S}(k)$] from simultaneous detections of photospheric velocity and magnetic fields. If, as indeed is the case, we only know very general features of these turbulent fields, like their power spectra, we can apply scaling techniques on equation (23). Scaling techniques were first used by Kolmogorov (1941) to describe the statistically stationary regime of hydrodynamic turbulence. In what follows, we use Kolmogorov's approach to find a relationship between the power spectrum $\mathcal{S}(k)$ and the corresponding power spectra for photospheric velocity and magnetic fields.

Let us assume that the nonlinear interaction of Fourier modes in the photospheric turbulent motions is essentially local (that is, all $k$'s in the sums of eq. [23] are of the same order of magnitude). This locality assumption, which basically means that the strongest nonlinear interactions occur between modes whose wavenumber magnitudes are similar, is supported by theoretical arguments (Kraichnan 1971) and numerical simulations (Kraichnan & Montgomery 1980) in a number of turbulent systems. It is usual in this kind of Kolmogorov-like theories to define "tiled" quantities related to the relevant random variables of the problem and having the same units. For instance, we define $\tilde{I}$ in such a way that $\tilde{I}^{2}$ represents the total power of the variable $I$ contained in a ring (in Fourier space) of mean radius $k$ and width $k$, so that $\mathcal{S}(k) = \tilde{I}_{k}^{2} / k$ (see, for instance, Hasegawa 1985). The usefulness of these "tiled" variables lies in the fact that they are not random variables but statistical averages of them.

Let us also assume that all relevant quantities display power-law spectra. In Paper I we have shown observational evidence in favor of this assumption for the X-ray intensity. There is also observational evidence supporting this hypothesis for the photospheric
fields (Knobloch & Rosner 1981; Chou et al. 1991; Hathaway et al. 1991; Tarbell 1992). For power-law spectra,

\[
\frac{U_k}{k} = \frac{D^2 k}{2\pi} \langle |U_k|^2 \rangle \approx k^{-\alpha}, \tag{24}
\]

\[
\frac{b_k}{k} = \frac{D^2 k}{2\pi} \langle |b_k|^2 \rangle \approx k^{-\alpha_b}, \tag{25}
\]

\[
\frac{I_k}{k} = \frac{D^2 k}{2\pi} \langle |I_k|^2 \rangle \approx k^{-\alpha_I}, \tag{26}
\]

From equations (20) and (21) we find that the scaling between the relevant variables goes as \( I_k \approx b_k U_k \), and therefore we obtain the following equation relating the indices of the different power spectra:

\[
\alpha_x = 2\alpha_b + \alpha - 2. \tag{27}
\]

Since the field \( b_{i,j} \) is advected by the photospheric velocity field like a passive scalar, the indices \( \alpha_b \) and \( \alpha \) are not independent. For the inertial region of a passive scalar (region of Fourier space where the scalar is not being externally pumped or dissipated), their relationship is (Tennekes & Lumley 1972),

\[
\alpha_b = \frac{5 - \alpha}{2}. \tag{28}
\]

Inserting this expression in equation (27), we finally find

\[
\alpha_x = 3. \tag{29}
\]

This theoretical derivation of the spectral index of the active region X-ray emission fits the observed values remarkably well (see Fig. 2). Also, its independence from the spectral structure of the photospheric velocity and magnetic fields is to be noted.

3. MHD TURBULENCE SCENARIO

The relative importance of nonlinearities compared with dissipative terms in the MHD equations (eqs. [1]–[3]), can be measured by two Reynolds numbers (\( R_e = u\ell/v \); Reynolds number; \( R_m = B\ell/\eta \); magnetic Reynolds number). Typical coronal values for these numbers range between \( 10^{10} \) and \( 10^{12} \), indicating that the dynamical evolution of this plasma will be subject to nonlinear effects. Therefore, these magnetically dominated fluctuations pumped into the system by footpoint motions (as discussed in § 2) are likely to couple nonlinearly to one another to produce a redistribution of energy (and other ideal invariants) in Fourier space. Since the magnetic field lines are supposed to follow the so-called line-tying condition, this additional component to the fluctuating fields, generated inside the corona by nonlinear interactions of the externally driven fluctuations, must have vanishing velocity components at the footpoints. Let us emphasize that we are not in a position to prove the occurrence of these interactions; we just affirm that, given the physical conditions of the coronal plasma, they are likely to take place, as has also been assumed by Heyvaerts &
Priest (1992). In the remainder of this section we simply assume their existence, describe the corresponding physical scenario, and discuss its potential relevance as an enhancing mechanism for Joule dissipation.

The effect of nonlinear terms is basically to redistribute excitations or fluctuations from one wavenumber to another in a virtually stochastic fashion. Only those excitations at sufficiently large wavenumbers decay as a consequence of dissipative effects. Therefore, a net flow of excitations in Fourier space is established, toward those regions which are deficient with respect to the values calculated from an ideal (without dissipation) model (Montgomery 1983). For instance, this nonlinear redistribution continuously replenishes the excitations being drained at the large-wavenumber region. An increase in Reynolds number only raises the value of the typical wavenumber at which dissipation begins to dominate, but it does not inhibit the excitation flow in Fourier space.

According to this scenario, three regions in Fourier space (wavenumber space) can be identified, each of them displaying a different turbulent behavior.

a) The energy-containing region.—This region comprises those modes that are being excited directly by the external driver, and is usually located toward the low-wavenumber zone. However, it does not necessarily include the very lowest wavenumbers, which are of the order of unity over the size of the system.

b) Dissipation region.—This region corresponds to those modes where fluctuations are being efficiently quenched by dissipative (viscous or ohmic) effects. This region is normally located at the largest wavenumbers, where the linear dissipative terms become comparable to the nonlinear terms.

c) Energy inertial region.—In this region both external forces and dissipation are negligible. Only nonlinearities play a role, transferring fluctuations from one mode to another, while keeping the total energy constant. This region normally bridges the gap between the low-wavenumber energy-containing region and the large-wavenumber dissipative zone.

Kolmogorov (1941) following heuristic arguments, has shown that when a three-dimensional incompressible fluid is submitted to external forcing with a narrow spectrum, a direct energy cascade is generated and a stationary energy spectrum is achieved, displaying the well known $k^{-5/3}$ distribution in the energy inertial region. Kolmogorov's ideas, mainly based on scaling properties of the ideal (nondissipative) equations and on the existence of a net energy flow through the corresponding inertial range, are usually known as cascade theory (Montgomery 1983; Hasegawa 1985) and have been applied to a number of turbulent systems, including two- and three-dimensional MHD turbulence. The power spectra predicted by cascade theory for the energy inertial range have in many cases been confirmed by experiments (Grant, Stewart, & Mollet 1962; Matthaeus & Goldstein 1982; Sommeria 1986) and numerical simulations (Lilly 1969; Herring & Kraichnan 1972; Fyfe, Montgomery, & Joyce 1977; Meneguzzi, Frisch, & Pouquet 1981; Matthaeus & Lamkin 1986; Biskamp & Welter 1989; Politano, Pouquet, & Sulem 1989).

Gómez & Ferro Fontán (1988) have shown that Joule dissipation of stationary MHD turbulence is a plausible mechanism for coronal heating. They assume that the subphotospheric forcing has a narrow spectrum whose wavelength is around the length scale of granular convection ($10^3$ km). Van Ballegooijen (1986) has calculated the evolution of a force-field driven by the turbulent subphotospheric velocity field and demonstrates the development of a direct energy cascade to the large-wavenumber spectral region. This scenario of energy cascades in coronal loops has recently been extended to broad-band photospheric power spectra (Gómez & Ferro Fontán 1992). Unfortunately, we are still unable to test these theoretical predictions observationally. Typical length scales for structures associated with the energy cascade region ($\leq 10^3$ km) are still beyond the present-day capabilities of spatial resolution.

In terms of the aforementioned classification of regions in Fourier space for a turbulent medium, the range of length scales covered by NIXT images seems to correspond to the energy-containing region rather than to the inertial range. The energy spectrum in the energy-containing region does not display the universal features of the spectrum in the inertial range and is in general strongly dependent on the characteristics of the external driving force. Therefore, in our theoretical analysis (see § 2) we avoided applying arguments of cascade theory, which are only appropriate for the inertial range.

In what follows we describe an alternative derivation of the result $\mathcal{S}(k) \approx k^{-3}$, based on scaling properties of the magnetohydrodynamic equations. The ideal MHD equations (eqs. [1]–[3] with $\eta = v = 0$) are invariant if we simultaneously scale the distance by $\lambda$, the field $u$ by $\lambda^4$, the field $B$ by $\lambda^8$ and time by $\lambda^{1/4}$. In the force-free approximation, the scaling exponents $h$ and $h_B$ can be arbitrarily different, while in the general case the scaling holds for $h = h_B$. These equations describe not only the evolution of the coronal part of the loops (in their limit $|u| \ll |B|$) but also the footpoint motions (in the opposite limit, $|u| \gg |B|$). In fact, equation (20), which describes the passive advection of the normal magnetic field component at the photospheric level, is nothing but a particular advection of the more general equation (1).

The first theory of scaling in turbulence was proposed by Kolmogorov (1941) for the Navier-Stokes equations and rests on the following assumptions (see Frish & Orszag 1990 for a comprehensive discussion). First, the scale invariance holds for average ("tiled") quantities, whereas it need not hold for detailed structures. Second, a finite flux of energy cascades from large scales to small ones, where it dissipates. Third and last, the energy flux at a particular wavenumber is assumed to depend on quantities evaluated in a neighborhood of that wavenumber (the energy flux is a local, diffusion-like process in Fourier space). The energy flux in Fourier space goes as $\mathcal{S} \propto k^2 \approx \lambda^{2h}$. Since this energy flux must be scale-independent, the scaling exponent must be $h = \frac{1}{3}$, which in turn leads to the well-known Kolmogorov power law $W(k) \propto k^{-5/3}$. For a scalar field $\theta(x, t)$ being passively advected by the velocity field, the corresponding flux in Fourier space is $\mathcal{S}_\theta = k^2 \overline{\theta^2} \approx \lambda^{2h+2h}$. Assuming that this flux of passive scalar is scale-invariant (i.e., we are in the inertial range of the passive scalar), we obtain $h_\theta = (1 - h)/2$.

The power spectrum for X-ray intensity goes as $\mathcal{S}(k) \approx k^2/k = (\overline{u B})^2/|k| \approx k^{-2}$. Let us make the following assumptions: (1) the scaling properties of $B$ and $h$ are the same, and therefore $h_B = h_\theta$ (this is true for potential extrapolations, and may also be valid for more general configurations); (2) $b$ is being passively advected by the photospheric velocity field; and (3) the range of scales that we are looking at ($10^3-10^5$ km) falls into the inertial range of $b$ and thus $h + 2h = 1$. Under these assumptions, we again obtain $\mathcal{S}(k) \approx k^{-3}$ as in equation (29). Note that we did not assume that we were working on either the photospheric kinetic energy or the...
coronal magnetic energy inertial ranges. However, our assumption 3 is dubious, because it means that no new magnetic structures (of sizes between $10^3$ and $10^5$ km) emerge to the photospheric level. But the observed relationship between the photospheric kinetic and magnetic energy does not depart much from the case of the inertial range of a passively advected magnetic field (Tarbell 1992).

The availability of simultaneous observations of photospheric velocity and magnetic fields would also allow a better computation of $\mathcal{S}(k)$ by using equation (23).

4. DISCUSSION

In Paper I, following a Fourier analysis on a sample of images from active regions obtained by NIXT, we found a broad-band, isotropic, power-law spectrum $k^{-3}$ for the spatial distribution of soft X-ray intensities, regardless of the different topological structures of the active regions.

In the present paper we explore theoretically the connection between coronal magnetic features and their driving agent, the photospheric convective motions, and present a theoretical derivation of the power-law spectra for the spatial distribution of X-ray emission. We first express the X-ray intensity in terms of the fluctuating velocity and magnetic field in the corona, assuming that the X-ray emission per pixel is proportional to the MHD Poynting flux entering that pixel. Then we relate the fluctuating coronal fields to the photospheric velocity and magnetic fields. Performing a statistical averaging procedure over these relationships, and assuming that the longitudinal component of the photospheric magnetic field is passively advected by footpoint motions, we find a power spectrum $\mathcal{S}(k) \approx k^{-2}$, which is in perfect agreement with the spectra of the active regions we studied.

We emphasize that the theory presented in § 2.4 is statistical and does not make predictions about individual events. For instance, our assumption of stationarity by no means excludes the possibility of time dependence for individual realizations (observations of time variations in a particular active region during a particular period of time). The variable $I_k(t)$ is probably a function of time, but $\mathcal{S}(k) = (D^2 k/2\pi)(|I_k|^2)$ is not.

We perform our theoretical analysis without specifying any particular coronal heating mechanism. However, our assumption that footpoint motions occur on time scales much longer than the Alfvén time of coronal loops tacitly implies the action of a DC heating mechanism. The features corresponding to dissipative structures are expected to become apparent at length scales well beyond present-day resolution capabilities. Thus it may be impossible to decide, on the basis of the observations analyzed here, which DC coronal heating mechanism applies.

The scenario for MHD turbulence that we explore further in § 3 is only one of the many possible heating mechanisms discussed in the literature. It predicts a natural cascade of energy from large magnetic structures (which receive this energy from the convective motions) to highly microscopic structures. This cascade then provides the energy released at very small length scales. The role of MHD turbulence in heating of the solar corona has been discussed in a number of papers (van Ballegooijen 1986; Similon & Sudan 1989; Gómez & Ferrón Fontán 1988, 1992; Heyvaerts & Priest 1992; see also Gómez 1990 for a recent review). Most of the theoretical models for coronal heating proposed in the literature require the formation of confined and strongly sheared magnetic zones where the associated electric currents resistively dissipate the magnetic energy. Examples of these strongly sheared regions throughout the literature are the tangential discontinuities assumed by models based on MHD wave dissipation (Heyvaerts & Priest 1983; Davila 1987) or a stochastic distribution of them to enhance dissipation of Alfvén wave packets (Similon & Sudan 1989), the current sheets thought to be responsible for impulsive events like solar flares (Heyvaerts, Priest, & Rust 1977), the spontaneously generated tangential discontinuities in the topological dissipation model (Parker 1972, 1983), or the resonant regions of models based on an analogy between coronal loops and LRC circuits (Jonsson 1978, 1982). We speculate that this direct cascade of magnetic energy associated with an MHD turbulent regime can be regarded as a source of the aforementioned highly sheared zones. In our opinion, magneto-hydrodynamic turbulence provides an appropriate theoretical starting point for detailed models of coronal heating.

The power spectra we have derived are not expected to hold for the largest wavenumber modes, which define the macroscopic shape of the active region. Loosely speaking, the mean magnetic field is oriented along the axis of the loop and is large compared with the turbulent fluctuations of the magnetic and velocity fields, thus causing the total distribution of fluctuations to be highly anisotropic. While the perpendicular motions may develop small dissipative scales giving rise to turbulent dissipation, spatial variations along the mean field generally remain smooth, their dynamics being determined by weak interaction of Alfvén waves (Biskamp & Welter 1989). In Paper I we investigated the division between the anisotropically distributed low modes and the more isotropic distribution of high-wavenumber modes by removing high-wavenumber modes from the observed images, and found that even if we remove up to 95% of the highest wavenumber modes, the filtered image looks very similar to the original one. This simple experiment verifies that only the very lowest wavenumber modes are responsible for the observed anisotropic shape of these coronal active regions. On the other hand, those modes contributing to the isotropic power-law spectrum in the intermediate-wavenumber range reflect the stochastic nature of the footpoint motions.

The question remains, of course, of what determines the distribution of the large coronal structures. We suggest that it is the underlying photospheric field, since potential extrapolations from magnetograms provide a good zeroth-order description of the coronal magnetic structure as evidenced by comparison with X-ray images (Poletto et al. 1975), although departures from the potential structure have also been reported (Krieger, de Feiter, & Vaiana 1976; Vaiana & Rosner 1978). A force-free extrapolation from vector magnetograms (which map the three components of the photospheric magnetic field) probably provides a better description. With respect to our approximation of a potential magnetic field, let us remark that we have only used this assumption on the truncated set of Fourier modes corresponding to scales between $10^3$ and $10^5$ km; we do allow for the existence of electric currents in modes whose wavenumbers are larger than our upper cutoff. These currents are indeed essential to provide for the dissipation of magnetic energy.

To get some insight into the extent to which small, almost spatially uniform electric currents would influence our results, we extend our calculations to the case of linear force-free field extrapolations, $\mathbf{V} \times \mathbf{B} = \beta \mathbf{B}$ in the limit $\beta \ll k$. In this case, the magnetic ...
field and its vector potential shown in equations (17) and (18) acquire an extra (force-free) term,
\[ \delta B_k = \frac{i \beta}{k^2} k \times z \ e^{-k z} b_k(t), \]
(30)
\[ \delta A_k = \frac{\beta}{k^2} z \ e^{-k z} b_k(t). \]
(31)

Keeping terms up to first order in \( \beta \), we find that \( \delta I(k) \approx I(k) | \delta B_k | / | B_k | \), and therefore
\[ I(k) \approx k^{-3} \left( 1 + \frac{\beta}{k} \right). \]
(32)

Hence, for \( \beta \ll k \), equation (32) implies no difference between the omnidirectional spectrum of a potential and a linear force-free field.

In summary, we developed a simple theoretical model to link the fluctuating fields in the corona with stochastic motions in the convective region. This model leads to the prediction of a power spectrum \( I(k) \approx k^{-3} \) that fits perfectly well with the ones reported in Paper I for a number of structurally different coronal active regions.

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