SEISMIC LIMITS ON THE SUN’S INTERNAL TOROIDAL FIELD

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The fine structure in the spectrum of the Sun’s five-minute acoustic vibrations is usually described by

\[ \nu_{n,l,m} - \nu_{n,l,0} = L \sum_{i=1}^{N} a_{i,n,l} P_i(\frac{m}{L}) = \sum_{i=1}^{N} \alpha_{i,n,l} P_i(\frac{m}{L}), \tag{1} \]

where \( \nu_{n,l,m} \) is the frequency of the oscillation, \( P_i \) is a Legendre polynomial, and the \( a_{i,n,l} \)s are the splitting coefficients and where \( n \), \( l \) and \( m \) are the radial order, angular degree and angular order of the oscillation, respectively. The symmetric splitting coefficients are sometimes defined as \( \alpha_{i,n,l} \)s because they tend to be dominated by near surface effects. \( N \) is 5 or 6 and usually \( L = \sqrt{l(l+1)} \). The antisymmetric part of the data arises from the linear effect of rotation. The symmetric part of the data contains contributions from shape distorting effects like those that would arise from an internal magnetic field and, as well, must contain a contribution from the second order effect of rotation.

Woodard, Kuhn, Murray and Libbrecht(1991) have shown that the time dependence of the symmetric part of the splitting data is directly associated with the solar surface activity. Here we report the results of inversions which simultaneously remove the near surface perturbation from the 1986, 1988, 1989 and 1990 data of Libbrecht and Woodard(1992) and enable us to place limits on the Sun’s internal toroidal magnetic field. One may anticipate that a sizeable toroidal field could follow from a dynamo action on a relatively small poloidal field inside the Sun. In this effort we exploit our earlier work, Dziembowski and Goode(1989,1991), and solve the inverse problem we posed for the Sun’s toroidal field,

\[ a_{zj,d} = \sum_{k \geq j} \int_0^R A_{k,j,d} \beta_k(r) dr + (a_{zj,d})_{rot} + \frac{I_{d} \gamma_j(\nu_d)}{I_d L_d}, \tag{2} \]
where \( a_{2j,d} \) is the symmetric part of the fine structure in the \( d^{th} \)-multiplet in the data and \( \beta_k(r) \) is a measure of the ratio of the magnetic pressure to the gas pressure. That is, the toroidal field, \( B_\phi \), is given by,

\[
B_\phi^2 = 4\pi psin^2\theta \sum_{k=1}^{2k-2} \beta_k(r) cos^{2k-2}\theta
\]

(3)

The factor \( I_* \), which is \( I \) for the \( n = 15, l = 20 \) mode was introduced to make \( \gamma' \)'s comparable in magnitude to the \( \alpha' \)'s of Libbrecht and Woodard(1990). The ratio of \( I/I_* \) depends primarily on the frequency, \( \nu \), and is \( \sim 40 \) at \( 2mHz \) and \( \sim 1 \) above \( 3mHz \). \( A_{j,k,d} \) is the toroidal field kernel as defined in equations(26) and (45) of Dziembowski and Goode(1989). The \( (a_{2j,d})_{rot} \) term represents the quadratic order effect of rotation as calculated by Dziembowski and Goode(1992).

![Graphs showing \( \gamma_1, \gamma_2, \gamma_3 \) vs. \( \nu \)](https://example.com/graphs)

**Fig. 1.** The \( \gamma' \)'s of equation (2) from inverting the 1986, 1988, 1989 1990 splitting data of Libbrecht and Woodard(1992).

The calculated near surface perturbation from the inversions of the four data sets are shown in Figure 1. The clear and strong time dependences are apparent. It is also clear that the near surface perturbation was essentially absent in 1986—the time of activity minimum. In fact, Dziembowski and
Goode (1992) pointed out, the only trend in the 1986 $a_2$ data is the due to the second order effect of rotation.

![Graph](image)

Fig. 2. The $\beta_2$ of equation (2) from inverting the 1986, 1988, 1989 1990 splitting data of Libbrecht and Woodard (1992).

In Figure 2, we show the average $\beta_2$ from the four sets. $\beta_2$ is the only significant $\beta_k$. No appreciable differences between the calculated fields were found for these four years. Taking the values indicated in Figure 2 to be seismic limits, the limit on the toroidal field near $0.55R_0$ is about $10MG$, near the base of the convection zone about $1MG$, and near the surface about $1KG$.

Our most statistically significant result, the megagauss field near the base of the convection zone, isless significant than we quoted, Dziembowski and Goode (1989, 1991) using earlier reductions of the Big Bear data. Even though a megagauss may seem huge, it is still too small to cause a detectable change in the structure parameters, like density as determined by means of a frequency inversion. On the other hand, such a field would have profound consequences for dynamo theories. We look forward to inverting the GONG data for $\beta_k$.

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Reference List