THE FORM OF THE ANGULAR VELOCITY IN THE SOLAR CONVECTION ZONE

D. O. GOUGH\textsuperscript{1}, A. G. KOSOVICHEV\textsuperscript{2}, T. SEKII
Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, England

K. G. LIBBRECHT, AND M. F. WOODARD
Big Bear Solar Observatory, California Institute of Technology, Pasadena,
CA 91125, USA

SPLITTING DATA AND THE REPRESENTATION OF ANGULAR VELOCITY

Rotational splitting data have been obtained as coefficients $a_k$, with $k$ odd, computed by projecting frequency multiplets onto truncated Legendre expansions of the form

$$\nu_{n,l,m} - \nu_{n,l,0} = l \sum_{k=1}^{2K-1} a_k P_k(m/l) ,$$

with $K = 3$. This suggests expanding the solar angular velocity in a series of convenient basis functions, such as

$$\Omega^h(r, \theta) = \sum_{k=1}^{K} \Omega_k(r) P_{2k}(\cos \theta) ,$$

with respect to spherical polar coordinates $(r, \theta, \phi)$ about the axis of rotation. To provide a well posed problem, the series (2) must be truncated at the same value of $K$ as the number of odd terms in series (1).

PREVIOUS INVERSIONS OF BBSO DATA

Inversion of coefficients $a_k(n, l)$ obtained from observations at Big Bear Solar Observatory by Libbrecht and Woodard (these proceedings) in 1986, 1988, 1989 and 1990 were reported by Gough et al. (1992). In particular, a contour diagram depicting the inversion for $\Omega(r, \theta)$ using the representation (2) was shown for 1986, and is reproduced here as Figure 1a. There is some indication that in the equatorial regions of the convection zone $\Omega$ was approximately constant on cylinders. In this respect, the inversions of the 1988, 1989 and 1990 data are similar. This result is contrary to indications from some earlier inversions. In view particularly of numerical simulations of the solar convection zone that exhibit the property of rotation on cylinders, we have therefore been led to ask how likely it might be that that property is shared by the sun.

\textsuperscript{1}and Department of Applied Mathematics and Theoretical Physics, University of Cambridge

\textsuperscript{2}and Crimean Astrophysical Observatory
FORWARD CALCULATIONS

We modified the angular-velocity distribution shown in Figure 1a in such a way that it became closer to being constant on cylinders in the region \( r \sin \theta > r_c \), except in a surface layer, yet maintaining the rotational splitting to be not very different from observation; here \( r_c \) is the radius of the base of the convection zone. We found a compromise between these two requirements, essentially by trial and error. An example \( \Omega^f \) of an angular velocity obtained in this way, and the coefficients \( a_k \) computed from \( \Omega^f \), are shown in Figures 1b and 1c respectively.

STRONG SEISMIC EQUIVALENCE

We define two angular velocities to be seismically equivalent in the strong sense with respect to observations \( D \) if they imply precisely the same observed splitting. Strong equivalence is thus dependent on which data \( D \) have been reported by observers, but not on the values of those data. In our case, the data are three expansion coefficients \( a_{2k-1} (k = 1, 2, 3) \).

A SIMPLE SEISMICALLY EQUIVALENT EXAMPLE

We seek a representation

\[
\Omega^c(r, \theta) = \begin{cases} 
\sum_{k=1}^{K'} U_k r^{2k} \sin^{2k} \theta, & r \sin \theta > r_c \\
\sum_{k=1}^{K''} W_k(r) \cos^{2k} \theta, & r \sin \theta < r_c 
\end{cases},
\]

where \( U_k \) are constants. Then coefficients \( U_k, W_k(r) \) can be sought that cause \( \Omega^c \) to yield identical expansion coefficients to those of the original angular velocity \( \Omega^p \). We have simplified the analysis firstly by demanding seismic equivalence with respect to a slightly different data set \( D' \): namely, the coefficients \( a'_k \) in an expansion of the form (1) but with \( l \) replaced by \( L = \sqrt{l(l+1)} \), and secondly by ignoring the small Coriolis term in the kernel weighting the average of \( \Omega \) that determines the rotational splitting. We have furthermore approximated that kernel by its asymptotic form: \( S^{-1} \psi(r) \Psi(\mu) \) where \( \psi = c^{-1} \left[ 1 - L^2 c^2 / r^2 \omega^2 \right]^{-1/2}, \)

\( S = \int d' \psi dr, \quad \Psi = 2 \pi^{-1} (M^2 - \mu^2)^{-1/2}, \quad M^2 = L^2 - m^2, \quad \mu = \cos \theta \) (Kosovichev and Parchevski, 1988; Gough, 1992). In that case it can be shown that representations (2) and (3) are equivalent if

\[
\sum_{k'=1}^{K''} J_{kk'} W_{k'} = \Omega_k - \sum_{k'=1}^{K'} I_{kk'} V_{k'},
\]

where

\[
V_k = \frac{(-1)^k}{k!} \sum_{k'=k}^{K'} \frac{k!' U_{k'}}{k'! (k' - k)!} \left( \frac{r}{R} \right)^{2k'}. 
\]

\[
I_{kk'} = \int_{-1}^{1} P_{2k}(\mu) I_{k'M}(\mu; \alpha) d\mu, \quad J_{kk'} = \int_{-1}^{1} P_{2k}(\mu) J_{k'M}(\mu; \alpha) d\mu
\]

\[
I_{k'M}(\mu; \alpha) = \int_{0}^{\alpha} \frac{\mu^{2k} d\mu}{\sqrt{M^2 - \mu^2}} / \int_{0}^{1} \frac{\mu^{2k} d\mu}{\sqrt{M^2 - \mu^2}}, \quad J_{k'M}(\mu; \alpha) = 1 - I_{k'M}(\mu; \alpha).
\]
FIGURE I  (a) Contours of constant $\Omega^h$ obtained by inverting BBSO solar rotational splitting data obtained in 1986. The dashed circle represents the base of the convection zone. (b) Example $\Omega^f$ of a smooth angular velocity obtained as a compromise between having $\Omega^f$ constant where $r \sin \theta > r_c$ and satisfying the splitting constraints; the splitting coefficients $\alpha_k$ implied by $\Omega^f$ (dots) are plotted in (c), together with the BBSO measurements, which are displayed as $\pm 1 \sigma$ error bars. (d) Contours of constant $\Omega^c$, which is seismically equivalent to $\Omega^h$ in the strong sense.
Equations (4) can be supplemented with continuity and smoothness constraints, if desired. However, we have not done so here. Consequently we take $K'' = K$. Evidently, we can choose $U_k$ at will. We have taken $K' = K = 3$, and have chosen $U_1 = 279.6$, $U_2 = 258.8$, $U_3 = -108.5$ (all in nHz). These values yield a somewhat gentler photospheric variation of $\Omega^c$ at low latitudes [$\theta > \sin^{-1}(r_c/r) \simeq \pi/4$] than the surface differential rotation inferred from Doppler measurements or the motion of magnetic tracers. They were chosen in this way partly because some numerical simulations of the solar convection zone exhibit a boundary layer near the surface that reduces latitudinal shear in the photosphere, and partly to restrain the variation of $\Omega^c$ near the poles. The resulting function $\Omega^c$ is illustrated in Figure 1d.

**DISCUSSION**

The prime purpose of this paper is to point out that currently available solar rotational splitting data are not inconsistent with an angular velocity that is constant on complete cylinders in the sun's convection zone. We have demonstrated the point by constructing an angular velocity $\Omega^c$, illustrated in Figure 1d, with just that property. We stress that we make no claim here that $\Omega^c$ is either a better or a worse representation of the sun's interior rotation than the function $\Omega$ depicted in Figure 1a. Seismically, the two functions are, in a sense, precisely equivalent. Any preference between them must be based on other criteria.

We stress also that there are infinitely many more seismically equivalent representations. Possibly more plausible examples of $\Omega^c$ can be constructed by increasing $K'$ and $K''$ and augmenting the seismic constraints with continuity and smoothness conditions. The realm of possibilities is increased yet further by admitting functions that are only weakly equivalent: namely, that they do not necessarily imply precisely the same seismic constraints, but satisfy them only within the observational errors.

**OUTLOOK**

Coefficients up to $K = 6$ have recently been obtained and are discussed by Libbrecht and Woodard (these proceedings). These will constrain possible functions $\Omega^h(r,\theta)$ and $\Omega^c(r,\theta)$ more tightly, and should make it easier to decide which is the more plausible. We intend to report on inversions of those data in due course.

**REFERENCES**

