THE HIGH-FREQUENCY P-MODE SPECTRUM

P.N. MILFORD, P.H. SCHERRER
Center for Space Science and Astrophysics, Stanford University, CA 94305

Z. FRANK
Lockheed Palo Alto Research Labs, Palo Alto, CA 94304

A.G. KOSOVICH2 AND D.O. GOUGH3
Institute of Astronomy, University of Cambridge, Cambridge, U.K.

ABSTRACT A 10-hour series of Doppler observations, carried out at the Swedish Solar Observatory with the Lockheed SOUP filter, has been analysed. The photospheric line Fe I 5576Å was observed with a temporal Nyquist frequency of 10.5 mHz and with a 220-arcsecond field of view. Observations of high-degree high-frequency modes above the acoustic cutoff show ridges continuing to frequencies up to 8 mHz.

The acoustic cutoff frequency \( \omega_c \) is usually considered as the height of a potential barrier near the solar surface. Acoustic waves with frequencies higher than \( \omega_c \) are only partially reflected by the barrier. However, even when waves are reflected partially, normal modes with complex eigenfrequencies exist. The real parts \( \omega \) of the frequencies are determined by properties of the subphotospheric acoustic cavity. The imaginary parts \( \eta \) are determined principally by leakage of wave energy into the chromosphere. It is clear that \( \eta \) should increase when \( \omega \) increases. That means that the higher frequencies are associated with greater line widths. We have developed a theoretical model of the high-frequency modes which yields their frequencies and lifetimes.

The theory also predicts atmospheric modes with nearly constant period, due to a partial reflection at the transition region. These have not yet been detected in this dataset.

OBSERVATIONS

This analysis is based on 10 hours of Doppler observations carried out at the Swedish Solar observatory, La Palma, using the Lockheed SOUP tunable filter. Active region AR6174, a small rounded sunspot, was observed on July 30, 1990.

1Visiting Scientist, Center for Space Science and Astrophysics, Stanford University, CA 94305
2Crimean Astrophysical Observatory
3Department of Applied Mathematics and Theoretical Physics, University of Cambridge
The region was near disk center when observed. Doppler observations were made in Fe I 5576, a non-magnetic photospheric line.

The observations covered a 220-arcsecond field of view, with a 512x512 element detector. The sampling cadence was 47.4 seconds. Filtergrams at -90 mÅ, -30 mÅ, +30 mÅ and +90 mÅ shifts from line center were observed. Normal CCD corrections, including dark-current subtraction, flat fielding and bad-pixel removal, were carried out.

To equalize the image-to-image seeing variations, a modulation transfer function equalization was carried out on the data. This forced the radially averaged spatial power spectra of each image to be, on average, the same. Each filtergram was derotated to a fixed angle. As the exact axis of rotation on the field of view is unknown (and slowly changing) the images were then rigidly aligned by cross correlating and shifting. Finally, to minimize the effects of atmospheric seeing, the images were destretched (Tarbell and Smithson 1981).

Doppler shifts at each temporal and spatial sample were calculated with a 4-point fit to the 4-filtergram observations.

The data were remapped to be linear in latitude and longitude. Prior to a 3D FFT the data were detrended by subtracting a 21-minute boxcar average, and were spatially and temporally apodized. The umbral data were found to be very noisy due to the low signal levels, and have been masked out of the data.

Figure 1 shows a comparison of the observed mode frequencies with the theoretical mode frequencies. It is a plot of the spatial wavenumber $k_h$ versus temporal frequency, where $k_h = \sqrt{l(l+1)}/R_\odot$. The grayscale indicates the logarithm of the observed power.

**THEORETICAL BASIS**

Acoustic waves vertically propagating in the solar atmosphere are reflected in high density-gradient zones, such as the photosphere and the transition region (Balmforth and Gough, 1990). There is also a potential contribution to the observed power spectrum due to coherent observations of waves travelling upwards from their site of excitation and of refracted waves that initially propagated downwards from the same site (cf. Kumar et al., 1990).

The smooth behavior of the observed ridges can be explained if the wave reflection coefficient is not as large as in the linear adiabatic approximation. Indeed, Goode et al. (1992) have argued that the reflection coefficient of the transition region is significantly reduced by nonlinearity and by nonadiabatic effects such as ionization, viscosity, radiative losses and lateral inhomogeneity.

We have computed the theoretical $k_h - \omega$ relation, assuming no significant wave reflection at the transition region. A standard equation for adiabatic oscillations of a spherically symmetrical star (Deubner and Gough, 1984) is used:

$$\psi'' + k_r^2 \psi = 0, \quad (1)$$

where $\psi = \rho^{1/2} c^2 \text{div} \xi$, a prime denotes differentiation with respect to the vertical coordinate $h$, and $\rho$ and $c$ are the density and sound speed of the equilibrium state; $\xi$ is the fluid displacement and $k_r$ is the vertical component of the local
FIGURE I Observed $k_h - \nu$ diagram, overlaid with the computed frequencies. Plot of frequency vs. $k_h$ with grayscale indicating log(power) in the observations. Note there is power observed at frequencies above the atmospheric acoustic cutoff. The f-mode spectrum was not calculated.

Wave number given by

$$k_r^2 = \frac{\omega^2}{c^2} \left( 1 - \frac{\omega_+^2}{\omega^2} \right) \left( 1 - \frac{\omega_-^2}{\omega^2} \right).$$  \hspace{1cm} (2)

The parameter $\omega_+$ plays a role of acoustic potential for the acoustic oscillations, and $\omega_-$ is a potential for gravity modes. Since the frequencies of the p modes are significantly higher than $\omega_-$,

$$k_r^2 \approx \frac{1}{c^2} \left( \omega^2 - \omega_+^2 \right).$$  \hspace{1cm} (3)

The acoustic potential has a strong peak just below the photosphere (because of a strong density gradient) and tends to a constant value $\approx 5$ mHz in the chromosphere. An almost nonreflecting boundary condition

$$\psi' + i k_{r_0} \psi = 0$$  \hspace{1cm} (4)

is applied at the top of the layer at a height $h_0$ above the photosphere, where $k_{r_0}$ is the value of $k_r$ at $h_0$. Equation (4) annihilates an incoming-wave component of the general solution of the wave equation (1) for an isothermal atmosphere, but not necessarily for other atmospheres. In the realistic solar model used in the computations, this boundary condition does not imply perfect transmission, and, therefore, atmospheric modes are likely to appear. The lower boundary condition $\psi = 0$ is applied far below the lower turning points of the modes.

The $k_h - \omega$ diagram for $h_0 = 1500$ km is shown in Figure II. The vertical bars indicate the magnitudes of the imaginary parts of the eigenfrequencies. The rising ridges correspond to p modes trapped beneath the photosphere. The
ridges are smoothly extended beyond the acoustic cutoff frequency, where the eigenfrequencies become complex. The typical lifetime of the modes, estimated from the imaginary parts, is about 3 hours at 10 mHz. Therefore, the high-frequency modes beyond the acoustic cutoff frequency appear to be explained by the partial reflection of the acoustic waves from the photosphere.

The horizontal ridges represent short-lived atmospheric modes, whose frequencies depend on the location of the upper boundary, \( h_0 \). The characteristic distances between the ridges can be estimated as \( \Delta \nu \sim \nu/2h_0 \), where \( \nu \) is the vertical component of a characteristic vertical phase velocity of the atmospheric waves; \( \nu \) is approximately equal to 3.5 km/s. Observations of the horizontal ridges on the \( k_h/\omega \) diagram would provide interesting information about the wave-energy absorption layer in the solar atmosphere.

REFERENCES