P-MODE FREQUENCY ShiftS AND CHROMOSPHERIC MAGNETISM

REKHA JAIN AND B. ROBERTS
Department of Mathematical and Computational Sciences, University of St. Andrews, St. Andrews, Fife KY16 9SS, Scotland (U.K)

INTRODUCTION

It has been established observationally that p-modes vary over the solar cycle (Elsworth et al. 1990, Libbrecht and Woodard 1990). Why? Theoretical work by Campbell and Roberts (1989), pre-dating the observations, suggested that p-modes would vary over the solar activity cycle as a consequence of changes in chromospheric magnetism. We pursue that suggestion here.

A key observational result has been presented by K. G. Libbrecht and M. F. Woodard (Libbrecht and Woodard 1990, 1992; Woodard and Libbrecht 1991) which demonstrates the shift in the frequency of p-modes for the years 1988-1990 (when the Sun was active), compared with 1986 (when the Sun was at a minimum). A theoretical explanation of this frequency variation is of considerable interest.

THE DISPERSION RELATION

To examine p-modes in a magnetic atmosphere we consider the model shown in Figure 1, which consists of an isothermal magnetic chromosphere surmounting a field-free polytropic convection zone.

Figure 1. The assumed thermal and magnetic structure of the atmosphere

This model leads (after detailed algebra) to the following dispersion relation (see also Evans and Roberts 1990, 1992; Jain 1992):

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\[ 2a \omega^2 c_{sp}^2 \frac{U(-a + 1, m + 3, 2kz_o)}{U(-a, m + 2, 2kz_o)} + \gamma g \omega^2 - kc_{sp}^2(\omega^2 + gk) \]

\[ = \frac{(c_{sc}^2 + \frac{3}{2} v_{Ac}^2)(g^2 k^2 - \omega^4)(\omega^2 - k^2 c_{sc}^2)}{g k^2 c_{sc}^2 + (c_{sc}^2 + v_{Ac}^2)(\omega^2 - k^2 c_{Tc}^2)} \left\{ k - \frac{\beta_1 A_1 A_3}{r} \frac{F\left(p+1,q+1;\tau+1;\frac{-A_1}{A_2}\right)}{F\left(p,q+1;\tau;\frac{-A_1}{A_2}\right)} \right\} \]  

(1)

Here \( c_{sc} = (\gamma RTc)^{1/2} \), \( v_{Ac} = B_c/(\mu \rho_c)^{1/2} \) and \( c_{Tc} = c_{sc} v_{Ac}/(c_{sc}^2 + v_{Ac}^2)^{1/2} \) are the sound speed, Alfvén speed and cusp speed at the base of the chromosphere. The sound speed at the top of the convection zone is \( c_{sp} = (\gamma RTp)^{1/2} \). The adiabatic index in the convection zone and chromosphere is \( \gamma \); \( m \) is the polytropic index in the convection zone and \( g \) is the local gravitational constant. The angular frequency is \( \omega \) and its horizontal wavenumber is \( k \) (and is directly related to the degree \( l \) of the mode); \( U \) is the confluent hypergeometric function and \( F \) is the hypergeometric function. The parameter \( a \) is a function of \( \omega^2/k \).

A numerical solution of equation (1) yields \( p \)-mode frequencies influenced by the chromospheric magnetic field \( B_c \) and temperature \( T_c \). It is of interest to examine the influence of variations in \( B_c \) and \( T_c \) on the \( p \)-mode frequencies \( \nu = \omega/2\pi \); see also Jain (1992), Jain and Roberts (1993).

Figure 2 shows the effect of varying the chromospheric temperature and field strength on frequencies \( \nu \) of degree \( l = 100 \). We have determined from equation (1) the frequency shift \( \Delta \nu \equiv \nu(B_c', T_c') - \nu(B_c, T_c) \), taking \( B_c = 40G \) and \( T_c = 4170K \) to correspond with conditions in 1986 (solar minimum) and \( B_c' = 50G, T_c' = 4170-6100K \) to represent conditions in 1988. Variations with degree \( l \) are shown in Figure 3.

![Figure 2](image.png)

**Figure 2.** The \( p \)-mode frequency shift \( \Delta \nu \equiv \nu(B_c', T_c') - \nu(B_c, T_c) \) as a function of frequency \( \nu \) as determined by equation (1), taking a chromospheric magnetic field \( B_c = 40G \) and temperature \( T_c = 4170K \) (corresponding to 1986) with \( B_c' = 50G \) (for 1988). The curves shown correspond to various chromospheric temperatures \( T_c' \). The degree \( l \) is set equal to 100.
Figure 3. The calculated frequency shift $\Delta \nu$ for modes of degree $l$ between 10 and 140. For all values of $l$ the shift increases at low $\nu$ before decreasing sharply at around $\nu = 3.7$-$3.9$ mHz. The base frequency $\nu(B_c, T_c)$ representative of 1986, is calculated for $B_c = 40$G and $T_c = 4170$K. The curves are drawn for (a) $B'_c = 50$G and $T'_c = 5900$K, corresponding to 1988 data, and (b) $B''_c = 55$G and $T''_c = 6700$K, corresponding to 1989 data. Note the change of scale in $\Delta \nu$.

We may compare our results with the observations. Figure 4 displays the calculated frequency shifts (shown as continuous curves) together with the data for 1988, (shown by $\times$), compared with 1986, and for 1989 (shown as $\bullet$).

Figure 4. The calculated frequency shift $\Delta \nu$ for modes of degree $l$ between 10 and 140 with base frequency $\nu(B_c = 40$G, $T_c = 4170$K). For all values of $l$ the shift increases at low $\nu$ before decreasing sharply at around $\nu = 3.7$-$3.9$ mHz. The curves are drawn for (a) $B'_c = 50$G and $T'_c = 5900$K, corresponding to 1988 data, and (b) $B''_c = 55$G and $T''_c = 6700$K, corresponding to 1989 data. Note the change of scale in $\Delta \nu$. 

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CONCLUSIONS

For the simple model of Figure 1, we have exhibited a turnover in frequency at 3.9 mHz, qualitatively similar to the remarkable turnover displayed in the observations of Woodard and Libbrecht (1991). The turnover in our model is a consequence of the combined effects of an increase in both chromospheric magnetic field strength and chromospheric temperature.

An increase in either magnetic field strength or temperature leads to an increase in the effective propagation speed of a p-mode in the magnetic atmosphere, and thus to an increase in frequency. On the other hand, an increase in temperature also leads to an increase in the density scale height in the atmosphere. Consequently the wave samples a region of increased gas density and so 'feels' an increase in inertia, with a reduction in frequency occurring. Altogether, these effects compete to produce the frequency shifts displayed in figures 2 to 4.

The lack of detailed observational evidence for rises in chromospheric temperature and the simplicity of our theoretical model limit our ability to determine the precise changes in the chromospheric magnetic field strength and temperature that might be causing the observed frequency shifts in p-modes. The chromospheric temperature rises we have invoked to explain the observed downturn in frequency shifts are rather large: 1700K from 1986 to 1988, and 2500K from 1986 to 1989. It may be that in a more realistic non-isothermal model atmosphere smaller temperature changes are able to produce frequency shifts of the observed magnitude. In any case, we feel our model offers guidance as to what physical processes conspire to produce the distinctive frequency shifts observed over the solar cycle.

REFERENCES

Libbrecht, K. G. and Woodard, M. F. 1992, these proceedings