Structure of coronal rays

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(Submitted July 21, 1992)
Astron. Zh. 70, 1092–1098 (September–October 1993)

We treat ray structures in the solar corona as folds (and pleats) of magnetic surfaces having a large gradient of electron density, which appear in the projection of magnetic surfaces onto the plane of the sky. We show that there is an even number of intersections of a ray with an arbitrary closed contour in the plane of the sky. This enables us to classify singular points of the observed structures. We carry out such a classification and give the results of an analysis of singular points in the corona observed in 1936–1984.

1. INTRODUCTION

Coronal structures in the form of a system of "helmets" or individual "rays" are associated with filaments and, as shown by Koutchmy,\(^1\) can be identified with lines of tangency of the line of sight to cylindrical surfaces whose generatrices are curves near magnetic field neutral lines. Sometimes the generatrices are circles, and then the comparison of the corresponding structures is simplified,\(^1\) but in general, the determination of lines of tangency of the line of sight to the magnetic surface (i.e., the "contact" lines) is a complicated problem requiring a great deal of numerical calculations.\(^2\) It is complicated by the fact that to calculate magnetic surfaces, one has to make certain assumptions about the nature of the field: to assume that the magnetic field is potential or else force-free [with some coefficient \(a(x, y, z)\) relating the field and the current]. The latter problem is complicated, and we are still far from developing a satisfactory algorithm for its solution. At the same time, a general theory for the projection of continuous surfaces onto a plane has now been developed. The features that appear in this case are associated with catastrophes of different types, as is well known.

In the case of coronal structures, the surface being projected is a magnetic surface, and the plane is the plane of the sky. The purpose of the present communication is to consider some consequences of such a representation.

2. CONTACTS OF A TWO-SIDED SURFACE

Coronal plasma structures are usually treated as equilibrium configurations in a plasma with a small parameter \(\beta\). The condition of equilibrium is necessitated by the fact that the magnetic energy density is so great that nonequilibrium structures can exist for about \(10^2\) sec, whereas those structures exist for at least several hours. The equilibrium equation contains a gravitational term \(\rho g\). The scale height \(kT/mg\) in the corona at \(T = 2 \times 10^6\) K is \(\sim 10^6\) cm, however, so for relatively fine structures (with a size of several arcseconds), equilibrium will have the form

\[
\frac{1}{c} [\mathbf{\beta}] - \nabla p = 0.
\]

In this case, magnetic surfaces in equilibrium coincide with surfaces of constant pressure. If the temperature is assumed to be constant, then magnetic surfaces coincide with surfaces of constant plasma density, as is usually the case in a laboratory plasma. The emission has been calculated\(^1\) for various radial density distributions in a circular cylinder. Results on the porosity of the emission from the solar corona have been summarized in Ref. 4. According to various data, it is 10-100. This means that rather than three-dimensional volumes, regions of fewer dimensions — one or two — actually radiate. Leaving aside the former case, as well as so-called fractals — structures with a nonintegral number of dimensions — we arrive at the concept that surfaces (magnetic surfaces, in accordance with the condition of equilibrium) actually radiate.

Adopting such a viewpoint, under fairly general assumptions we can show that the number of intersections of contact lines (rays) with an arbitrary closed contour in the plane of the sky is even. That statement has the nature of a theorem, and it is easy to prove by introducing a parameter on which the angle between the normal to the surface and the line of sight depends continuously. We assume the magnetic surfaces to be smooth.

We isolate a closed contour in the plane of the sky and assume that it is the directrix of a cylindrical surface consisting

![Diagram 1](image1)

**FIG. 1.** Different types of behavior of curves inside a contour in the plane of the sky (see text).

![Diagram 2](image2)

**FIG. 2.** Continuous functions \(\cos \gamma(z)\): curves I and II pertain to two-sided surfaces and curve III to a one-sided surface.
of straight lines parallel to the line of sight. The point A is the beginning of a circuit around the contour in the direction indicated by the arrow in Fig. 1. We introduce the distance x from A to the current point on the contour. At the point of intersection of the line of sight with the magnetic surface, the normal to the surface forms an angle $\gamma(x)$ with the line of sight. We construct a graph of $\cos \gamma(x)$, noting that the surface is tangent to the line of sight at $\cos \gamma(x) = 0$.

The graph has the form shown in Fig. 2: the $\cos \gamma(x)$ curve starts with some value $\cos \gamma(0)$, and at $x = L$, where $L$ is the length of the contour, $\cos \gamma(L) = \cos \gamma(0)$. The function $\cos \gamma(x)$ is continuous for any smooth magnetic surface, so the $\cos \gamma(x)$ curve either does not intersect the x axis or has 2, 4, ... intersections if the magnetic surface is two-sided. The function $\cos \gamma(x)$ is not necessarily single-valued. In Fig. 2 we show curves I and II, the first of which is a single-valued function of x while the second is multivalued in some segments. This obviously pertains also to the case in which the line of sight intersects several surfaces — the total number of


FIG. 5. Models of folds (and pleats) in an arbitrary surface (see text).
intersections (which are the only things accessible to the observer) is even.

Finally, one-sided surfaces have the property that the sign of the unit vector normal to the surface changes in a circuit around them along a closed contour. Here \( \cos \gamma(L) = -\cos \gamma(0) \) (see curve III in Fig. 2) and the number of roots of the equation \( \cos \gamma(x) = 0 \) is odd. Such surfaces (of which a Möbius strip is an example) cannot separate regions of high and low plasma pressure, however, since they do not separate space into interior and exterior regions, as closed two-sided surfaces do. The surface in this case is not a magnetic surface in an equilibrium plasma (with different densities on its two sides). Those surfaces can be excluded from consideration, and the basic statement is then proven.

Let us turn to the different types of the behavior of rays in the plane of the sky. In Fig. 1 we show the following cases: 1) the ray intersects the contour twice; 2) the ray has a reversal point; 3) the ray has a point of intersection with another ray. These cases do not contradict what has been said: the number of intersections is even. We then have 4) cutoff of a ray; 5) a triple point on a ray. The number of intersections in these cases is odd, and such rays therefore cannot exist in the model under consideration.

3. STRUCTURE OF THE CORONA IN 1936-1984

Before turning to the observations, we note that only observational data with limited angular resolution can be obtained. This means that two or several rays may merge (within the resolution), as a result of which the observed picture may differ from reality. In Fig. 3 we show cases of such merging: a) a curve with two reversal points may be observed as a segment of a curve; b) a curve with a reversal point and a curve with a continuous derivative may be observed as one curve with a triple point.

The first case is commonplace, since small folds have just such a structure, whereas the second case is the result of a special arrangement of two curves in space, and the configuration breaks up upon a small "stirring" of the curves. This may mean that the first type of violation of the theorem should be widely represented in coronal structures, whereas the higher the resolution, the more rarely the second should be observed.

Let us turn to Ref. 5 and count the number of singular points of different types in coronal structures. In the histogram given in Fig. 4, we give the total number of singular points of different types in coronal structures in 1936-1984.

Singular points of the Y and A types cannot always be distinguished in structural drawings, but the intersection of coronal rays is well determined, and it is clear from Fig. 4 that this number exceeds the sum of the numbers of both Y and A points. The number of cutoff rays was not counted (it is large), but according to the foregoing, this may be explained by the existence of small-scale folds in a magnetic surface.

From the histogram in Fig. 4, it is clear that the number of "feasible" singular points (171) exceeds the number of "infeasible" ones, and the latter are 22% of the total (222). If our presentation is correct, then that percentage should decrease with increasing spatial resolution.

4. CHANGES IN CORONAL STRUCTURES

Drawing the contour so that it encompasses the visible solar disk, on the basis of the foregoing, we can conclude that the total number of "rays" in the corona is even. At the same time, coronal structures are subject to variation. Those variations must conserve parity. Loop structures, in particular, can move in a way similar to that observed for loop systems; their motion has a different nature (and a different scale) in this case, however. The well-known structure called a "tennis racket" (Los Alamos observations?) can be interpreted, on the basis of the foregoing, as a change in the shape of the magnetic surface involving the development of a circular continuous fold. Parity conservation is not violated in this case.

5. MODELS OF FOLDS

The foregoing suggests the construction of a model that satisfies the following requirements. Light from some source is scattered by a surface. The surface has a complicated structure as \( r \to R_\odot \), and it changes into an almost flat "heliospheric sheet" as \( r \to \infty \).

Such a model can be made from an elastic film stretched over a hoop \( (r \to \infty) \) and gathered into folds as \( r \to R_\odot \). The corresponding structure is given in Fig. 5. The "porosity" corresponds approximately to the values indicated above (the film thickness is 0.3 mm and the radius is 3 cm).

It is seen in Fig. 5a that this structure is complicated. It consists of "rays," most of which begin at \( r = R_\odot \), and some of them end at A points. There are many X points and no Y points. It should be noted that Fig. 5a was obtained using a stiff, nonstretching film, in which folds were formed. We note that a stiff film is not an arbitrary surface. In the upper left-hand quadrant one can see a structure with a twist. If such a structure is observed in the corona, it is usually considered to be a current-carrying braid. As Koutchmy has noted, the absence of self-intersections in such structures casts doubt on the conclusion that a current exists.

In Figs. 5b, c we give a structure obtained in experiments using a soft, stretchable film (Fig. 5b). A local twist is also observed (Fig. 5c, upper right-hand quadrant). Moreover, in the lower right-hand quadrant of Fig. 5c we see a "loop" system, which is merely a round fold near the "limb." As for the ends of rays in these photographs, under careful examination one sees that the ends most often have the form of self-intersecting structures of the \( \gamma \) type.

The effect of finite spatial resolution is demonstrated by Figs. 5d, e. These are photographs of the same structures obtained out of focus: In the first, the resolution is equal to the film thickness, and in the second, the scale of the resolved structures is much larger than the film thickness. Here we cannot describe the ends of rays: they are dissolved in the dark surrounding field.

We can draw the following conclusions from Fig. 5.

1. The model of an emitting surface confirms the presentation in Sec. 2, with the addition that points of type A are a special case of structures of type \( \gamma \).

2. If the estimate of \( 10^2 \) porosity is correct, then for a complete investigation of the structure of rays, we must have photographs of the corona with resolution higher than \( R_\odot \cdot 10^{-2} = 10^9 \). Singular points can thus be resolved and one can answer many questions about the structure of a magnetic surface.