DEVELOPMENT OF A TOPOLOGICAL MODEL FOR SOLAR FLARES

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Abstract. The main theoretical studies of the process involved in solar flares have been made in the two-dimensional approximation. However, the preliminary studies made with three field components suggest that reconnection could take place in the separatrices, the separator (intersection of separatrices) being a privileged location for this process. As a consequence the sites of flare kernels must be located on the intersections of the separatrices with the photosphere. Therefore, in order to understand the role of interacting large-scale structures in solar flares, we have analysed the topology of three-dimensional potential and linear force-free fields. The magnetic field has been modelled by a distribution of charges or dipoles located below the photosphere. This modelling permits us to define the field connectivity by the charges or the dipoles at both ends of every field line.

We found that the appearance of a separator above the photosphere is more likely when a parasitic dipole emerges outside the axis that joins the main polarities and when the field lines are characteristic of a field created by dipoles. The separatrices derived in the potential and force-free hypothesis have different shapes. However, in the strong field regions where flares usually occur, the separatrices of the potential and force-free field models become closer. This property makes possible the use of the potential field, as a first estimate, for computing the location in the photosphere of the separatrices and for comparing this location with the position of observed Hz kernels. Displacements of the separatrices of a force-free field result from modifications of the free energy of the field. Then force-free fields have the further capability of predicting the kernel displacement. In all cases a configuration suitable for prominence support is found above the separator.

1. Introduction

1.1. Main properties of the magnetic field in flaring active regions

Even if many aspects of the process are still puzzling, the analysis and confrontation of data obtained in different parts of the spectrum have greatly advanced our understanding of solar flares in the past decade. It is widely admitted that solar flares result from the release, in the corona, of the energy stored in stressed magnetic fields. Coronal magnetic fields are not measured yet, but the observation of the stress of the field at the photospheric level gives very useful information about the process involved in the corona. The evolution of strong magnetic fields of opposite polarities and magnetic stress are the main keys. Flares occur:

(1) When opposite polarities are pushed together. Martres et al. (1968) have first shown the importance of new emerging flux for the flare process. The collision between


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a preceding and a following spot of opposite polarity can also lead to flares (Zirin, 1988). The majority of flares takes place when flux of opposite polarity emerges (or submerges) in a given region (Heyvaerts and Hagyard, 1991; and references therein). The interpretation of both soft and hard X-ray observations leads to the same conclusion (Machado et al., 1988), while the exact location of energy release has not yet been identified.

(2) In the presence of highly sheared magnetic field near a neutral line (Hagyard, 1988). This is interpreted as the need to store magnetic energy above the minimum (and not available) energy of a potential field. This kind of magnetic configuration is also generally related to filament formation (Martin, 1990) and a close correspondence between filaments and flares is observed (Zirin, 1988).

As illustrated by the observed shape of chromospheric ribbons, the geometry of the field is generally very complex and very far from a two-dimensional (2D) approximation. Reflecting the intrinsic difficulty of the problem, no necessary and sufficient conditions for solar flare occurrences have yet been derived (Heyvaerts and Hagyard, 1991).

1.2. MHD MODELS OF SOLAR FLARES

A great number of physical processes have been explored. Even constraining ourselves to the magnetohydrodynamic (MHD) processes involved in a flare, a survey of the abundant literature could not be reported here. We refer to general reviews like Priest (1981, 1982), Zwingmann (1988), and Schmieder and Priest (1991). Many MHD models have been proposed to explain solar flares. However, modelling has now reached a state of maturity where strong constraints can be set on the physical processes involved.

Observations suggest that the interaction of magnetic structures and the presence of electric currents are the main parameters in the physics of flares. Heyvaerts, Priest, and Rust (1977) have worked out an emerging flux model where a new dipole appears close to an older one. A current sheet develops and becomes unstable, either because of a thermal instability or because the threshold of turbulence is reached. This model has been tested numerically by Priest and Forbes (1986). The results show a quasi-static phase followed by a dynamic one. Simulations of Biskamp and Welter (1989) have shed new light on the interaction of three arcades. The ejection of a plasmoid is found only in such complex magnetic configurations.

A crucial feature in flare theory, related to the interaction of magnetic field structures, is the separator, which is a singular line (Syrovatskii, 1981) at the intersection of the separatrices – the separatrices are surfaces that separate flux systems, each identified by its unique field line connectivity. Separators develop current sheets in response to changes in boundary conditions (Sweet, 1958; Low, 1987, and references therein) and are the locations of field line interchange.

The possibility of generating separatrices depends on the magnetic field geometry. Aly (1991) put strong limitations on the possibility of generating a separatrix by opening the magnetic field. He showed that the energy of any line-tied, force-free field is not larger than the energy of the totally open field which has the same photospheric vertical field component. Then an open field configuration (with a current sheet) cannot be reached
by reducing the magnetic energy of a stressed field. This seems to rule out the process of Barnes and Sturrock (1972) where the field is first opened by an instability and then reconnects to give post-flare loops.

Assuming a current-free (potential) magnetic field, Baum and Bratenahl (1980) were the first to analyse the separatrix generation and location in a magnetic field created by four magnetic charges. They showed that it is neither sufficient to have a magnetogram, nor to compute field lines with footpoint density proportional to field strength or randomly selected to get the topological properties of a magnetic field. This work was developed by Gorbachev and Somov (1988, 1989). While the magnetic field is represented by the potential field of only four magnetic charges, their model interprets well the observed location of the chromospheric ribbons of one flare. Moreover, while 2D models require a quadrupolar region to create a separator, a bipolar region with an S-shaped neutral line is sufficient in 3D. Recently, Gaizauskas and Harvey (1991) and Schmieder et al. (1991) applied this model qualitatively to explain their observations. In the present paper, we develop this promising method by building an algorithm that can compute systematically the location of separatrices in a complex region (Section 2). The description of the observed magnetic field by potential charges is extended to dipoles in Section 3.

Potential magnetic fields have no free energy. Aly (1990) showed that magnetic fields can be stressed and hold in a stable way until a large amount of energy has been accumulated. In an unbounded region of space, he studied the stability of a line-tied, force-free magnetic field characterized by a scale length $A$ and by a DC current density with a magnitude $\alpha_0$ times the magnetic field intensity. He concluded that a field with $\alpha_0 A = O(1)$ is stable and contains free energy of the order of the energy of the potential field having the same normal component on the photospheric boundary. All studies in 3D on the separators and separatrices have been made using the hypothesis of a current-free magnetic field. However, Hénoux and Somov (1987) suggested that reconnection along the separator would interrupt currents flowing along lines of force, releasing the energy stored in these currents. Therefore, the case of force-free field configurations is investigated in Section 4 for the linear case. A summary and discussion of the results are given in Section 5.

2. Magnetic Field Line Connectivity

2.1. Limits and Inadequacy of the Use of Full Magnetic Regions to Define the Connectivity

Separatrices are surfaces that separate different magnetic flux systems, each identified by its unique field line connectivity. Finding the separatrices’ location is easy in a strictly 2D field. The simplest example, with a separator, is a quadrupolar region where the flux linkage between the four magnetic regions is defined without ambiguity. The generalization to a 3D field is not trivial.

Within a 2D field, the separatrices separate field lines coming from different photo-
spheric polarity regions, and different positive (or negative) regions cannot merge unless the opposite polarity between them disappears. In 3D, the addition of a third dimension can allow the merging between two regions leaving only three polarity regions. This can happen at great distances or in low field strength regions, so that this merging has little physical meaning. An example is shown in Figure 1. If we start with a 3D field with enough symmetry, for example with four aligned charges where both $OX$ and $OY$ are axes of symmetry (as in Figure 2(a)), four distinct polarity regions are present at the photospheric level and the lines of force connecting these regions can have four kinds of connectivity localized in four different cells. However, a small change in the position or the intensity of the charges produces only three distinct polarity regions (noted I, II, and III in Figure 1). In this case only one single separatrix, with no separator, is present. This is also clear in the case studied by Gorbachev and Somov (1989) where they have four magnetic field concentrations, but only two polarity regions are present at the photospheric level. Using only the polarity observed on the magnetogram to define the connectivity gives only one connectivity cell. Then, computing the field line connectivity just by considering the polarity at the field line ends and ignoring the physical differences between enhanced field regions is misleading, so we need to explore another way to generalize the 2D approach to 3D.

Generally, if we move one of the photospheric footpoints of a field line, the other one moves smoothly. In cases where a finite jump is present, the field line changes its magnetic connectivity. This is a first and general way to compute the location of a separatrix if we repeat this all over the photospheric plane. But it is very difficult to develop an algorithm to find automatically such kind of separatrix in a complex

![Diagram](image_url)

**Fig. 1.** Top view of the magnetic field distributions on a horizontal ($Ox$, $Oy$) plane for four aligned charges (noted 1, 2, 3, 4) with a slightly asymmetric intensity ($Q_1 = 0.9$, $Q_2 = -1$ and $Q_3 = -Q_4 = 0.6$). Isocontours of $B_z$ are drawn with continuous (respectively dashed) light lines for positive (respectively negative) values. Their distribution is uniform and proportional to $B_z$. The neutral line is enhanced by a thick continuous line. Here the photospheric magnetic regions of a given polarity (noted I, II, III) are used to define the connectivity; consequently, only two cells of different connectivity are present, while there are four for a symmetric field (because there are four distinct photospheric magnetic regions). Some field line connections from region I to II (respectively, II to III) are represented with dashed-dotted (respectively dotted) lines. A single separation surface is present between connectivity cells I–II and II–III.
configuration. We must also notice that the separatrices defined using this technique are generally not closed surfaces (Aly, private communication). Moreover, in Gorbachev and Somov’s configuration no finite jump can be found because the photospheric region is bipolar. In that case, starting with a field line near one of the principal flux concentrations, we can move its footpoint to another region of the same polarity without discontinuity in its footpoint end. We have to compute the separatrix in a different way and we need to separate the fluxes coming from regions that are physically disconnected in the convective region.

2.2. ADVANTAGES OF MODELLING THE MAGNETIC FIELD BY A DISCRETE NUMBER OF MAGNETIC SOURCES

A convenient way to compute the connectivity is to take into account the magnetic flux concentrations and their time evolution. These compact structures are present on the Sun since the magnetic field is created in the subphotospheric layers and concentrated at the border of several convective cells. Down in these deep layers the magnetic flux tubes are in different velocity-field regions. Consequently the differences in the evolution of the tubes are transmitted to the corona by currents flowing parallel to them, even if the field distribution becomes relatively uniform there. The frozen-in condition leads to the formation of current sheets between magnetic fields of different origin. Then computing the field line connectivity using photospheric field concentrations has a physical meaning: current sheets are present on the separatrix (whose size depends on the photospheric motions and on the local plasma physics).

Clearly a magnetograph is not sufficient to give all the information, because magnetic sources are masked. The time evolution of the structures can greatly help to separate regions of different origin since they show different motions. On the other hand, a clear distinction between two close polarities of the same sign is not easy. The separation into structures could seem arbitrary. However, this separation is difficult to make, mainly in the low field regions. Since flare kernels are not going to be in low field regions, these ambiguities are not important.

A simple way to take into account both the flux concentrations and their independence is to model the magnetic field by a few magnetic charges located below the photosphere. This approach has been developed previously by Gorbachev and Somov (1989). The connectivity of one point of the magnetogram is defined by the charges reached at both ends by the field line passing through this point. This separates adjacent regions of the same polarity even if the photospheric field is bipolar. As shown in Figures 2, the cell connectivity changes smoothly with the evolution of the photospheric field, avoiding the problems outlined in Section 2.1.

The use of a potential field below the photosphere is certainly a wrong approximation because the field is dominated by the plasma motions. In future developments it will be interesting to stop integrating as soon as the photosphere is reached, because the magnetic field is provided by extrapolation codes only above the photosphere. However, in a first step, provided that the field lines approximately concentrate as they do near a magnetic charge, their exact position in the convective region is not needed.
Fig. 2. Intersection of the cupolae with the photosphere \((Z = 0)\) for four aligned charges (noted 1, 2, 3, 4). Here the connectivity is defined by the two charges located at each end of every line of force. This definition is used in all the following figures. The drawing convention used in all figures is as follows: – Regions of connectivity are indicated by numbers like 1–2. – The thickest lines show the separatrices’ intersection with the horizontal plane. – The transverse field is shown by arrows whose length is proportional to the logarithm of the field. – Singularities, located in the plane \(Z = -0.2\), are marked with a \(\oplus\) symbol. The charge distribution is symmetrical in (a) \(Q_1 = -Q_2 = 1\) and \(Q_3 = -Q_4 = 0.6\) and with a weak asymmetry in (b) \(Q_1 = 0.9\), while other charges are unchanged.

2.3. NUMERICAL METHOD

Even if we limit ourselves to the largest scale lengths, the observed photospheric field of a flaring region is generally more complex than the field modelled by four charges. We model the magnetic field either by an appropriate number of charges or by dipoles and study the linear force-free case. Notice that in the following we use the term singularity for both charges and dipoles at the location of which the field becomes infinite.

We develop an automatic algorithm in which the connectivity of a point is defined by the singularities (charges or dipoles) reached at both ends by the field line passing through this point. The integration was tested by reversing the integration direction and comparing the result to the initial point coordinates. The computation is initiated with a coarse grid whose spatial step must be lower than the smallest connectivity cell size that we want to detect. (It can be the size of the smallest polarity region and, in ambiguous situations, computations with an initial finer mesh can be done.) Results are stored in an array and the computations are repeated with a new mesh twice finer. To save computation time the calculation of the connectivity is done only between adjacent points having a different connectivity. Finally, the cell boundaries are detected in the array. This method has the advantages of being both automatic and very fast, since it computes only the field lines close to the separatrix in the second and further iterations.

To improve the representation of the observed magnetogram, more than one field singularity can be needed to describe an elongated photospheric zone. In this case the singularities belong to the same region of the convective layer. Therefore, their relative
motions and the current sheet formed in the separatrix of two fluxes can be neglected. We then use a group of singularities to define the connectivity cells. The numerical code was used in Mandrini et al. (1991) to model the magnetic field of an observed active region by an ensemble of dipoles and to relate the sites of the Hα kernels of a flare to the location of the separatrices. Here, we limit ourselves to the analysis of the topological properties of the connectivity cells in simple configurations with a reduced number of singularities.

3. Topological Properties of Simple Magnetic Field Configurations – Modelling the Field by a Potential Field

3.1. MODELLING THE MAGNETIC FIELD BY CHARGES

The topological cells have been computed for a configuration formed by a principal bipole (external) and a parasitic one (internal). The magnetic field is given by

\[ \mathbf{B}(R) = \sum_{i=1}^{4} Q_i \frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} \]  

(1)

with

\[ \mathbf{r}_i = \mathbf{R} - \mathbf{R}_i \]  

(2)

where \(Q_i\) and \(\mathbf{R}_i\) are, respectively, the intensity and the position of charges, and \(\mathbf{R}\) is the position vector. This field representation is identical to previous work on the subject.

Fig. 3. Intersections of the cupolae with two horizontal planes at altitudes \(Z = 0\) (the photosphere) and \(Z = 0.2\). The parasitic polarities are shifted away from the axis of symmetry by a distance \(Y_p = 0.4\). The charge intensities are the same as in Figure 2(b) \((Q_1 = 0.9, Q_2 = -1\), and \(Q_3 = -Q_4 = 0.6\)). In (b) some field lines are drawn with a starting point, in the left polarity, close to the separatrix at photospheric level. Dotted and dashed-dotted lines show the flux linkage differences when the starting point is moved across the left cupolae.
It differs from Baum and Bratenahl (1980) by the algorithm used to compute the separatrix and from Gorbachev and Somov (1989) by the charge disposition.

We first considered the topology of the magnetic field of two aligned symmetrical bipoles. As shown by comparison of Figures 2(a) and 2(b), the shape of the separatrices is not significantly affected by a small asymmetry. The separation between the dipole axes has an important effect on the sizes of the separatrices and the separator. Figures 3 and 4 show the evolution of the separatrices at two different heights above the photosphere (located at $Z = 0$, being the charges at $Z = -0.2$). When the parasitic bipoles is located far from the principal one, its own field dominates in a more extended region. Consequently, an increase of the distance between two non-aligned bipoles is favourable for the emergence above the photosphere of a separator by requiring a lower parasitic magnetic flux. However, when the separation is too large the separator lies in a low field region where not enough magnetic energy is available to feed the flare. Moreover, in this case the volume involved in the eruption is much larger; then, the chromospheric ribbons may no longer be visible. If a filament is present before the flare, this type of event can be observed as a dynamical dispersion brusque of a quiescent filament.

The influence of the intensity of the parasitic charges has been studied by Gorbachev and Somov (1988) for a different configuration. We obtained similar results for all the

Fig. 4. Same as in Figures 3 but with $Y_p = 2$. The extension of the domes and of the separator increases greatly with $Y_p$, both in the horizontal and vertical directions. (a) and (c) show one field line per connectivity cell, starting close to the separator summit. In (c) the abscissa $S_h$ corresponds to the projection of the field line on a horizontal plane, then field lines are unwound. A dip is present in field line (a) where dense plasma can be supported.
configurations we studied, i.e., the separator reaches the photosphere when the parasitic polarity exceeds a minimal intensity (which depends on the configuration), then its size increases both in the vertical and horizontal directions. This is shown in Table I for the charge disposition of Figure 3.

When the integration starting point is close to the separatrix, a discontinuity in the positioning of the end footpoints of the lines is present. An example of one field line per cell is given in Figure 4(a). The starting points are close to the separator top and integration is performed in both directions until the photosphere is reached. The dependence with height is drawn in Figure 4(c) using as abscissa the projection of the field line on the photosphere (as shown in Figure 4(a)). Field lines (b) and (d), which turn back nearly 180° in Figure 4(a), are unwound in Figure 4(c). Taking into account the convention used for the representation and needed for three-dimensional fields, Figure 4(c) shows the classical picture of an X-point in a two-dimensional field.

Field line (a) in Figure 4(c) has a magnetic dip where ionized and dense material can be accumulated to form a prominence. This is the same local configuration as studied by Démoulin and Priest (1992b) in a quadrupolar, two-dimensional field. It gives a logical explanation of the link, which is usually observed (Zirin, 1988; Martin, 1990), between prominences and flares. Moreover, when the parasitic bipole has a low strength the dip can reach the photosphere; then the prominence is formed at low heights, as observed. The prominence grows in height with the emergence of the parasitic polarity until it becomes detached from the photosphere. At this stage an ideal instability can lift up the structure and force reconnection mainly at the separator location. This process describes well the observed global evolution. Since prominences usually reach the chromosphere only at particular locations, called the feet of the prominence, a future development must take into account lower magnetic scale length variations, like the one associated with the supergranulation.

### TABLE I

<table>
<thead>
<tr>
<th>$Y_p$</th>
<th>Parasitic singularity intensity</th>
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<th>Dipole representations</th>
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<td>Horizontal extension</td>
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3.2. Modelling the Magnetic Field by Dipoles

The field connectivity has been computed in the previous section and in references quoted therein using a localized charge representation. The physical meaning of such magnetic charge is not clear because magnetic fields are created by currents. The first term in a multipole expansion is, then, a dipolar one. However, a magnetic field with the spatial dependance given in Equation (1) can be obtained by a continuous distribution of dipoles located on a segment along the negative part of the vertical axis. The size of the region above the segment, where Equation (1) is valid, is of the order of the segment length. In conclusion, a distribution of current can locally give a magnetic field with a magnetic charge behavior. The relevance of charges or dipoles must be tested with observations. We give below two reasons why we prefer dipoles to charges.

– First, magnetic knots of opposite polarity are abundant in the vicinity of sunspots. For an isolated spot the total magnetic flux of these knots is of the same order as the flux of the spot. With low spatial resolution, this gives an opposite field distributed around the spot. Such configuration can be modelled by vertical dipoles located below the photosphere.

– Second (and related to the previous one), the photospheric field is observed to be nearly horizontal in the outer penumbral boundary of a simple spot. With a magnetic charge located at $Z_0$ below the photosphere, the angle $\theta$ between the field and the horizontal plane is less than $10^\circ$ only for $R \geq 6 |Z_0|$, where $R$ is the distance to the center. With a vertical dipole, the field becomes horizontal at $R = \sqrt{2} |Z_0|$.

Observations of the photospheric region AR 2372 support this view (Mandrini et al., 1991).

3.2.2. Results

The magnetic field of the dipole distribution is

$$B(R) = \sum_{i=1}^{4} \frac{(3(M_i \cdot r_i)r_i - M_i r_i^2)/r_i^5}{},$$

where $M_i$ is the dipole momentum. Vertical dipoles are used for simplicity to limit the number of free parameters.

For comparison, the dipoles have been placed in the same location as the charges used previously. If the condition $M_i = 0.5Q_iD_i$ is satisfied, dipoles and charges located at a depth $D_i$ below the photosphere produce an identical photospheric magnetic field on their axis. As a matter of fact, since all charges in Figures 1–4 are at the same depth and since multiplying all the fields by a common factor does not affect the topology, we simply take $M_i = D_i$. Locally, above each singularity, there is a photospheric field concentration for both cases. It is only far from the concentrated field regions that the two distributions differ qualitatively (as it can be seen by comparing Figure 5(a) to Figure 2(b) and Figure 6(a) to Figure 3(a)).

The greatest difference between charges and dipoles is that field lines can close back
to the same dipole. As shown in Figures 5 and 6, the dipole representation introduces new connectivity cells (2–2 and 1–1) and their related separatrices formed by the boundaries between the cells 1–2, 2–3, and 1–4. The extra separatrices present a new potential site for the formation of current sheets. Within the existing limitations of the present three-dimensional approach, we estimate that they do not play a significant role for the following reasons:

- Their intersections involve only field lines common to three cells, such as cells 1–1, 1–2, and 1–4 (Figures 5 and 6), so now new privileged places for reconnection like separators are created.

- The emergence of the parasitic polarity is supposed to be the main disturbance to the initial field; therefore, we can neglect the relative motion of dipoles 1 and 2. Only weak current sheets are formed along the extra separatrices.

![Diagram](image)

Fig. 5. Intersection of the cupolae with horizontal planes at heights $Z = 0, 0.2,$ and $0.4$ in the dipole representation. The dipole moments have the same intensities as the charges in Figure 2(b), therefore the photospheric field strength at the vertical axes of the singularities is the same. The photospheric field has similar strong flux concentrations but different neutral line locations. There are two more connectivity cells present (1–1, 2–2) than with the charge representation (Figure 2(b)). The cells simplify with height and look like the ones obtained with charges, but with a greatest extension of the separator.
Fig. 6. Same as Figure 3 but with dipoles and with a cut at an additional height, Z = 0.4. The asymmetry increases the size of the cells. Some field lines are drawn in (c) starting in the left principal polarity. These dashed-dotted and dotted field lines belong to two different connectivity cells. (b) and (d) give another set of field lines (dashed-dotted) to show that a dip is localized above the separator. In (d) the origin of $S_n$ is chosen at the right field-line footprint.

- The separatrices between cells 1–2, 1–1, and 2–2 are located in low field regions.
- Observations (Hagyard, 1988) show that currents are localized close to the neutral line where the parasitic polarities are present. The new separatrices are then in near potential field regions.

Mandrini et al. (1991), using a dipole representation to analyse observations of a
particular flare, have found no chromospheric counterpart of such extra separatrices. Obviously, other studies are needed to confirm this result.

The extension of the separator is larger for a dipole distribution than for a charge distribution. This can be seen by comparing Figure 5(a) to Figure 2(b) and Figure 6 to Figure 3 and, also, in Table I. For a parasitic charge intensity 0.08 below the main charge intensity, the separator is at the photospheric level; while its top height for dipoles with the same relative moments is 0.24. A comparable height can be reached with charges only if the parasitic charge intensity is around 0.6 of the main one (Figure 3(b)). Therefore, fields which behave like dipoles give configurations where flares are easier to create, if the presence of a separator is a decisive property for flares to occur (if we extend 2D reconnection results to 3D).

A dip above the separator is still present with dipoles (Figure 6(d)). A prominence can be supported on the neutral line between the two main dipoles even with very low intensities of the parasitic dipoles. This is, in fact, a general property of field configurations where two separatrices’ cupolas intersect. It implies that the presence of a prominence in regions of flare occurrence is a natural consequence of the magnetic topology.

4. Topological Properties of Simple Magnetic Field Configurations – Modelling the Field by a Force-Free Field

4.1. Reasons for modelling the field by a force-free field

Both Hα observations of fibrils and observations of the transverse magnetic field component show that flare occurrence is often associated with a deviation of the transverse field of more than 70° with respect to the direction of a potential field with the same vertical component (Zirin, 1988; Hagyard, 1988). After the flare, this shear decreases. For example, post-flare loops are nearly orthogonal to the neutral line as potential magnetic field computations predict. Recently, Gaizauskas and Harvey (1991) have observed an active region where magnetic shear was necessary for flares to occur.

It is then believed that the flare energy comes from the free-energy of the currents distributed in the corona. This has been set recently on a firm mathematical ground by Aly (1990), who showed that a three-dimensional, force-free field can store as much free energy as is contained in a potential field with the same vertical photospheric component. Distributed currents are more suited for energy storage than current sheets (Priest, private communication) and Hénoux and Somov (1987) have suggested that reconnection along the separator would lead to current interruption and to the release of the free energy.

The potential field approximation implies the hypothesis that current systems will distort the configuration negligibly. This is a first step to estimate the separatrix in a topological model. We make below, for the first time, a quantitative estimate of how the presence of distributed currents modifies the location and shape of the separatrices, using the force-free field approximation.
4.2. Modelling the Field by a Force-Free Field

Force-free fields satisfy the equations

\[ \text{curl} \, \mathbf{B} = \alpha(\mathbf{r}, \mathbf{B}) \mathbf{B}, \]  

(4a)

\[ \text{div} \, \mathbf{B} = 0. \]  

(4b)

Equations (4) look simple but it is a challenge to solve them, since they are nonlinear with \( \alpha(\mathbf{r}, \mathbf{B}) \) determined by the field. Only some particular solutions are known in 2D and recently Low and Lou (1991) provided a powerful semi-analytical solution for a 3D-like field. Though being of great interest, nonlinear force-free fields are generally very difficult to construct. Moreover, since Equation (4a) is nonlinear, a particular solution does not help us to obtain a general one.

Several numerical techniques have been used to solve the problem, but they all fail when complex topologies are considered. One main difficulty comes from the natural discontinuity of the field at a separatrix. This arises directly from the definition of a separatrix, i.e., it separates two magnetic fluxes of different origin and so with different current systems. Recently, a direct integration, using the three photospheric field components as boundary conditions, has been performed (Cuperman, Ofman, and Semel, 1990). While the photospheric boundary limit used comes from analytical models, the integration fails to go above one-tenth of the photospheric map size.

The above-reported difficulties in dealing with nonlinear force-free fields show that we are still far from being able to derive general force-free solutions from observed data. The problem simplifies greatly if we assume a constant \( \alpha \). Equation (4a) becomes linear and the field can then be computed from photospheric data in a similar way as it is done with potential fields.

However, in order to understand the results, we must keep in mind the intrinsic limitation of linear force-free fields. Since currents are imposed on all field lines (with an intensity proportional to the field strength), they have a strange behaviour at great distances. The field changes direction periodically forming closed magnetic cells. Such

Fig. 7. Same as Figure 6(a) but with a low, sheared, force-free field (\( \alpha = 0.4 \)). While the photospheric field is weakly affected by the \( \alpha \) value, the separatrices are greatly distorted except in strong field regions.
features cannot represent a coronal field and linear force-free fields must be used only with the constraint $\alpha L \ll 1$, where $L$ is the size of the analyzed region. Their use is limited to moderate sheared fields (unless we confine them by artificial boundaries). In a nonlinear force-free field, such field reversals are avoided by decreasing $\alpha$ with the distance to the sheared region.

Démoulin and Priest (1992a) have generalized the classical multipole extension of a potential field to a linear, force-free field. The charge or dipole solution can be used to look for the influence of $\alpha$ on the field topology. Since the evolution of the magnetic field topology with increasing $\alpha$ is qualitatively similar in the charge and dipole representations, we will report here only the last one.

### 4.3. Magnetic Dipoles in a Linear Force-Free Field

Using usual spherical coordinates $(r, \theta, \phi)$ centered on the dipole vertical $Z$-axis, the field components for a linear, force-free, dipolar field are (Démoulin and Priest, 1992a):

$$B_r = 2F \cos(\theta)/r,$$

$$B_\theta = -\partial(rF)/\partial r \sin(\theta)/r,$$

$$B_\phi = \alpha F \sin(\theta),$$

with

$$F = \frac{M}{r^2} \left( \cos(\alpha r) + \alpha r \sin(\alpha r) \right).$$

In the limit $\alpha r \ll 1$ Equation (5) reduces to (3). A summation over all dipoles (as in Equation (3)) gives the field of the dipole distribution.

![Fig. 8. Force-free field results with $\alpha = 1$ at two heights. The photospheric field is greatly distorted compared to the potential case (Figures 6). The distortion of the separatrices comes from both a different photospheric field distribution and $\alpha$ value. However, comparison with Figures 9, which have the same vertical photospheric field but an opposite value for $\alpha$, shows that the effect of $\alpha$ is really present and important. The field line starting points in the left polarity are approximately the same as in Figures 8 and 9. The large-scale distortion of field lines is clearly seen.](image)
While the photospheric vertical field is weakly affected by a low \( \alpha \) (compare Figures 7 to 6(a)), the connectivity cells are greatly distorted. For greater \( \alpha \) (Figure 8(a)) the vertical photospheric field distribution is distorted, but is independent of the sign of \( \alpha \). Comparison of Figures 8 and 9 shows only the effect of \( \alpha \). New connectivity cells and separatrices are present. The same analysis of Section 3.2 can be applied to the new separatrices to show their low influence in energy release.

The effect of \( \alpha \) is more important far from the field concentrations. There is a great distortion and an increase in size of the connectivity cells with shear. The maximum rotation of the transverse magnetic field, relative to the potential direction, is of the order of 70° close to the neutral line in weak field regions; while it is only 45° near parasitic field concentrations. This is a property of linear, force-free fields which shear the field only on great scale lengths. Increasing \( \alpha \) to greater values introduces a limitation in the linear approximation. The field lines of one dipole (or charge) are contained in a sphere whose radius is proportional to \( \alpha^{-1} \) (Démoulin and Priest, 1992a). For great \( \alpha \) values it implies a decrease of the cell size with \( \alpha \) and the appearance of an extra neutral line in the field map. Such cases are not represented because of their intrinsically low physical meaning.

Considering only the high field regions, where flare ribbons are more likely to occur, we see that the potential field approximation gives a correct localization of the separatrices. This provides a straightforward explanation for the good fit obtained between potential computations and observations. Besides, the field evolution following a decrease in the absolute value of \( \alpha \) produces kernel motions by modifying the separatrix location (Figures 8 and 9). It will be worthwhile to compare these motions to observed ones, keeping in mind that this approach is only a first attempt to evaluate the kernel

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Fig. 9. Same as Figures 8 but with an opposite shear (\( \alpha = -1 \)). Note that the distributed currents have a greater effect at large distances from the dipole locations. The small portions of the separatrices in concentrated field regions are displaced and rotated by approximately 45° with respect to the potential case. The evolution of these parts of the separatrices could be observed only in Hz, since the flare kernels are more likely to be located in concentrated field regions.
evolution in a three-dimensional field, since it implies that reconnection affects the whole volume (because of the linear, force-free approximation).

5. Conclusions

At flare onset the field is supposed to be forced to evolve by a growing ideal instability. The frozen-in condition of the plasma implies the formation of current sheets on surfaces that separate fluxes of different origin. Because recent theoretical developments show that reconnection occurs rapidly in such circumstances, the separatrices are privileged regions for the release of the free magnetic energy stored in distributed currents. At the intersection of the separatrices (separator) the parallel field is a minimum and so this region is believed to be a major location for energy release. Due to the intrinsic difficulties in solving the equations involved, this model and related ones are developed in two dimensions. An extension to 3D is needed. Taking into account the major points derived from 2D studies, we want to compare the resulting theoretical developments to observations.

Keeping in mind that the field topology features are the main physical ingredients, we have developed an automatic algorithm to compute separatrix surfaces in a magnetic field created by several magnetic singularities such as charges or dipoles. The singularities are used to model the flux concentrations present at the photosphere. This field representation implicitly uses the linearity of the equations governing the field and so is limited to potential or linear, force-free fields.

We have computed the separatrices in a region formed by a main bipole and a parasitic one. While the charge and dipole representations have similar field concentrations at photospheric level, the latter is found to be more suited for flare occurrence because the separator emerges above the photosphere for a much lower intensity of the parasitic polarity than with charge representations. In both cases, the separator emergence is favoured by a separation between the axis of the parasitic bipole and the axis of the main one. Such parasitic polarity may not be observed in low-resolution magnetograms. The spatial dependence of the solar field must be tested on observed magnetic data.

We find that field lines passing just above the separator have a dip, an appropriate configuration to support dense plasma. When the emerging parasitic polarity intensity is low, the dip is formed just above the photosphere on a neutral line. Active region prominences are shown to exist at such low heights. The prominence height increases with the parasitic polarity flux and the dip location becomes detached from the photosphere. This corresponds to the emergence of the separator above the photosphere, a condition favourable for flare occurrence. So our model shows a natural link between prominences and flares.

The use of dipoles or of linear force-free fields introduces extra separatrices but no new separator compared to the configuration computed by charges. We have argued that such new surfaces play only a minor role in the reconnection process. Then the same common active separatrices are present in all field modellings. The main differences are
limited to the shape of these surfaces. When compared to observations, the potential approach is useful to localize approximately the separatrices in relation to photospheric flare kernels. If the kernels are on the separatrices, motions of these kernels can be predicted by following a sequence of computations in the linear force-free approximation with decreasing $|z|$ values.

Comparison of the present developments to observations will be the object of forthcoming papers. The first analysis of a few observations has already confirmed the validity of our present approach. By following the field lines that end in flare kernels, we have found that these lines pass close to the separator (Mandrini et al., 1991). In order to improve the modelling by the use of nonlinear force-free fields we must solve two problems. First, the charge or dipole representation is no longer useful and we need to develop an algorithm to integrate the field only up to the photospheric level. Second, we must be able to compute the field from photospheric data. The first point is under study and it requires us to define magnetic regions at the photosphere. No major obstacle is present at this stage. The second point is far from being achieved. Improvements are needed both in Zeeman observations (transverse field in low field regions, transverse field sign ambiguity) and in numerical methods. One basic difficulty in the computation of nonlinear force-free fields comes from the separatrices themselves. Currents on both sides have distinct origins and so they are in general discontinuous at these surfaces. Due to all these difficulties, well-known but not yet solved, we must realize that potential and linear force-free fields will be the only way to model the three-dimensional magnetic fields of flaring active regions for at least some years.

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References