Three-Dimensional MHD Simulation of the Parker Instability in Galactic Gas Disks and the Solar Atmosphere

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Abstract

Three-dimensional (3D) magnetohydrodynamic simulations were performed in a study of the nonlinear evolution of the Parker instability in galactic gas disks and/or accretion disks, as well as in emerging flux regions (EFR) of the Sun. The initial magnetic fields are parallel to one of the horizontal coordinates in magnetostatic equilibrium. The effect of coupling between the Parker (undular) instability and the interchange instability was mainly studied. In both the galactic and solar cases, adjacent flux tubes move independently, so that a highly interleaved structure is created, although the expansion of magnetic loops induced by the Parker mode is similar to that found in 2D models. In galactic gas disks magnetic loops evacuate regions of interstellar space by accumulating interstellar gas in magnetic pockets. The accumulated gas is compressed by both infalling gas and horizontally expanding magnetic loops, forming dense, thin spurs. That is, the 3D nonlinear Parker instability creates a large-scale void-shell-spur structure, which is very similar to the large-scale structure of the universe as well as the interstellar gas and/or dust distribution. In the solar EFR model, magnetic loops expand into the corona, while the gas slides down along the magnetic field lines. An approximate self-similar expansion and shock wave formation at the loop footpoints (which were previously found in 2D loops) were observed in 3D loops. A horizontal expansion of the rising flux tubes produces vortex motions, which then generate torsional Alfvén waves.

Key words: Interstellar matter — Magnetic fields — Magnetohydrodynamics — Parker instability — Sun

1. Introduction

A vertically decreasing horizontal magnetic field in a gravitationally stratified gas layer can become unstable against two types of perturbations. One is an interchange mode, the wave vector of which is perpendicular to the unperturbed field lines. The other is an undular mode, in which magnetic loops rise buoyantly while the gas slides down along the magnetic field lines. In astrophysical plasmas, the latter is known as the Parker instability.

It has been suggested that the Parker instability plays important roles in many astrophysical situations: the formation of interstellar cloud complexes (Parker 1966; Mouschovias 1974; Mouschovias et al. 1974; Blitz and Shu 1980; Elmegreen 1982a,b; Shibata and Matsumoto 1991), spurs which extend perpendicular to the Galactic plane observed in radio continuum (Sofue 1973, 1976) and in H I 21-cm line emission (Heiles and Jenkins 1976; Heiles 1984), the rise and emergence of magnetic flux tubes in the Sun and other stars (Parker 1979; Acheson 1979; Spruit and van Ballegooijen 1982; Moreno-Insertis 1986) as well as in accretion disks (Galeev et al. 1979; Stella and Rosner 1984; Kato and Horiuchi 1986; Horiuchi and Kato 1990; Chagelishvili et al. 1989).

Although the linear stage of the Parker instability has been studied extensively, no one had previously studied the multidimensional nonlinear evolution of the Parker instability. In preceding papers we reported on the results of two-dimensional nonlinear magnetohydrodynamic simulations of the Parker instability in accretion disks and/or galactic gas disks (Matsumoto et al. 1988, 1990; Shibata et al. 1990b) as well as in the emerging flux regions (EFR) of the Sun and other stars (Shibata et al. 1989a,b; Shibata et al. 1990a,c).

In this paper we present some typical results concerning three-dimensional numerical simulations of the Parker instability in gas disks and EFRs. The effect of coupling between the Parker instability and the interchange instability was mainly studied based on the assumption that the initial perturbation is sinusoidal (∼ eigen mode). We discuss here how previous 2D results can be modified in 3D situations. More detailed analysis
2. Formation of Void-Shell-Spur Structures in Interstellar Space

We consider here a local model for part of the magnetized gas layer around the equatorial plane of a galactic disk. Using local Cartesian coordinates \((x, y, z)\), we took the \(x\)-direction to be in the disk azimuthal direction and the \(y\)-direction to be in the disk radial direction; the \(z\)-direction was vertical to the equatorial plane of the disk.

The gravitational acceleration could be approximated by

\[
g(z) = GMz/(R^2 + z^2)^{3/2},
\]

where \(R\) is one characteristic length which determines the height at which the gravitational acceleration is maximum.

We neglect the effects of self-gravity, the curvature of unperturbed field lines, shear flow, and the Coriolis force.

The basic equations are then as follows:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = 0,
\]

\[
\frac{\partial (\rho V_x)}{\partial t} + \frac{\partial}{\partial x}[\rho V_x^2 + \frac{1}{8\pi}(B_y^2 + B_z^2 - B_x^2)] + \frac{\partial}{\partial y}\left(\rho V_x V_y - \frac{1}{4\pi} B_z B_y\right) + \frac{\partial}{\partial z}\left(\rho V_x V_z - \frac{1}{4\pi} B_y B_z\right) = 0,
\]

\[
\frac{\partial (\rho V_y)}{\partial t} + \frac{\partial}{\partial x}(\rho V_x V_y - \frac{1}{4\pi} B_z B_y) + \frac{\partial}{\partial y}\left[\rho V_y^2 + p + \frac{1}{8\pi}(B_x^2 + B_z^2 - B_y^2)\right] + \frac{\partial}{\partial z}\left(\rho V_y V_z - \frac{1}{4\pi} B_x B_z\right) = 0,
\]

\[
\frac{\partial (\rho V_z)}{\partial t} + \frac{\partial}{\partial x}(\rho V_x V_z - \frac{1}{4\pi} B_y B_z) + \frac{\partial}{\partial y}(\rho V_y V_z) + \frac{\partial}{\partial z}\left[\rho V_z^2 + p + \frac{1}{8\pi}(B_x^2 + B_y^2 - B_z^2)\right] + \rho g = 0,
\]

\[
\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial y}(V_y B_z - V_z B_y) + \frac{\partial}{\partial z}(V_z B_x - V_x B_z) = 0,
\]

\[
\frac{\partial B_y}{\partial t} + \frac{\partial}{\partial x}(V_x B_y - V_y B_x) + \frac{\partial}{\partial z}(V_z B_y - V_y B_z) = 0.
\]
Fig. 1. Time evolution of the Parker instability on the $y = 0$ plane, for $\epsilon = 6$ and $\beta = 1$. The solid curves represent isocontours of the vector potential ($A_y$). The isocontours of $A_y$ approximately show the magnetic field lines if the field configuration is approximately two-dimensional. The arrows show the velocity vectors. The numbers at the left bottom of each figure indicate the time in units of $\tau = R/C_s = 2.4H/C_s$, where $H$ is the scale height of the disk. If we adopt $H \approx 100$ pc and $C_s \approx 10$ km s$^{-1}$ for galactic gas disks, the length of the loop is $8 \times 2.4H \approx 1.9$ kpc, the time for the bottom figure is $8.6\tau \approx 2.1 \times 10^8$ yr. A reference length for the velocity vectors is given at the right bottom.

The solution is symmetric about a rigid conducting wall at $z = 0$. This boundary condition will be removed in subsequent work, since the most unstable Parker mode breaks the inversion symmetry of the disk (Horiuchi et al. 1988; Chagelishvili et al. 1988).

To initiate dynamical evolution, small velocity perturbations having the form

$$V_x = AC_s \sin[\pi(x - X_{\text{max}})/X_{\text{max}}] \cos(2\pi y/Y_{\text{max}}) \tag{14}$$

were imposed on the equilibrium disk model. Their amplitude ($A$) was taken to be 0.03.

Equations (2)–(9) were solved numerically by a modified Lax-Wendroff scheme (Rubin and Burstein 1967), with artificial viscosity (Richtmeyer and Morton 1967). The grid spacing was uniform in the $x$- and $y$-directions, but slowly coarser along $z$.

Figure 1 shows the results of a three-dimensional simulation of the Parker instability in a gas disk with $\epsilon = 6$ and $\beta = 1$. The ratio of specific heats ($\gamma$) was taken to be 1.05. The density ratio between the equatorial plane and at infinity is $\rho(z = \infty)/\rho(z = 0) = \exp(-\epsilon) \approx 2.5 \times 10^{-3}$. Although $\epsilon$ is of the order of 1000 in galactic gas disks, the density ratio can be as large as $10^{-3}$ if a hot corona
exists above a cold disk. Thus, $\varepsilon = 6$ is not unreasonable when simulating galactic gas disks. The model volume is $(X_{\text{max}}, Y_{\text{max}}, Z_{\text{max}}) = (9.6H, 7.2H, 10.8H)$, where $H$ is the scale height at a point where the gravity is maximum [$H = (1 + 1/\beta) C_s^2/(\gamma g_{\text{max}}) = 0.41R$]. When applying these results to the galaxy, we use the following units: $R = 2.4H = 240$ pc, $C_s = 10$ km s$^{-1}$; $\tau = R/C_s = 2.4H/C_s = 2.4 \times 10^7$ yr. Note that the local scale height increases with $z$ when $z > 0.71R$. This simulates the situation that a hot corona exists above a cold disk. Thus, $\alpha$ loses its original meaning, the “radius” of the disk, but represents the thickness of the Parker unstable disk. The number of grid points used in this simulation was $(N_x \times N_y \times N_z) = (83 \times 63 \times 62)$.

As the instability grows, the magnetic field rises buoyantly as gas slides down the forming magnetic loops, accumulating in magnetic pockets at the base of the loops.

Figure 2 shows the magnetic field lines (solid curves in the upper panel), velocity vectors, and density distribution (lower panel) in the $y = 0$ plane at $t = 10.1\tau$. Dense blobs are created at the bottom of magnetic pockets, and spur-like structures form perpendicular to the disk plane above these dense regions. Shock waves are created at the magnetic loop footpoints where the speed of gas downflow exceeds the sound speed. These characteristics are the same as those found in previous two-dimensional simulations (Matsumoto et al. 1988, 1990).

Figure 3 shows the three-dimensional structure in the nonlinear stage of the Parker instability. Density isocontours are shown for (a) the $x$-$y$ plane ($z = 2$), (b) the $x$-$z$ plane ($y = 1.5$), and (c) the $y$-$z$ plane ($x = 4$). Magnetic loops evacuate the gas within the loop and create expanding bubbles (voids), while infalling gas creates dense thin spurs perpendicular to the magnetic field lines. In the boundaries of expanding magnetic loops, a dense shell-like structure is formed due to compression. Domain structures comprising voids, shells, and dense spurs created by the Parker instability are analogous to the large-scale structure of the universe (e.g., de Lapparent et al. 1986); they are also similar to the interstellar gas and/or dust distribution (e.g., Heiles and Jenkins 1976). It has been suggested that a large-scale void-shell-spur structure of interstellar gas is created by superbubbles (Tomisaka and Ikeuchi 1986; Ikeuchi 1988). We propose here another possibility: that they are produced by the Parker instability. A filamentary distribution of dense gas is also found in numerical simulations of a self-gravitating gas layer (Miyama et al. 1987). Figure 3 shows that such a filamentary structure can be created, even if we do not take into account the self gravity.
The spurs created in the nonlinear stage of the Parker instability are compressed by both the supersonic downflow along the field lines and by horizontally expanding magnetic loops. A schematic diagram of the three-dimensional structure is given in figure 4. The dense spurs created during the nonlinear stage of the Parker instability may explain the H I spurs or worms (Heiles 1984), as well as the ionized thermal spurs observed in radio continuum emission (Müller et al. 1987). The ionization mechanism of the thermal spurs may involve shockwave heating at the sides of the spurs. The spurs are created within $8\tau \approx 1.9 \times 10^8$ yr, and exist for at least $2\tau \approx 4.8 \times 10^7$ yr in this model.

In order to clarify the three-dimensional effects, figure 5 shows the results of a two-dimensional simulation for the same parameters as those in figures 1 through 3. The growth rate of the Parker instability and the maximum speed of the downflow are smaller in the two-dimensional models than in the three-dimensional models; the shock waves and spur-structures are not as clear.

Figure 6 shows the linear growth rate of the Parker instability for the same parameters used in nonlinear simulations ($\epsilon = 6$ and $\beta = 1$). The dashed curve represents the most unstable two-dimensional ($k_y = 0$) fundamental mode (Horiuchi et al. 1988). This mode was excluded in the numerical simulations described in this paper, based on the symmetric boundary condition at the equatorial plane. The solid curves show growth rates of the first-harmonic mode for various $k_y$. Since the numerical model shown in figures 1–3 corresponds to $k_y = 2.1$, its linear growth rate when $k_x = 0.785$ ($\lambda = 2\pi/k_x = 8$) is more than twice that of the two-dimensional ($k_y = 0$) first harmonic mode. The three-dimensional modes ($k_x \neq 0$ and $k_y \neq 0$) can be considered as being the Parker mode mixed with the interchange mode. The growth rate of the mixed mode is larger than the pure Parker mode ($k_y = 0$), although the unperturbed state described by equations (10)–(13) is stable for the pure interchange mode ($k_x = 0$). These results of a linear stability analysis explain why the Parker instability in a three-dimensional

Fig. 3. Three-dimensional structure in the nonlinear stage of the Parker instability ($t = 10.1\tau$) when $\epsilon = 6$ and $\beta = 1$. Density isocontour curves (a) on the $z = 2$ plane (b) on the $y = 1.5$ plane, and (c) on the $x = 4$ plane are shown. The contour step widths are 0.25 on the logarithmic scale.

Fig. 4. Schematic diagram of the three-dimensional configuration of magnetic fields and matter distribution during the nonlinear stage of a Parker instability.
Fig. 5. Results of a two-dimensional simulation of the Parker instability when $\epsilon = 6$ and $\beta = 1$. The upper panel shows the magnetic field lines (solid curves) and velocity vectors (arrows) at $t = 16.1\tau$. The lower panel shows the density isocontours. The step width of the density isocontours is 0.25 on the logarithmic scale.

Fig. 6. Linear growth rate of the Parker instability when $\epsilon = 6$ and $\beta = 1$. The dashed curve indicates the growth rate of the most unstable fundamental mode for $k_y = 0$. The solid curves represent the growth rates of the first harmonic modes for various $k_y$. The units of the growth rate are $C_s/R = 1/\tau$.

Simulation grows faster than in a two-dimensional simulation. When the mixed mode perturbation grows, the adjacent flux tubes move differently and interleaved structure of magnetic field lines is created.

3. Three-Dimensional Model of Emerging Magnetic Flux in the Solar Atmosphere

It is now well established that sunspots and active regions are formed by the emergence of magnetic flux tubes from the interior of the Sun into the solar atmosphere (e.g., Zwaan 1987). Newly emerged bipolar active regions are called emerging flux regions (EFRs). One of the major issues in EFR research is to clarify how the magnetic flux is carried from the convection zone to the photosphere, and the nature of its subsequent expansion into the upper atmosphere. Recently, Shibata et al. (1989a,b, 1990a) performed a two-dimensional magnetohydrodynamic simulation of EFRs and explained the upward velocity of the arch filaments, the downflow velocity along the filaments, the small rise velocity of the photospheric magnetic flux, and the strong downdrafts observed in the photosphere.

We present here the results of three-dimensional simulations of the emerging magnetic flux in the solar atmosphere. The basic equations are again equations (2)–
The gravitational acceleration $g(z)$ is taken to be constant. The unperturbed model comprises a gas layer in magnetostatic equilibrium. This layer comprises a cool chromospheric/photospheric layer and a hot coronal layer. The initial temperature distribution is assumed to be

$$T(z) = T_{ch} + (T_{cor} - T_{ch})\left( \frac{\tanh\left( (z - z_{cor})/w_{tr} \right) + 1}{2} \right),$$

(15)

where $T_{cor}/T_{ch}$ is the ratio of the temperature in the corona to that in the chromosphere/photosphere, $z_{cor}$ is the height of the base of the corona, and $w_{tr}$ is the temperature scale height in the transition region. In this paper we take $T_{cor}/T_{ch} = 25$, $z_{cor} = 11H$, and $w_{tr} = 0.6H$, where $H$ is the pressure scale height of the chromosphere/photosphere.

We assume that the magnetic field is initially horizontal (in the $x$-direction) and is localized at the bottom of the photosphere. Its strength is $B_x(z) = [8\pi p(z)/\beta(z)]^{1/2}$, where we take $\beta(z)$ to be 1 for $0 \leq z \leq 4H$, and $\beta(z) = \infty$ for $-4H \leq z \leq 0$ and $4H \leq z$. The initial density and pressure distributions are numerically calculated by solving the static pressure balance equation.

Small velocity perturbations of the form

$$V_x = AC_s \sin[2\pi(X_{max} - x)/\lambda] \cos(2\pi y/Y_{max})$$

(16)

are initially imposed on the magnetic flux sheet ($0 \leq z \leq 4H$) within the finite horizontal domain ($X_{max} - \lambda/2 \leq x \leq X_{max} + \lambda/2$), where $\lambda = 20H$ is the horizontal wavelength of the small velocity perturbation and $C_s$ is the sound speed in chromosphere/photosphere. In the following we take $A = 0.1$. The perturbations are imposed in a finite horizontal domain in order to simulate the evolution of a single magnetic loop (see Shibata et
al. 1989a for the case when periodic perturbations are imposed). In two-dimensional simulations of the pure Parker instability, the results do not change very much even if random perturbations are imposed (Nozawa et al. 1992). More general cases including three-dimensional random perturbations will be discussed separately (Matsumoto et al. 1992).

The boundary conditions are the same as those in the gas disk model, except that the lower boundary is at \( Z_{\text{min}} = -4H \) in this model. The model volume is \((X_{\text{max}}, Y_{\text{max}}, Z_{\text{max}} - Z_{\text{min}}) = (40H, 19.2H, 24.5H)\), and the grid used in the simulation is \( (N_x \times N_y \times N_z) = (53 \times 51 \times 82) \). In the following, we use units \( H = 200 \) km, \( C_s = 10 \) km s\(^{-1}\), and \( \tau = H/C_s = 20 \) s.

Figure 7 shows the results of a three-dimensional simulation of the solar emerging flux model (the Parker instability in an isolated flux sheet at the bottom of the photosphere). The solid curves and arrows in the upper panel of each part of the figure show the isocontours of the vector potential and velocity vectors, respectively. The solid curves in the lower panels show the density isocontours.

As the instability grows, magnetic loops expand into the upper layer as the gas slides down along the magnetic field lines. The interleaved structure of the magnetic field lines created by the interchange mode is similar to that in the galactic-disk model.

Although the interchange mode was coupled with the Parker mode in the three-dimensional situation, the numerical results of the 3D simulation are similar to those of the 2D simulation. An approximate self-similar expansion and shock-wave formation at the loop footpoint, which have been found for 2D loops (Shibata et al. 1989a,b, 1990a,c) were again observed for 3D loops.

The most interesting result inherent to the three-dimensional simulation is that the horizontal expansion of the rising flux tubes generate vortex motion (figure 8), which produces torsional Alfvén waves. In figure 8, downflow and rising flow coexist in the chromosphere within a region of \(< 5H = 1000 \) km \( \approx 1''5 \). It may be difficult to detect such a small-scale velocity structure from observations. In a subsequent paper (Matsumoto et al. 1992), we shall analyze the 3D simulations in more detail, and compare the numerical results with observations of EFRs.

Self-similar expansion and vortex motion are more prominent in the solar emerging flux model than in the galactic-disk model, since the pressure ratio between the corona and the photosphere is small \( (\leq 10^{-4}) \) in the EFR model.

4. Discussion

Recent progress involving supercomputers has enabled us to perform a three-dimensional MHD simulation of the Parker instability in gas disks, as well as in emerging
flux regions of the Sun. We have confirmed that the main results from previous 2D simulations are still valid in 3D simulations. We have further found that a "void-shell-spiral structure" is formed during the nonlinear stage of the Parker instability in the case of galactic gas disks.

The fundamental findings have been confirmed in more general cases, such as with random perturbations (Matsumoto et al. 1992). When the perturbation is random, short-wavelength interchange modes are excited. In such cases, the "void-shell-spiral structure" becomes less prominent. When a magnetic shear exists, however, the interchange modes are suppressed and a large-scale structure can be formed.

By using high-speed (≥ 5 GFLOPS) supercomputers with large main memory (≥ 500 MB), three-dimensional MHD simulations with 100 × 100 × 100 grid points can be performed within several CPU hours. This gives us the opportunity to simulate essentially three-dimensional phenomena with sufficient spatial resolution. Interesting problems, such as the emergence of twisted magnetic flux tubes and the effects of the galactic rotation on the evolution of the Parker instability, will be studied in the near future.

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