STEEL CORONAE AND THEIR RELATION
TO CONVECTION ZONES AND ROTATION RATES

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1. Introduction

There have been many papers in the literature discussing the appropriate quantities
for correlating stellar coronal properties with convection zone properties, some of
which have been summarized by Dobson and Radick (1989). For example, do X-ray
fluxes or luminosities correlate better with rotation periods \( P \) or Rossby numbers
\( (Ro) \)? \( (Ro = P/\tau_c, \) where \( \tau_c \) is the turnover time at the base of the convection zone).
Similarly, how do coronal temperatures depend on convection zone parameters and
the rotation rate? Here we use simple scaling laws and available data to derive

2. The dependence of coronal temperatures on stellar rotation rates

We have used coronal temperatures, \( T_c \), measured with the \textit{Einstein} Observatory,
from the sample analyzed by Schmitt \textit{et al.} (1990), from stars for which one-
temperature fit to the spectrum is satisfactory. These are supplemented by a few
other measurements from Schrijver \textit{et al.} (1984), Golub \textit{et al.} (1982), Giampapa
\textit{et al.} (1985) and Schmitt \textit{et al.} (1987). Figure 1a shows \( \log T_c \) plotted against
\( \log P \), for the whole sample. All the stellar data used are tabulated in Jordan and
Montesinos (1991). It is clear from Figure 1a that there is not a unique dependence
of \( T_c \) on \( P \). The F dwarfs (open squares) lie on average below the trend shown by
the G/K dwarfs (circles). The evolved stars lie above the dwarf stars of the same
rotational period, as found earlier by Montesinos and Jordan (1988).

3. Scaling laws from energy balance arguments

If the energy density in the coronal gas originates from the coronal magnetic field,
one can argue that if the energy dissipation occurs at a fixed value of \( <\delta B_z^2> / B_z^2 \)
then \( B_z^2 \) scales as the coronal pressure, \( P_c \). In an isothermal, hydrostatic corona,
where the emission is formed over the first pressure-squared scale height, the coronal
emission measure is given by

\[
\text{Em}(T_c) = 7.1 \times 10^7 \frac{P_c}{T_c g_*}
\]  

so that

\[
B_z \propto P_c^{1/2} \propto (\text{Em}(T_c) T_c g_*)^{1/4},
\]  

The \textit{Einstein} X-ray fluxes are almost proportional to \( \text{Em}(T_c) \) because the temper-
ature sensitivity of the observing bands tends to cancel that of the radiative powe-
losses. Since this may not be true for other instruments we work in terms of \( \text{Em}(T_c) \),
rather than \( F_X \). \( \text{Em}(T_c) \) can be found from the volume emission measures given by
Schmitt \textit{et al.} (1990) by dividing by \( 4\pi R_*^2 \).

\textbf{Mem. S.A.It., 1992 - Vol. 63^* - N° 3 e 4} 735
Figure 1. (a) Observed coronal temperatures, $\log T_c(K)$, plotted against the stellar rotational period, $\log P$ (days). F dwarfs are indicated by squares, G/K dwarfs by circles, F subgiants by inverted triangles, G/K giants or subgiants by normal triangles. Filled symbols indicate RS CVn systems or close binaries.

(b) $\log(Em(T_c)T_c g_*)^{1/4}$ plotted against $\log Ro$, for dwarf stars.

We have postulated that $B_c$ might depend simply on the Rossby number, say as $Ro^{-x}$. Figure 1b shows $(Em(T_c)T_c g_*)^{1/4}$ plotted against $Ro$ for the F, G and K dwarfs. The values of $Ro$ were calculated from $\tau_e$ as given by Noyes et al. (1984). The best fit relation, using a least-square reduced major axis method (Isobe et al. 1990), is

$$0.25 \log(Em(T_c)T_c g_*) = 9.78(\pm0.03) - 0.79(\pm0.09) \log Ro.$$  \hspace{1cm} (3)

Excluding the worst fitting star, 110 Her, an F IV star which is unusually active for its rotation rate, increases the value of $x$ to $0.85(\pm0.07)$. Thus $x$ is close to but slightly less than 1.0. A larger sample is required to determine the power more accurately.

Further scalings result if the coronal energy losses are dominated either by thermal conduction or radiation, or if the ratio of these is fixed as in a minimum energy loss solution (see Hearn 1977, Jordan 1980, Jordan et al. 1987). These are

$$Em(T_c) \propto T_c^2 g_* \quad \text{and} \quad P_c \propto T_c^2 g_*.$$  \hspace{1cm} (4)

and in terms of $Ro$
\[ \text{Em}(T_e)g_*^{1/2} \propto B_e^3 \propto R_{\odot}^{-3z} \quad \text{and} \quad T_e g_*^{1/2} \propto B_e \propto R_{\odot}^{-z}. \] (5)

The first of relations (4) holds well for the whole sample (see Jordan and Montesinos 1991) and agrees with the results of Schrijver et al. (1984), except for the important \( g_* \) term. The first of relations (5) is shown for the dwarfs in Figure 2a, where \( z = 0.84(\pm 0.09) \), or 0.96(\pm 0.07) excluding 110 Her, in agreement with the values found from (3), to within the error bars.

Thus, including a term in \( g_* \), there are physical reasons why \( \text{Em}(T_e) \), or the Einstein X-ray fluxes should scale with \( R_{\odot} \). If a sample covering a small range of values of \( g_* \), \( R_{\odot} \) and \( T_{\text{eff}} \) is used then other quantities such as \( L_X \) or ratios such as \( L_X/L_{\text{bol}} \), \( F_X/F_{\text{bol}} \) will also scale with \( R_{\odot} \). Because of the fundamental relations between \( R_{\odot}, B-V \) and \( \tau_\lambda \), a scaling between \( \log L_X \) and \( P \) is also expected, provided \( \tau_\lambda \) does not vary by a large factor. Thus F dwarfs will not fit the same \( \log L_X-P \), \( F_X-P \) or \( \log L_X-\sin i \) relations as the G/K dwarfs. This explains the results found by Schmitt et al. (1985) and Dobson and Radick (1989). Since there are close correlations between X-ray fluxes and chromospheric and transition region fluxes, for all types of stars, this also explains why \( F(\text{Ca II}) \) and \( F(\text{C IV}) \) do not depend only on \( P \), when a sample including F dwarfs, G/K dwarfs and various giants are included, as has been found by Rutten and Schrijver (1987) and Simon and Fekel (1987).

In summary, given the basic relation between \( \text{Em}(T_e) g_*^{1/2} \) and \( R_{\odot} \), from energy balance arguments, the success, or otherwise, of finding correlations between

![Graphs](image-url)

**Figure 2.** (a) \( \log \text{Em}(T_e) g_*^{1/2} \) plotted against \( \log R_{\odot} \), for dwarf stars. (b) \( \log T_e g_*^{1/2} \) plotted against \( -\log N_d \), for dwarf stars and RS CVn systems.
parameters such as \( F_X, L_X, F_X/F_{bol} \) or \( L_X/L_{bol} \), and parameters such as \( P, R_0 \) or \( v \ sin \ i \), can be understood.

4. Evolved stars

Values of \( \tau_c \) have been calculated for slightly evolved stars by Gilliland (1985) and Rucinski and Vandenberg (1986). Basri (1987) has used the former, with an adjusted \( T_{eff} \) scale, to find \( R_0 \) for RS CVn stars. (Some values require correction for a systematic confusion, Basri, private communication). The RS CVn systems do not fit the correlation of \( T_c g_0^{1/2} \) (or the other quantities above) with \( R_0 \), found for the F, G, K dwarfs, allowing for the systematic differences between the values of \( \tau_c \) found for the dwarfs by Gilliland (1985) and Noyes et al. (1984), but are offset by about a factor of 3 in \( R_0 \). We have also explored the use of the dynamo number, \( N_d = (R_c/H)^{1/2} R_0^{-1} \), where \( R_c \) and \( H \) are the radius and pressure scale height at the base of the convection zone, respectively. The factor \( R_c/H \) found by Gilliland (1985) or Rucinski and Vandenberg (1986) is \( \sim 1 \) for sub-giants but \( \sim 2.5 \) to 3 for dwarf stars. Figure 2b shows \( log T_c g_0^{1/2} \) plotted against \( N_d \) from Gilliland 1985, with Basri’s modifications for the RS CVn’s). Although correlations with \( R_0 \) or \( N_d \) give the same powers for a sample of dwarfs alone, using \( N_d \) gives the best correlation if a sample of dwarfs and sub-giants is considered together.

References