MODELLING OF SOLAR CORONAL LOOPS

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Summary Solar active regions are observed to be composed of loop structures, with a wide range of maximum temperatures and lengths. There has, over the past fifteen years or so, been considerable interest in understanding these structures in terms of the possible energy input processes, and in terms of their MHD and thermal stability. This review concentrates on what is known about the observable parameters, and how these are related to the theoretical energy balance equation. Although it is difficult to derive the form of the heating function, because of the uncertainties inherent in the observations, very few sets of observations exist in which all the useful parameters have been measured simultaneously. In order to investigate heating processes other observations are also required, i.e. of non-thermal velocities and magnetic fields in the photosphere and in loops (through microwave data). The pressure variation across loops is still poorly known, and is also important for studies of loop stability.

1. Introduction

Modelling of coronal active regions in terms of loop structures was stimulated by images obtained in the X-ray (Vaiana et al. 1973) and UV parts of the spectrum, although the presence of loops was known from earlier optical eclipse images. The transition from modelling active regions in terms of loops rather than as coronal condensations occurred in 1974, as can be seen from the review by Jordan (1975). The properties of active regions as a whole were studied from the OSO series of satellites and reviews of this work can be found in Noyes (1971). The Skylab missions provided far more extensive imaging and spectral data over a wider wavelength range and the results of a variety of analyses are summarized in the monograph Skylab Solar Workshop III (Orrall, 1981).

Since then a large number of papers have been written on the structure, heating, dynamics and stability of loops. Mewe (1991) has given a recent review with a substantial list of references to earlier work. Jordan (1988) summarized some aspects of active region structure, evolution and heating. This review will concentrate on observations that allow physical parameters to be determined, and which must be reproduced by models, on the relation between structure and energy balance, and problems yet to be solved. The emphasis will be on plasma at transition region temperatures and above. Transient phenomena, of particular interest to studies of flares, and post-flare loops will not be discussed. Observations are reviewed in Section 2. Section 3 summarizes methods used to model the temperature and density structure of loops, and the energy balance assumed or
implied. Heating mechanisms are discussed briefly in Section 4.

2. Observations

The monochromatic images obtained during the Skylab missions still give the best simultaneous information on the spatial distribution of material in active regions as a function of temperature. The spectroscopic instruments allowed measurements of emission line intensities, and intensity ratios from which electron densities could be found. Although large quantities of data exist from the Solar Maximum Mission the spatial resolution is lower in the soft X-ray region.

There has been increasing interest in correlating the EUV and X-ray emission from active regions, and active region plasma parameters, with the underlying photospheric magnetic field (Schrijver et al. 1985, Schrijver 1987). Studies combining X-ray imaging, microwave and photospheric magnetic field measurements have also been made (Webb et al. 1987, Schmeltz et al. 1988, Nitta et al. 1991), and attempts are being made to deduce the magnitude of non-potential fields (e.g. Gary et al. 1987). Although we do not review methods of measuring magnetic fields, these are a vital part of investigating the structure, heating and stability of loops. Studies of active regions as a whole are also required, and the matching of computed field lines in the the presence of currents, to observed structures (Levine 1976), is an important aspect of the modelling process.

A new era of high resolution imaging has been opened by the sub-arcsec X-ray observations of active regions using the Normal Incidence X-ray Telescope (NIXT) (Golub 1991, Gomez and Golub 1992).

2.1. Observations of the density and temperature structure

The articles in the monograph Skylab Solar Workshop III (Orrall 1981) review observations relating to the structure of active region loops and the diagnostic techniques used in their analysis. For more recent reviews of active region structure and energy balance, and of atomic data related to density diagnostics, see Mewe (1991) and Mason (1991).

2.1.1. Temperature structure. Complete loop structures extending into the corona are commonly observed in emission lines formed at $T_e > 2 \times 10^6$ K but less frequently in lines whose "optimum" temperature of formation is lower than this. An individual loop usually has an extensive isothermal region, along the loop, at its maximum observed temperature.

Simultaneous images of loops in lines formed at different temperatures show that within an active region there are loops with a wide range of maximum temperatures, which are not spatially coincident. This was apparent in the active region observed in UV forbidden lines of Si VIII, IX, Fe XI, XII and S XI, during the 1970 total eclipse (Gabriel and Jordan 1975), and in many active regions observed from Skylab (e.g. Levine 1976, Levine and Withbroe 1977, Sheeley 1981, Dere 1982a).

Observations both on the disc and at the limb show that the loops fan out at different angles to the vertical, as shown in Figure 1, from Levine (1976). Observations at the limb give the impression that the most strongly emitting hot loops lie at lower heights than cool loops (see e.g. Touyse 1976, Gabriel and Jordan 1975, Foukal 1975), but in other images shown by Levine (1976) there are loops in Mg X of a similar length to those in Ne VII, although at different angles to the vertical. On balance, the strong emission from high ions, such as Fe XV, appears to come from the cores of active regions (Sheeley 1981) and loops that appear complete in cool lines, such as Ne VII, are usually the longest. The systematic behaviour of loop parameters and their spatial distribution within a given active region does not appear to have received much attention in the recent literature.
Figure 1. EUV image showing loops in O VI 1032 Å. (From Levine, 1976).

Figure 2. Images in XUV lines. Dashed lines indicate the centre of emission of the three Ne VII loop segments. (From Dere, 1982a).
Projection effects are also important in studies of the total area covered by an active region as a function of temperature and of the underlying magnetic field (e.g. Schrijver 1987, Schrijver et al. 1985). Loops with a range of angles to the vertical may give the impression of a spatial extent increasing with temperature, even when individual loops have small expansion factors between the transition region and high temperatures.

The images obtained at 3–4 arcsec resolution by Golub (1991) in a passband dominated by Fe XVI emission ($T_e \sim 3 \times 10^6$ K), show that loops are resolved into more numerous, thinner structures, so that the length to radius aspect ratio of individual loops is larger than deduced from lower resolution observations. Also, although the loops maintain an almost constant cross section area within the corona, they taper down rapidly at low heights to connect into regions of enhanced network, or into sunspot penumbrae, rather than umbrae. Ideally, such observations are needed in a range of lines, formed at different temperatures. Further observations at 0.75 arcsec resolution have recently been obtained (Gomez and Golub 1992). These are important for constraining theoretical models of the structure and MHD stability of loops.

Although the plasma in a loop appears to be isothermal over a large proportion of its length, material at lower temperatures can be observed extending partially up the legs of loops. In an active region studied by Levine (1976) and Levine and Withbroe (1977), O VI and Ne VII emission extends partially along loops that are complete in Mg X. The analysis of AR 12702 by Dere (1982a) (see Figure 2) shows that emission in Mg VI, Ne VII and Mg VIII, apparently occupying the same loop, extends successively further up the legs, from the same base height. Dere (1982a) shows that the decrease of the emission with height is for each line comparable with the isothermal density-squared scale height, at the relevant ‘optimum’ temperatures of line formation. This argues strongly against there being one uniform loop structure, with a unique temperature gradient. Instead, it suggests the presence of individually structures, each with its own temperature distribution. Other studies have shown the hot plasma in loops can be in hydrostatic equilibrium (e.g. Gabriel and Jordan 1975, Habbal et al. 1985).

Several sets of independent observations have suggested that within a given loop the cooler material lies on average inside the hotter. This was argued by Gabriel and Jordan (1975) from monochromatic images obtained in UV forbidden lines. Foukal (1975) studied the widths of loops in different temperatures from Skylab observations. The widths in low stages of ionization (e.g. C II, C III) appear to be less than those in higher stages (e.g. O IV, O VI). Dere (1982a) also found that in the loops discussed above the width of the Mg IX and Mg VIII emission was larger than that of the cooler lines. The width of the partial loops observed in H Ly $\alpha$ by Tsiropoula et al. (1986) is also small, $\sim 2000$ km. Similarly, Hanaoka et al. (1988) conclude that the cooler emission lies within the hotter, on the basis of optical monochromatic images in [Fe X] and [Fe XIV] obtained during the 1980 total eclipse. Thus the structure within presently unresolved loops appears to consist either of sheaths of plasma with lower temperatures near the loop axis, or of small flux tubes, with a larger number of cool flux tubes closer to the axis. Although quite low temperatures do seem to exist within a loop, it is difficult to determine the actual temperatures without detailed modelling from simultaneous images covering a wide range of temperatures. In particular, the mere existence of emission from a low ion at a large height cannot by itself be used as evidence of departures from hydrostatic equilibrium. The ‘optimum’ temperature for the line formation is determined by the ion balance population, and the temperature dependent terms in the excitation rate, weighted by any variation of $N_e^2 dh/dT_e$ across the line forming region. But the extent to which emission is observable at higher temperatures depends critically on the variation of these factors with $T_e$. The analysis of partial H Ly $\alpha$ loops observed by Tsiropoula et al. (1986) illustrate this point. In a detailed treatment they conclude that the emission is formed at $T_e \sim 2.6-3.6 \times 10^5$ K.
far higher than would be the case in the normal transition region. However, it is clear that some loops are not in hydrostatic equilibrium. Changes in emission with time, suggesting flows along loops are observed, as discussed by Foukal (1976), Habbal et al. (1985) and Haisch et al. (1988). Dramatic cases of material draining from loop systems have also been observed (e.g. Levine and Withbroe 1977). The least stable loops, in this respect, seem to be cool loops terminating in sunspots.

2.1.2. Pressures in loops. The distribution of the electron pressure within loops is less well known, but is important in understanding their stability (see e.g. Chiuderi, Giachetti and Van Hoven 1977, Priest 1981). Gabriel and Jordan (1975) used the density sensitive lines of Fe XII to determine a pressure of \( \sim 6 \times 10^{15} \) cm\(^{-3}\) K, at a height of 30,000 km, about an order of magnitude larger than in the quiet corona. They used normalized level populations which are to some extent supported by recent calculations by Tayal et al. (1991). The other forbidden lines suggested that at a given position lower temperatures are associated with lower pressures. Foukal (1975) also deduced that there was a lower pressure in cool loop cores, but from arguments involving the apparent contrast in intensity between the loops and the surrounding corona, not from direct measurements. The method should give reasonable results for Mg X, which is formed close to the temperature of the ambient corona and a pressure contrast (loop/corona) of about a factor of six results. The absolute intensity of the Ne VII line at 465 Å, combined with the upper limit to the path length, certainly leads to a pressure an order of magnitude lower than that derived from Mg X, if it is assumed that the temperature is \( \sim 5.2 \times 10^5 \) K, the 'optimum' for the line formation when \( N_e^2 \frac{dh}{dT_e} \) is constant. However, if the Ne VII emission was being formed at a temperature of \( 8 \times 10^5 \) K, the pressure deduced would be larger by an order of magnitude. Alternatively, the true path length for the Ne VII emission could be smaller, giving a locally larger pressure. Thus the pressure distribution within a loop is still uncertain and further measurements using density sensitive line ratios in an identifiable active region loop (as opposed to post flare loops) are needed. High pressures at relatively low temperatures (\( \sim 3.6 \times 10^{16} \) cm\(^{-3}\) K at \( \sim 2 - 3 \times 10^5 \) K) were found by Tsiropoula et al. (1986), in their study of loops emitting in H Ly \( \alpha \).

It does, however, seem clear that active region loops contain material at pressures that are around an order of magnitude larger than in the ambient corona at the same height. Studies of density sensitive line ratios in the transition region do show higher pressures in active regions (Dupree et al. 1976). Images in C III line ratios confirm that the highest densities occur in small patches where the absolute intensity is high, and which seem to correspond to the footpoints of a loop structure seen in Mg X. Dere (1982a) has summarized a variety of measurements, as shown in Figure 3. In view of the advances made in calculating atomic data over the past ten years it would be worth reassessing the density from all available density sensitive line ratios.

Dere (1982a) notes that the measurements of densities using line ratios tend to give higher values than those from the emission measure, divided by the extent of the region (see Figure 3). The filling factor implied appears to increase with \( T_e \), suggesting that most of the loop could be filled with hot (\( \geq 10^6 \) K) plasma, with the cooler material being in some filamentary structure, but again we point out that a small increase in the temperature of line formation would raise the density deduced from the emission measure.

2.2. The emission measure distribution

To establish the emission measure distribution over the whole temperature range from \( 10^5 \) K to the maximum temperature requires the combination of at least UV and EUV observations, and ideally also X-ray observations. Because of the limitations in spatial resolution it is difficult to do this for individual loops. There is more information available
Figure 3. Summary of measurements of pressures in active regions (dots, line ratios; open circles, emission measure/volume). (From Dere, 1982a).

Figure 4. The emission measure distribution for an active region observed with OSO-VI (From Dere, 1982b, and see Dupree et al. 1973).
on the emission measure distribution for spatially integrated active regions, including results from the OSO satellites. For example, Dere (1982b) determined the distribution from spectra obtained with OSO-6, which were used by Dupree et al. (1973) to measure active/quiet intensity ratios, and this is shown in Figure 4. The overall shape of the distribution is basically similar to that found for the quiet sun. The difference between the quiet and active distributions, at the 3 arcsec resolution of OSO-6 is best seen from the ratios as a function of temperature. When the difference in density is taken into account, below $T_e = 2 \times 10^5$ K the ratio is constant to within the uncertainties, and above this temperature the ratio increases slightly with $T_e$, roughly as $T_e^{1/2}$. It can be seen from Figure 4 that the gradient of the emission measure in the active region lies between $T_e^{3/2}$ and $T_e$, implying that the spatially averaged quiet sun gradient is around 1 to 3/2. Schrijver et al. (1985) made a detailed analysis of a number of active regions and some quiet regions. In particular, they examined how the absolute intensities of transition region and coronal lines scaled with each other, and how the emitting areas scaled. The results are consistent with there being an intrinsic shape to the emission measure distribution, with the shape of the observed distribution then being determined by the scaling of the areas. The observed distribution would then appear to have a gradient with a power $T_e^{3/2}$ larger than the intrinsic gradient. Earlier studies (Gabriel 1976, Jordan 1976) also concluded that when integrated over the network and cell interiors the quiet sun distribution has a gradient steeper than 3/2, ($\sim 5/2$) which is reduced to about 3/2 only when the contrast and relative areas of the network is taken into account. Theoretical considerations, discussed below, suggest that ignoring area expansion factors, the gradient of the emission measure distribution should be around 3/2. The above results also seem to imply that any small scale area factors, of an unresolved size, scale as those observed at low resolution.

The emission measure distributions found by Pallavicini et al. (1981a,b), for a active regions observed with the Skylab instruments, have a lower gradient, but fewer lines were observed and the abundances of oxygen and neon were adopted. With the photospheric abundance of oxygen (and the same oxygen/neon ratio) the gradient is again close to 3/2. The emission measure distributions are derived using intensities averaged over the whole active region, or over the core of the region.

3. Structure and energy balance

Two approaches can be used to model the density and temperature with height. The first uses the observed emission measure distribution to obtain a model in hydrostatic equilibrium, which requires the pressure to be known at some $T_e$, as a boundary condition. The radiation losses are found from the emission measure distribution, but the net conductive flux must be calculated from the model. The required form of the heating function can then, in principle, be found. The second assumes a particular form for the heating rate per unit volume. Assumptions must also be made about the boundary conditions at the temperature chosen to represent the “base” of the loop. The pressure and maximum temperature must be specified. The pressure has often been assumed to be constant, but hydrostatic models have also been made. The energy balance equation is solved for the conductive flux, and then the temperature is found as a function of height above the base of the loop. The emission measure distribution can also be calculated and compared with that observed. Area expansion factors can be introduced in either approach.

3.1. Hydrostatic models based on the observed emission measure distribution.
The energy balance equation can be written as

\[
\frac{dF_m}{dT_e} = -\frac{dP_r}{dT_e} - \frac{dF_c}{dT_e}
\]  

(1)

where \(F_m\) is the flux of non-thermal energy, \(P_r\) is the radiative flux and \(F_c\) is the conductive flux. The radiation losses are given by

\[
\frac{dF_c}{dr} = N_e N_H P_{rad}(T_e) \approx \frac{P_e^2 \alpha T_e^3}{T_e^2}
\]  

(2)

where \(P_e\) is the electron pressure \((/k)\) and \(P_{rad}(T_e)\) can be approximated by power law fits, depending on the range of temperature considered. Here we put

\[
P_{rad}(T_e) = \alpha T_e^3
\]  

(3)

with \(\alpha = 1.5 \times 10^{-19} \text{ erg cm}^3 \text{ s}^{-1} \text{ K}^{1/2}\) and \(\beta = -1/2\), for \(T_e > 2 \times 10^8 \text{ K}\). The classical conductive flux is given by

\[
F_c = -\kappa T_e^{5/2} \frac{dT_e}{dr}
\]  

(4)

where we take \(\kappa = 1.0 \times 10^{-6} \text{ erg cm}^{-1} \text{ K}^{-7/2} \text{ s}^{-1}\). The emission measure is defined as

\[
Em(T_e) = \int_{\Delta r} N_e^2 dr
\]  

(5)

where \(\Delta r\) is the region over which the line is formed, and corresponds to a temperature range which is typically \(\Delta \log T_e = \pm 0.15 \text{ dex}\), about the optimum temperature for the line formation. The emission measure can be re-written as

\[
Em(T_e) = \frac{P_e^2}{2^{1/2} T_e} \frac{dr}{dT_e}
\]  

(6)

where the pressure and temperature are the local means. (This form is valid only up to \(T_m/\sqrt{2}\), where \(T_m\) is the maximum temperature). Combining equation (6) with the equation of hydrostatic equilibrium gives

\[
P_e(T_e)^2 = P_e(T_{ref})^2 \pm 5.5 \times 10^{-4} \int_{T_{ref}}^{T_e} Em(T_e) dT_e
\]  

(7)

where \(P_e(T_{ref})\) must be measured. Observed values of \(Em(T_e)\) can be used without assuming a specific functional form. The temperature gradient, and variation of \(T_e\) and \(P_e\) above a chosen height can be found from equation (6). The main uncertainty is in the absolute, rather than the relative, values of \(dT_e/dr\), through the dependence on \(P_e^2\), which does not vary by a large factor as a function of \(T_e\).

The radiative losses over \(\Delta \log T_e = 0.3 \text{ dex}\) can be written as

\[
\Delta F_r = Em(T_e) \alpha T_e^{-1/2}
\]  

(8)

and do not depend on the model. The total radiation losses can then be found by summation. The conductive flux is found from equation (4), and has the same source of uncertainty as \(dT_e/dr\).
The energy input required can be expressed explicitly as

\[
\frac{dF_m}{dT_e} = -\frac{\alpha E m(T_e) T_e^{21/2}}{T_e^{21/2}} \left[ \frac{3}{2} \frac{d \log E m}{d \log T_e} + \frac{2d \log P_e}{d \log T_e} \right]
\]

(9)

where

\[
\frac{2d \log P_e}{d \log T_e} = -5.5 \times 10^{-4} \frac{E m(T_e) T_e}{P_e^2}
\]

(10)

It is simple to examine the magnitude of the terms in square brackets in equation (9), using equation (10), to see whether an assumption of constant pressure can be justified. For example if we require \(dr/dT_e \leq 0.1H/T_e\), where \(H\) is the local isothermal scale height, then for typical quiet sun or active region values of the parameters, one could use a constant pressure for \(T_e \leq 7 \times 10^5\) K or \(9 \times 10^5\) K, respectively. Similarly, considering a loop of length \(L\), for \(P_e^2\) at the top to be at least half that of \(P_e^2\), requires \(L \leq 0.35H\). Loops with \(T_m < 3 \times 10^6\) K usually have lengths that do not satisfy this requirement.

In the region where \(P_e \simeq \text{constant}\), the gradient of the emission measure is the main source of uncertainty in \(dF_m/dT_e\), since the observations show that it is close to 3/2. However, the form of equation (9) is useful in that it shows that, when \(P_e\) is constant, conduction is an energy loss term if \(d \log E m(T_e)/d \log T_e > 3/2\), and vice-versa. A gradient of 3/2 corresponds to the case of constant conductive flux. The observations show that the emission measure passes through a minimum at \(T_e \sim 10^5\) K, so that energy carried by conduction is certainly deposited at and below this temperature. Above this temperature, but in the region where \(P_e \simeq \text{constant}\), it can be shown that the gradient of the emission measure distribution is not very sensitive to the functional form of the heating function, and is predicted to be about 3/2.

The energy carried by conduction at \(2 \times 10^5\) K can be found from the absolute value of the emission measure, through

\[
F_c(T_o) = -\frac{\kappa T_o^{3/2} P_o^2}{21/2 E m(T_o)}
\]

(11)

which, plus the total radiative losses, gives the minimum energy flux above \(T_o\).

The above approach is useful for making hydrostatic models, for determining the radiation losses and for finding the minimum energy required above \(T_o\), but the net conductive flux cannot be determined in the region where \(P_e\) is constant, given the observational uncertainties in the emission measure gradient at a given \(T_e\).

3.2. Models using a functional form for the heating

This approach was used in early work by Schmeleva and Syrovatskii (1973) and Landini and Monsignori Fossi (1975). Scaling laws for global loop properties were derived by the above authors, by Craig, McClymont and Underwood (1978) and Rosner, Tucker and Vaiana (1978) (hereafter, RTV). Hood and Priest (1979) include similar material, but also considered the loop MHD and thermal stability, and they cover many of the points discussed below. Vesecky, Antiochos and Underwood (1979) introduced a variable loop cross-section area, and hydrostatic equilibrium (Underwood, Antiochos and Vesecky 1981). Since then a variety of model conditions have been explored, including further studies of thermal stability and time dependent solutions (see review by Mewe 1991).

In this approach the energy input is expressed as

\[
dF_m/dr = -\epsilon_o T_e^\gamma = -E_h
\]

(12)

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The energy balance equation can then be re-written in terms of $F_c(T_e)$ and $dF_c(T_e)/dT_e$, to give, for a constant pressure,

$$\int F_c(T_e) dF_c(T_e) = k \alpha P_o^2 \int T_e^{\beta+1/2} dT_e - k \epsilon_o \int T_e^{\gamma+5/2} dT_e \quad (13)$$

Most authors have assumed that $dF_c(T_e)/dT_e$ is zero at some temperature, $T_*$, between $T_o$ and the maximum temperature, $T_m$, so that

$$\epsilon_o = P_o^2 \alpha T_0^{\beta-2-\gamma} \quad (14)$$

(Note that there is no unique value of $T_*$ if $\gamma = -5/2$, which corresponds to the case of constant conductive flux). The boundary condition at $T_m$ is $F_c(T_e) = 0$, at some base temperature, $T_b$, we instead work above $T_o = 2 \times 10^5$ K, since $F_c(T_o)$ can be found from observations of $Em(T_o)$ and $P_o$, and also a constant value of $\beta = -1/2$ can then be used.

Equation (13) can then be integrated from $T_o$ to $T_*$ and from $T_*$ to $T_m$, to give functional forms for $F_c(T_e)$, e.g.

$$F_c(T_e)^2 = F_c(T_o)^2 - \frac{2k \epsilon_o (T_e^{7/2+\gamma} - T_o^{7/2+\gamma})}{7/2 + \gamma} + 2k \alpha P_o^2 (T_e - T_o) \quad (15)$$

or

$$F_c(T_e)^2 = \frac{2k \epsilon_o (T_m^{7/2+\gamma} - T_e^{7/2+\gamma})}{7/2 + \gamma} - 2k \alpha P_o^2 (T_m - T_e) \quad (16)$$

The temperature gradient and the emission measure (or differential emission measure, $DE = N_e^2 dr/dT_e$) can then be calculated as a function of temperature.

Attention has been concentrated on the important question of whether or not the observations can be used to find $\gamma$, given their uncertainties. The emission measure distribution can be written as

$$Em(T_e) = Em(T_o) \left(\frac{T_e}{T_o}\right)^{3/2} \frac{F_c(T_o)}{F_c(T_e)} \left(\frac{P_e}{P_o}\right)^2 \quad (17)$$

up to $T_e = T_m/\sqrt{2}$. $F_c(T_o)^2$ is defined through equation (16), so that in general $\epsilon_o$ and $T_m/T_e$ depend on both $F_c(T_o)$ and $\gamma$ (for fixed $P_o$ and $T_m$). Only if $F_c(T_o) = F_c(T_b) = 0$ do $\epsilon_o$ and $T_m/T_e$ depend solely on $\gamma$. Thus, when working with a finite value of $F_c(T_o)$, in general equation (14) does not provide a independent estimate for $\epsilon_o T_m^\gamma$, and another estimate must be made. Following Craig et al. (1978),

$$E_h(T_m) \approx \frac{\kappa T_m^{7/2}}{L^2} + \frac{\alpha P_o^2}{T_m^{5/2}} \quad (18)$$

However, to match the numerical results of RTV (1978) for the case of $F_c(T_b) = 0$ and $\gamma = 0$, it is more appropriate to use

$$E_h(T_m) \approx \frac{2}{3L^2} \kappa T_m^{7/2} + \frac{\alpha P_o^2}{T_m^{5/2}} \quad (19)$$
This can then be substituted into the expression for $F_c(T_o)/F_c(T_e)$ giving

$$
\frac{F_c(T_o)^2}{F_c(T_e)^2} = \frac{R \left( 1 - \left( \frac{T_o}{T_e} \right)^{\gamma + 7/2} \right) - (\gamma + 7/2) \left( 1 - \frac{T_o}{T_e} \right)}{R \left( 1 - \left( \frac{T_o}{T_e} \right)^{\gamma + 7/2} \right) - (\gamma + 7/2) \left( 1 - \frac{T_o}{T_e} \right)}
$$

(20)

where

$$
R = \frac{2}{3} \frac{\kappa T_m^6}{\alpha F_c^2 L^2} + 1 = \left( \frac{T_m}{T_e} \right)^{\gamma + 5/2}\left( \frac{dF_m}{dr} \right)_{T_m} = \left( \frac{dF_e}{dr} \right)_{T_m}
$$

(21)

Note that $F_c(T_o)/F_c(T_e)$ tends to $T_o/T_e$ when $R = 3.5$ and $\gamma = 0$, in agreement with the results of RTV (1978). The absolute value of $Em(T_o)$ is given by

$$
Em(T_o) = \frac{1}{2} \left( \frac{\kappa}{\alpha} \right)^{1/2} \frac{P_o T_o^{3/2} (\gamma + 7/2)^{1/2}}{T_m^{1/2}} \left[ R - (\gamma + 7/2) \right]^{1/2}
$$

(22)

The maximum emission measure,

$$
Em(T_m) \propto P_o^2 L/T_m^2
$$

(23)

can also be used to give approximate values of $P_o$ or $L$, and in any case should be reproduced by any independent measurements of $P_o$, $L$ and $T_m$. Thus coronal measurements alone define $R$, but to find $\gamma$ either the form of the emission measure distribution, or preferably, $Em(T_o)$ must be measured.

We have above introduced $L$ through the estimate of $E_h(T_m)$. Alternatively, $L$ can be calculated by integrating $dr/dT_e = -\kappa T_e^{5/2}/F_c(T_e)$. However, since in general $R$ depends on $F_c(T_o)$, $\gamma$ and $L$, an iterated solution would be necessary. Only when $F_c(T_o) = 0$ does $R$ depend solely on $\gamma$.

### 3.3. Comparison with observations

The general behaviour of $Em(T_e)$ as a function of various combinations of fixed parameters and variables has been explored in the early literature (e.g. Craig et al. 1978, RTV 1978, Underwood et al. 1981, Withbroe 1981), but the full range of $\gamma$ has not always been illustrated, and comparisons with observed values of parameters are not always complete.

We now use some available data to explore the range of values of $R$ and $\gamma$ allowed. The range of values of $R$ can be estimated from observations of active regions (see e.g. Fig. 8.3 in Withbroe 1981). For a given value of $T_m$, the values of $P_o L$ have a spread of about a factor of 2 around that given by the scaling law derived by RTV (1978). Assuming this is a real spread of values, it implies $11 \geq R \geq 1.6$. (Note that because here $F_c(T_o)$ is $\geq 0$, for $T_m \gg T_o$, $R$ must be $> \gamma + 7/2$). Thus $R = 11$ does not place useful limits on $\gamma$, but $R = 1.6$ limits $\gamma$ to $-1.9 \leq \gamma \leq -2.5$, which is a tight constraint. For a fixed value of $T_m$ and $P_o$ (i.e. $L$ variable), and a given value of $\gamma$, then $F_c(T_o)$ decreases as $R$ decreases. Since $R > 1$, $P_o L$ cannot exceed the RTV value by more than a factor of 2.5. If there is a unique value of $\gamma$ (which must then be small), and the spread in in values of $R$ is real, then large values of $R$ imply large values of $F_c(T_o)$. From equation (21), the ratio of the heating rate, and the conductive loss rate, to the radiative loss rate at $T_m$ then varies, as opposed to the case of $F_c(T_o) = 0$ and $\gamma = 0$, when $R$ and these ratios are fixed.
The effects on the shape of $E_m(T_e)$ when $R$ is varied between 11 and 1.6 are as follows. (A typical value of $T_m = 3 \times 10^6$ K is used). For $R$ large, and $\gamma = 0$, $E_m(T_e)$ does not deviate strongly from the basic $T_e^{3/2}$ power law, (up to $T_e/T_m = 0.71$). At most, for $\gamma = -2.4$, $E_m(T_e)$ is larger than the the $T_e^{3/2}$ power law by a factor of 1.7. Thus for large $R$, the shape of $E_m(T_e)$ with $T_e$ does not usefully constrain $\gamma$. However, as $R$ decreases, the shape of $E_m(T_e)$ begins to vary significantly, and as discussed above, high values of $\gamma$ become increasingly excluded. For example, with $R = 3.5$, at $T_e = 0.7 T_m$, the value of $E_m(T_e)$ is less than the $T_e^{3/2}$ law by a factor of 2.5, for $\gamma = 0$, but is greater than the $T_e^{3/2}$ law by a factor of 1.7 for $\gamma = -2.4$. Although one could distinguish between such distributions, the difference of either from the $T_e^{3/2}$ law would be only marginally detectable, and $\gamma$ can be found by this method only if $R \leq 3.5$.

The absolute value of $E_m(T_e)$ is potentially more useful. Using the same range of values of $R$ as above, equation (22) shows that even for the largest value of $R = 11$, $E_m(T_e)$ differs by a factor of 2 between $\gamma = 0$ and $\gamma = -2.4$. At $R = 3.5$, for these values of $\gamma$, $E_m(T_e)$ differs by a factor of 5.6, and for smaller values of $R$, a measurement of $E_m(T_e)$ places even tighter constraints on $\gamma$. Thus the minimum set of observable parameters needed is $E_m(T_e)$, $P_o$, $T_m$ and $L$. At present there seem to be very few data sets in which all 4 parameters have been measured independently, e.g. Doyle et al. (1985), Raymond and Foukal (1982). Some other sets of data can be used, making use of equation (23) to estimate $P_o$ or $L$. The values of $E_m(T_m)$, $T_m$ and $L$, or $P_o$, in Pallavicini et al. (1981a,b), and Haisch et al. (1988) lead to $R < 3.5$, thus excluding $\gamma = 0$. $E_m(T_o)$ can also be found from Pallavicini et al. (1981a,b) (correcting the abundances used for oxygen and neon), and give $\gamma$ between -0.8 and -2.5. Dere (1982b) gives $E_m(T_o)$, $E_m(T_m)$ and $T_m$, and $P_o$ can be found from the relative intensities of O V in the original paper by Dupree et al. (1973), using up-to-date atomic data. $L$ must be estimated from equation (23). Again, a value of $R \leq 3.5$ is implied and $\gamma \simeq -2.3$. The data in Doyle et al. (1985) and Raymond and Foukal (1982) lead to large values of $R$ but values of $\gamma$ near the limit of -2.5. Thus the small amount of data that exist suggest that $\gamma$ is close to the limit of -2.5.

Ideally, several independent measurements of $P_o$ are needed for a given loop, since this is the most difficult parameter to measure with sufficient accuracy. $T_m$ also needs to be well defined using several lines formed near $T_m$. Many loops should be examined to find whether or not there really is any systematic behaviour in the values of $\gamma$ found.

The above approach uses a fixed value of $\gamma$ at all temperatures. In practice there may be heating via the magnetic field only near the top of the loop, say above some temperature $T_u$. Although $\gamma$ may be small below $T_u$, with the conductive flux being almost constant, this situation obviously cannot extend to $T_m$, since the energy input near $T_m$ must account for the energy lost by conduction and by radiation at $T_m$. In this case there seems to be no way of finding the local value of $\gamma$ near $T_m$; only the total energy input $E_h(T_m)$ can be estimated in the same way as above.

Using equations (111) and (23) for $E_m(T_o)$ and $E_m(T_m)$, and $F_c(T_e)$ constant gives $F_c(T_m) \approx \kappa T_m^{3/2}/\sqrt{2L}$, essentially the same as used above. $F(T_o)$ can also be expressed as

$$F_c(T_o) = \alpha \kappa P_o^2 T_m (R - 1)/\sqrt{2}$$

(24)

to match the general form from equations (17) and (20). Any value of $R > 1$ is allowed, but in practice $F_c(T_o)$ will be limited by the ability of the layers below $T_o$ to radiate away the energy deposited.

Another simple analytical case occurs when there is no heating between $T_o$ and some $T_u$, and the radiative losses are balanced by the net conductive flux from above. Then

$$F_c(T_e) = F_c(T_o)^2 + 2\kappa\alpha P_o^2 (T_e - T_o)$$

(25)
or with the same definition of $R$,

$$\frac{F_c(T_o)^2}{F_c(T_e)^2} = \frac{(R - 3)(1 - T_o/T_m)}{R(1 - T_o/T_m) - 3(1 - T_e/T_m)}$$

(26)

For $R \gg 3$ and $T_o < T_m$, this tends to 1, and $Em(T_e)$ again depends mainly on $(T_e/T_o)^{3/2}$. For small $R$, and large $T_e$, $Em(T_e)$ tends to depend only on $T_e/T_o$. The absolute value of $Em(T_o)$ in the cases of constant conductive flux and no heating, between $T_o$ and $T_u$, are similar for $R \gg 3$. This is because the conductive flux is large and, in comparison, very little energy is removed by radiation. The above two cases could, in principle, be distinguished through the different limiting values of $R > 1$ (constant conductive flux) and $R > 3$ (no heating), and through the form of $Em(T_e)$, and $Em(T_o)$, when $R$ is small.

Throughout the above discussions, the variation of the pressure in hydrostatic equilibrium can be taken into account through the term $P_e^2/P_o^2$ in equation (17) (other terms are not significant), which reduces the gradient at high temperatures. This is easily allowed for in numerical calculations.

In conclusion, the shape of the emission measure distribution is similar for all forms of heating when $R$ is large, because of the increasing importance of $F_c(T_o)$ in comparison with the radiation losses. However as $R$ decreases the differences in $Em(T_e)$ increase, and larger values of $\gamma$ are successively excluded. The only single value of $\gamma$ that could fit a real range of values of $R$ between around 11 and 1 is close to -5/2, i.e. close to constant conductive flux. The absolute value of $Em(T_o)$, for a given $R$, is more sensitive to $\gamma$, and it is the lack of simultaneous measurements of this fourth parameter that at present limits our knowledge of the systematic behaviour of $\gamma$ in active region loops.

If the emission measure gradient is fundamentally close to 3/2, then any substantial deviations from this can be attributed to a variable area factor, or filling factor, between $T_o$ and $T_m$. If a small filling factor were present at $T_o$ the value of $Em(T_o)$ would need to be increased, leading to a larger deduced value of $\gamma$.

### 3.4 Scaling laws

From the above it is obvious that a scaling law between $T_m$, $P_o$, and $L$ only exists when $R$ is fixed. The scaling law of RTV is obtained when $F_c(T_o) = 0$ and $\gamma = 0$, giving $R = 3.5$, and $(dF_c/dr)T_m/(dF_e/dr)T_m = (R - 1) = 2.5$.

The scaling law derived by Hearne and Kuin (1981) from a minimum energy loss solution gives similar results, when the difference in the definition of parameters is taken into account. Hearne and Kuin (1981) use the case of $F_c(T_o) = 0$, and no heating between $T_o$ and $T_u$, to find $F_c(T_u)$, and then minimize $F_m(T_u)$ w.r.t. $T_m$ at constant $\rho$ and $L$; the minimum value of $R$ is then 1.14.

Equation (19) gives a minimum energy loss solution irrespective of the energy balance below $T_m$. The constant in the scaling and the minimum value of $R$ depend on which parameters are held constant. If we fix $T_m$ and $P_o$, but allow $L$ to vary, to match the discussion in previous sections, then the minimum value of $R$ is 2. If the spread in values of $R$ is real then loops do not seem to follow the minimum energy loss solution. Alternatively, one can fix $P_o$ and $L$, in which case the minimum value of $R$ is 12/7. Although we will not discuss the thermal stability of loops in general, we point out that the values of $T_m$ and $P_o$ then correspond to the critical values in the stability analysis. (See e.g. Priest 1981). Thus values of $R < 1.7$ are not thermally stable in this approach.

When applied to an average corona, with the isothermal scale height replacing $L$, the relatively small range of $F_c(T_m)/F_e(T_m)$ allows the observed scalings between stellar coronal parameters to be understood (see Jordan and Montesinos 1991 for details).
4. Heating processes

If the heating is related to the magnetic field then two types of process are of interest (i) heating by MHD waves, and (ii) heating by magnetic field reconnection. (See Gomez 1991 for a recent review of possible processes, and Ulmschneider et al. 1991). Here we examine the scalings expected in a few simple examples, to see how $R$ might depend on magnetic field parameters.

If the heating is by MHD waves then one can put

$$Eh(T_m)L = \rho < V_T^2 > V_A$$  \hspace{1cm} (27)

and

$$\beta = P_T 8\pi/B^2$$

to give

$$< V_T^2 > = 9.5 \times 10^6 \beta^{1/2} T_m R/((R - 1)^{1/2}$$  \hspace{1cm} (28)

Saba & Strang (1991) have recently measured non-thermal velocities in an active region corona and to $< V_T^2 >^{1/2}= 68$ km s$^{-1}$, at $T_m \sim 3 \times 10^6$ K. The other plasma parameters in these appear to be estimates rather than measurements lead to a value of $R = 1.1$, near the lower limit of 1, and hence $\beta = 0.22$. With their estimate of $P_T \sim 3 \times 10^{16}$ cm$^{-3}$ K (which is in fact a rather large value), this gives $B \sim 30$ Gauss, comparable with values computed from the fitting of potential fields (e.g. Nitta et al. 1991). Thus these particular measurements do not exclude MHD wave heating.

With further measurements of $< V_T^2 >$, as well as $P_o$, $T_m$ and $L$, the systematic behaviour of $\beta$ and $B$ with $R$ could be examined. However, if $\beta$ and $T_m$ are fixed, $< V_T^2 >$ is expected to change by only a factor of 1.7 for $R$ between 11 and 1.1.

For an MHD wave $\rho < V_T^2 >$ can be replaced by $< \delta B^2 > /4\pi$, which can also be regarded as the magnetic energy released during reconnection. Then one finds

$$\frac{< \delta B^2 >}{B^2} = \frac{3.5 \times 10^{-2} \beta^{3/2} R}{(R - 1)^{1/2}}$$  \hspace{1cm} (29)

With the numbers above a value of $< \delta B^2 > /B^2 = 1.3 \times 10^{-2}$ is found. For fixed values of $\beta$ and $T_m$, $< \delta B^2 > /B^2$ changes by less than a factor of 2 between $R = 11$ and $R = 1.1$, with a minimum value of $10^{-2}$ at $R = 2$.

The more specific process of heating through non-linear mode reconnection was discussed by Galeev et al. (1981). They propose that

$$E_h L = \frac{5 \times 10^{-2} V_A B_j^2 B_\phi}{4\pi B_z}$$  \hspace{1cm} (30)

where $B_\phi$ and $B_z$ are azimuthal and vertical field components, $B_j$ is the non-potential field and the numerical constant is an efficiency factor. Then

$$\frac{B_\phi}{B_z} = \frac{0.69 R B_j^{3/2}}{(R - 1)^{1/2}}$$  \hspace{1cm} (31)

If we associate the non-thermal motions with the factors in equation (30) other than $V_A$, then $B_j = 0.22$ as before and $B_\phi/B_z = 0.25$. If $< V_T^2 >$ and $T_m$ are fixed, both $\beta$ and $B_\phi/B_z$ vary by only a factor of 1.3, for $R$ in the range 11 to 1.1. With the previous

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Mem. S.A.It., 1992
estimate of \( P_e = 3 \times 10^{16} \text{ cm}^{-3} \text{ K} \), \( B_2 \) is 30 Gauss. Neither \( B_\phi \) nor \( B_z \) can be found unless it is assumed that \( B_\phi = B_j \), or \( B_z \) is found from matching potential field calculations, based on photospheric measurements. Overall, the above observations give some hope of placing reasonable constraints on \( \beta \) and \( B_\phi/B_z \). The variations of \( \langle V_T^2 \rangle \) and \( R \) with temperature need to be known before any conclusions can be drawn concerning the systematic behaviour of \( B_\phi/B_z \) and \( \beta_j \). Jordan (1988) found a tendency for \( R \) to increase with \( T_m \), which for fixed \( \beta \) would imply \( B_\phi/B_z \) increasing with \( T_m \).

Conclusions

Our understanding of the heating of active region loops is at an early stage. At present very few complete data sets have been published from which the energy balance between can be determined. This requires knowledge of \( T_m \), \( P_o \), \( L \) and \( Em(T_o) \). The small amount of information available suggests that the energy carried by thermal conduction is almost constant. If heating is occurring only near \( T_m \), then expressing \( E_h \) as \( \epsilon_j T_m^4 \) is not helpful. Given the advances in collision cross-sections during the past ten years a systematic re-analysis of emission measures and density sensitive line ratios would be worthwhile.

To investigate specific heating processes other parameters must be measured. In particular many more measurements of non-thermal velocities near \( T_m \), and as a function of \( T_e \), are needed. Measuring \( \langle V_T^2 \rangle \) can allow the plasma \( \beta \) and \( B \) to be found, or in theories involving twisted fields, the non-potential \( \beta \) and \( B_\phi/B_z \). It is also important to measure the surface B fields, so that fields higher in loops can be calculated, and non-potential fields estimated from comparisons with observed structures. Measurements of B fields with the VLA are also valuable in this respect. Thus a coordinated multi-wavelength approach is vital in planning any future observations of active region loops.

References