ON THE INTERACTION BETWEEN DUST AND GAS IN LATE-TYPE STELLAR ATMOSPHERES AND WINDS

K. B. MacGregor
High Altitude Observatory, National Center for Atmospheric Research, Boulder, CO 80307

AND

R. E. Stencel
Joint Institute for Laboratory Astrophysics and Center for Astrophysics and Space Astronomy, University of Colorado, Boulder, CO 80309

Received 1991 December 23; accepted 1992 April 6

ABSTRACT

An assumption inherent to most models of dust-driven winds from cool, evolved stars is that the radiative and collisional drag forces acting on an individual dust grain are in balance throughout the flow. We have checked the validity of this supposition of “complete momentum coupling” by comparing the grain motion obtained from such a model with that derived from solution of the full (i.e., including inertia) grain equation of motion. For physical conditions typical of the circumstellar envelopes of oxygen-rich red giants, we find that silicate grains with initial radii smaller than about 5 × 10⁻⁶ cm decouple from the ambient gas near the base of the outflow. The implications of these results for models of dust-driven mass loss from late-type giants and supergiants are discussed.

Subject headings: circumstellar matter — dust, extinction — stars: atmospheres — stars: late-type

1. INTRODUCTION

A long-standing problem in stellar physics concerns the dynamics of the extended atmospheres of cool evolved stars: namely, what processes are responsible for the initiation and maintenance of the mass loss such stars are observed to undergo? While many mechanisms have been proposed, few (if any) are capable of consistently accounting for the inferred properties of the winds from late-type giants and supergiants (Holzer & MacGregor 1985). In particular, it is difficult to devise a means for accelerating the flow such that its asymptotic speed is significantly lower than the gravitational escape speed from the stellar surface, as observations of many red giants and asymptotic giant branch (AGB) stars indicate (see Judge & Stencel 1991).

One mechanism which has received considerable attention and has been extensively applied relies upon the radiative acceleration of dust grains to induce overall expansion of a gaseous, circumstellar envelope. Dust, as evidenced by excess emission at infrared wavelengths, appears to be nearly ubiquitous among stars which are sufficiently cool and/or evolved. Although the physics of dust formation is presently not well understood (see, e.g., Draine 1981; Gail & Sedlmayr 1988), once condensed, individual grains can experience a net outward acceleration by absorbing and scattering photons which originate in the stellar photosphere. The subsequent motion of a grain is impeded only by collisional encounters with the microscopic constituents of the background gas through which it drifts. Such collisions provide the means for transmitting a portion of the momentum acquired by the grains to the ambient atmosphere. The momentum so transferred is then diffused throughout the envelope by collisions between gas molecules. If the frequency of grain-gas collisions is sufficiently high, the collective “force” exerted on the gas by

1 The National Center for Atmospheric Research is sponsored by the National Science Foundation.
small size (radius \( \lesssim 5 \times 10^{-6} \) cm) decouple near the base of the flow.

2. MODEL AND METHOD

We consider a steady, spherically symmetric wind emanating from a star of mass \( M_* \), radius \( R_* \), luminosity \( L_* \), and effective temperature \( T_{\text{eff}} \). Because the thermal pressure gradient force plays an subordinate role in the envelope dynamics, it is sufficient for the present purpose to assume that the outflow is isothermal. In spherical polar coordinates, the radial component of the gas momentum equation can then be written in the form

\[
du = 2c_s^2 (\frac{GM_*}{r}) (1 - \Gamma),
\]

where \( u(r) \) is the gas velocity and \( c_s \equiv (kT/\mu)^{1/2} \) is the isothermal sound speed in a plasma having temperature \( T \) and mean mass per particle \( \mu \). Under the assumption of complete grain-gas momentum coupling, the quantity

\[
\Gamma = \frac{N_{\text{gr}} a^2 \rho \nu_{\text{th}} L_*}{4GM_* c},
\]

appearing in equation (1), is the force (in units of the local gravitational force) exerted on a unit volume of gas as a result of collisions with radiatively accelerated dust grains. In equation (2), \( a \) is the grain radius, \( \nu_{\text{th}}(T_{\text{eff}}, a) \) is the Planck mean radiation pressure efficiency factor, and \((N_{\text{gr}}/\rho)\) is the number density of the grain material to the gas mass density.

In order to evaluate \( \Gamma(r) \), it is necessary to determine the local values of \( a \) and \((N_{\text{gr}}/\rho)\). For this purpose, we adopt the following schematic model for grain formation and growth. We choose the reference radius \( r = r_0 \) marking the inner boundary of the flow to be coincident with the grain condensation radius (see below). The grains are taken to be solid, spherical particles of density \( \rho_{\text{gr}} \) and mass \( m_{\text{gr}} = (4/3)\pi a^3 \rho_{\text{gr}} \). The grains are further supposed to be predominantly composed of material of chemical species “x,” an individual atom of which has mass \( m_x \). We assume that all grains condense at \( r_0 \) with the same initial radius \( a_0 \) and that once formed, a grain is never destroyed. If the abundance (by number) of species “x” relative to the sum of all gas constituents is \( \gamma \) and if a fraction \( f_0 \) of the available mass of grain-forming material condenses at \( r_0 \), then conservation of mass yields

\[
N_{\text{gr}} = \frac{3f_0 \gamma m_x m_\text{gr}}{4\pi a_0^3 \rho_{\text{gr}}} \left[ \frac{u(r)u(r)}{u_{\text{gr}}(r)/u_{\text{gr}}(r_0)} \right],
\]

where \( u_{\text{gr}}(r) \) is the equilibrium grain velocity, obtained by equating the radiative and drag forces acting on a single grain at location \( r \), as described below. As an individual grain drifts relative to the background atmosphere, it will occasionally encounter atoms of chemical species “x.” Collisions with and absorptions of such particles will cause the grain radius to grow according to

\[
da = \frac{4\pi a^2 \rho_{\text{gr}} u_{\text{gr}}}{dN_{\text{gr}}/dt},
\]

where \( dN_{\text{gr}}/dt \) is the number of “x” atoms incident on the grain surface per second and \( q_a \) is the fraction of all such collisions in which the atom adheres to the grain. In the limit of rarefied gas dynamics \( (a \ll \text{mean free path for collisions between gas molecules}) \) and assuming that the background gas is characterized by a Maxwellian distribution function with a single temperature \( T \), it is straightforward to derive the following approximate (but accurate) expression for \( dN_{\text{gr}}/dt \) namely,

\[
dN_{\text{gr}}/dt = \frac{2\pi a^2 N_x \nu_{\text{th}}}{u_{\text{rel}}^2} \left[ 1 + \frac{u_{\text{rel}}}{4 \nu_{\text{th}}/a} \right]^{1/2},
\]

where \( u_{\text{rel}} \equiv (u_{\text{gr}} - u) \) is the grain velocity relative to the gas and \( \nu_{\text{th}} = (2kT/m_\text{gr})^{1/2} \). Because the grains are assumed to remain spherical, the fraction of grain-forming material actually in grains increases with \( a \) (and \( r \)) as \( f = f_0 (a/a_0)^3 \).

The equation of motion for a single, uncharged giant is

\[
m_{\text{gr}} \frac{du_{\text{gr}}}{dt} = m_{\text{gr}} u_{\text{gr}} \frac{du_{\text{gr}}}{dr} = \frac{a^2 \nu_{\text{th}} L_*}{4\pi c^2} - f_{\text{drag}},
\]

where the first term on the right-hand side is the radiative force \( f_{\text{rad}} \) appropriate to an optically thin circumstellar envelope. For reasons discussed in the Appendix, we neglect grain charging and the Coulomb drag force associated with it. The collisional drag force can be expressed as

\[
f_{\text{drag}} = \frac{\pi a^2 \rho_{\text{gas}} F}{6},
\]

where \( \rho_{\text{gas}} = (2kT/m_\text{gr})^{1/2} \) and \( F \) is a function of \( u_{\text{rel}} \). The exact form of \( F \) depends upon the nature of the grain-gas interaction. With the same assumptions used in deriving equation (5), we have computed \( F \) supposing that each collision of a gas molecule with a grain can be described by one of the following limiting cases (see also Baines, Williams, & Assebiemi 1965): (i) specular reflection of the molecule at the grain surface; or (ii), absorption, thermalization to the grain temperature \( T_{\text{gr}} \), followed by reemission in a random direction. In the former case, we find

\[
F_{\text{spec}} = \frac{1}{2} \left( \frac{u_{\text{rel}}}{c_s} \right) \left[ \frac{128}{9\pi} + \left( \frac{u_{\text{rel}}}{c_s} \right)^2 \right]^{1/2},
\]

while in the latter case \( F \) assumes the form

\[
F_{\text{abs}} = \frac{1}{4} \left( \frac{u_{\text{rel}}}{c_s} \right) \left[ \frac{128}{9\pi} \left( 1 + \frac{T_{\text{gr}}}{c_s^2} \right)^2 + \left( \frac{u_{\text{rel}}}{c_s} \right)^2 \right]^{1/2}.
\]

In the limit \( T_{\text{gr}} \ll T, F_{\text{abs}} = F_{\text{spec}} \) as expected. As in the case of equation (5), expressions (8) and (9) are approximations to the true functional dependences of \( F_{\text{spec}} \) and \( F_{\text{abs}} \) on \( u_{\text{rel}}, T_{\text{gr}}, \) and \( T_{\text{gr}} \).

The methods employed in conducting our analysis are as follows. The formation radius \( r_f \) for silicate (Mg,SiO$_4$) grains of specified size \( a_0 \) in a hydrostatic stellar atmosphere is determined using Draine's (1981) parametric representation of the condensation calculations of Lattimer, Schramm, & Grossman (1978). In applying these results, we assume that the dominant processes contributing to the thermal equilibrium of a single grain are radiative heating and cooling, the former occurring by absorption of radiation from the stellar photosphere. The equation describing the grain energy balance is then

\[
\tilde{Q}_{\text{abs}}(T_{\text{eff}}, a) W T_{\text{eff}}^4 = \tilde{Q}_{\text{abs}}(T_{\text{gr}}, a) T_{\text{gr}}^4,
\]

where \( \tilde{Q}_{\text{abs}} \) is the Planck-mean absorption efficiency and \( W(r) \) is the spherical dilution factor. Because condensation is more likely to occur under conditions of high density and low temperature, we adopt an atmospheric density distribution which is extended relative to one for which the thermal pressure gradient is the sole supporting force (see, e.g., Jones et al. 1981;
TABLE 1

<table>
<thead>
<tr>
<th>$a_0$ (cm)</th>
<th>$(r_0/r_{cr})$</th>
<th>$M$ (M$_\odot$ yr$^{-1}$)</th>
<th>$u_{\infty}$ $(\text{km s}^{-1})$</th>
<th>$u_{\infty, \infty}$ (km s$^{-1}$)</th>
<th>$u_{\infty, \infty}$ (km s$^{-1}$)</th>
<th>$(f_{\text{drag}}/f_{\text{coll}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.5 \times 10^{-5}$</td>
<td>1.019</td>
<td>1.102 $\times 10^{-6}$</td>
<td>60.26</td>
<td>101.22</td>
<td>100.89</td>
<td>0.985</td>
</tr>
<tr>
<td>$1.4 \times 10^{-5}$</td>
<td>1.034</td>
<td>8.794 $\times 10^{-7}$</td>
<td>55.50</td>
<td>95.48</td>
<td>95.12</td>
<td>0.983</td>
</tr>
<tr>
<td>$1.3 \times 10^{-5}$</td>
<td>1.056</td>
<td>6.618 $\times 10^{-7}$</td>
<td>50.86</td>
<td>90.65</td>
<td>90.24</td>
<td>0.980</td>
</tr>
<tr>
<td>$1.2 \times 10^{-5}$</td>
<td>1.087</td>
<td>4.556 $\times 10^{-7}$</td>
<td>46.66</td>
<td>87.75</td>
<td>87.26</td>
<td>0.977</td>
</tr>
<tr>
<td>$1.1 \times 10^{-5}$</td>
<td>1.128</td>
<td>2.775 $\times 10^{-7}$</td>
<td>43.58</td>
<td>88.78</td>
<td>88.10</td>
<td>0.971</td>
</tr>
<tr>
<td>$1.0 \times 10^{-5}$</td>
<td>1.172</td>
<td>1.452 $\times 10^{-7}$</td>
<td>42.22</td>
<td>95.69</td>
<td>94.62</td>
<td>0.961</td>
</tr>
<tr>
<td>$9.0 \times 10^{-6}$</td>
<td>1.214</td>
<td>5.479 $\times 10^{-8}$</td>
<td>42.99</td>
<td>118.16</td>
<td>115.96</td>
<td>0.942</td>
</tr>
<tr>
<td>$8.0 \times 10^{-6}$</td>
<td>1.247</td>
<td>1.924 $\times 10^{-8}$</td>
<td>44.22</td>
<td>151.67</td>
<td>146.71</td>
<td>0.910</td>
</tr>
<tr>
<td>$7.0 \times 10^{-6}$</td>
<td>1.286</td>
<td>5.868 $\times 10^{-9}$</td>
<td>45.10</td>
<td>200.35</td>
<td>187.45</td>
<td>0.841</td>
</tr>
<tr>
<td>$6.0 \times 10^{-6}$</td>
<td>1.331</td>
<td>1.537 $\times 10^{-9}$</td>
<td>45.77</td>
<td>279.97</td>
<td>239.64</td>
<td>0.686</td>
</tr>
<tr>
<td>$5.0 \times 10^{-6}$</td>
<td>1.393</td>
<td>2.922 $\times 10^{-10}$</td>
<td>45.84</td>
<td>430.36</td>
<td>278.34</td>
<td>0.366</td>
</tr>
<tr>
<td>$4.0 \times 10^{-6}$</td>
<td>1.459</td>
<td>6.450 $\times 10^{-11}$</td>
<td>45.00</td>
<td>571.38</td>
<td>239.59</td>
<td>0.137</td>
</tr>
<tr>
<td>$3.0 \times 10^{-6}$</td>
<td>1.534</td>
<td>1.348 $\times 10^{-11}$</td>
<td>44.01</td>
<td>835.27</td>
<td>205.26</td>
<td>0.042</td>
</tr>
</tbody>
</table>

* Derived from momentum-coupled wind solutions.

Jura 1986; Cuntz 1990). Specifically, for $R_* \leq r \leq r_0$, the gas density distribution is obtained by solving

$$\frac{dp}{dr} = -\frac{\rho GM_*}{r^2} \left[ 1 - \gamma \exp \left( \frac{r - R_*}{L} \right) \right],$$

(11)

where $p = \rho c_s^2$, and $\gamma$ and $L$ are specified constants. On the right-hand side of equation (11), the term in addition to the gravitational acceleration is intended to simulate the supporting force provided by an outwardly propagating mechanical energy flux. Such a flux may plausibly be present in red giant atmospheres, in the form of acoustic waves or shocks generated by pulsation or convection in subphotospheric layers.

For $r \geq r_0$, we set $\gamma = 0$ so that radiatively accelerated dust grains are the sole source of outward momentum of the gas flow. We then solve for $u(r)$ and $a(r)$ from equations (1) and (4). This is done using a simple shooting method to continuously refine an initial guess for $u(r)$ until a solution is found which passes smoothly through the sonic critical point apparent in equation (1). Under the assumption of complete momentum coupling, the grain velocity is determined by evaluating the right-hand side of equation (6) to zero, selecting one of equations (8) or (9) to specify $f_{\text{drag}}$ or solving for $u_{\infty}$. The values so derived are then used to solve for the force factor $f_{\text{drag}}$ in the gas equation of motion (1) (see eqs. [2] and [3]). Once a full wind solution is obtained, the validity of the perfect coupling assumption is tested by solving the grain equation of motion including inertia. That is, we use the distributions of $a$, $u$, $\rho$, and $T$ from the wind model to fix the local values of these quantities and integrate equation (6) to determine $u(r)$. Comparison of these results with the grain velocity distribution derived in conjunction with the wind calculation then enables us to determine whether the supposition of complete momentum coupling is justified.

3. RESULTS

We have used the model described in the preceding section to study the interaction between gas and dust in the circumstellar envelope of a star having $M_* = 2 M_\odot$, $R_* = 200 R_\odot$, and $T_{\text{eff}} = 3000$ K ($L_* \approx 2900 L_\odot$). In the results presented below, we have set $T = T_{\text{eff}}$ throughout the atmosphere, and adopted the values of $\xi_{\text{cog}}$ and $\xi_{\text{dust}}$ for silicate grains computed by Gilman (1974). The structure of the region $R_* \leq r \leq r_0$ interior to the condensation radius was determined from equation (11) with $\gamma = 0.95$ and $L = R_*$. For an assumed photospheric number density $N(R_*) = 7 \times 10^{13}$ cm$^{-3}$, condensation of silicate grains with sizes $a_0 \lesssim 1.5 \times 10^{-5}$ cm occurs at $r_0 = 1.647 R_*$, at which location $N(r_0) = 1.674 \times 10^{10}$ cm$^{-3}$. The following values were assigned to parameters governing grain formation and growth: $\gamma = 10^{-4}$, $m_0 = 28m_{\text{H}}$, $\rho_{\text{eff}} = 3$ g cm$^{-3}$, $f_{\odot} = 0.5$, $q_0 = 1$. Results corresponding to the use of equation (8) in computing the collisional drag force are summarized in the accompanying table and figures.

In Table 1, we enumerate some of the physical properties of wind solutions obtained for initial grain radii in the range $3 \times 10^{-6} \leq a_0 \leq 1.5 \times 10^{-5}$ cm. Tabulated quantities bearing the subscript "\infty" are evaluated at $r = 100 r_0$. Note that the sonic critical point $(r_0/r_*)$ is located closer to the condensation radius for larger grains, and that the mass loss rate $M$ is a sharply increasing function of increasing $a_0$ (see also Fig. 1). In fact, a factor of 5 increase in $a_0$ leads to an increase in $M$ of

![Figure 1](https://example.com/fig1.png)
almost five orders of magnitude. Much of this behavior is attributed to the magnitude and variation of the force factor $\Gamma$ (cf. eq. [2]) near the base of the flow. For $u(r_0)/c_s \ll 1$, the gas momentum equation (1) can be formally integrated to yield

$$\frac{u(r_0)}{c_s} \approx \left( \frac{r_0}{r} \right)^2 \exp \left[ \frac{\epsilon(r_0)}{c_s^2} - 1 - \frac{GM_*}{2 c_s^2 r_0} \left( 1 - \frac{r_0}{r} \right) \right],$$

where

$$\epsilon(r) = \int_{r_0}^r dr' \frac{GM_*}{r'^2} \Gamma(r')$$

is the work done on the gas by radiatively accelerated dust grains over the interval $r_0 \leq r \leq r$. For all of the solutions listed in Table 1, $\Gamma(r) \approx 1$; however, considerable variation exists in the value of $\Gamma(r_0)$. By virtue of the assumptions made concerning grain condensation at $r_0$, for the fixed amount of grain forming materials available there, more small grains can be made than large ones [$N_{d}(r_0) \propto a_0^{-3}$, other things being equal]. Nevertheless, because $\dot{Q}_{sp}$ is more than a factor of $10^{2}$ larger for $a_0 = 1.5 \times 10^{-5}$ cm than for $a_0 = 3 \times 10^{-6}$ cm, this effect is more than compensated for by the enhanced efficiency with which large grains interact with the radiation field. The net result is that $\Gamma(r_0)$, and hence the value of the integral in equation (13), is significantly less for small grains than for large ones. For the solutions under consideration, $\Gamma(r_0) = 0.024$ and $u(r_0)/c_s = 9.24 \times 10^{-6}$ for $a_0 = 3 \times 10^{-6}$ cm, while $\Gamma(r_0) = 0.797$ and $u(r_0)/c_s = 0.756$ for $a_0 = 1.5 \times 10^{-5}$ cm. It is this dependence of $u(r_0)$ on $a_0$ which is responsible for the variation in $\dot{M}_{d} = 4\pi r_0^2 \rho_{d}(r_0) u(r_0)$ seen in Figure 1.

Solutions obtained under the assumption of complete momentum coupling have the property that the quantities $u$, $u_{g}$, $u_{gr}$, and $\Gamma$ all approach constant values for $r \gg r_0$. The asymptotic gas flow speed $u_{g}$ (cf. Table 1) is largely determined by the value $\Gamma_\infty$ assumed by the force factor at radii greater than the sonic radius. For grains with the largest ($a_0 \geq 10^{-5}$ cm) and smallest ($a_0 \sim 5 \times 10^{-6}$ cm) initial sizes, $\Gamma_\infty$ (and hence $u_{g}$) increases with increasing $a_0$. From Table 1, it is also evident that for grains of intermediate initial size ($5 \times 10^{-6} \leq a_0 < 10^{-5}$ cm), $u_{g}$ decreases with increasing $a_0$. This dependence is a consequence of the fact that for $a_0$ in this initial size range, $\Gamma_\infty$ is a decreasing function of $a_0$. Such behavior arises because the rate of grain growth is lower for larger grains. Because $u_{gr}(r_0)$ increases with increasing initial grain radius, $\dot{m}_{d}(r_0)/\dot{m}_{d}(r_\infty)$ (i.e., eq. [4]) decreases; for $r \geq r_0$, larger grains undergo less of a fractional increase in size, and as a result, there is less of an increase in $\Gamma$. For example, at $r = 100 r_0$ grains with $a_0 = 7 \times 10^{-6}$ cm have increased in size by a factor $a/a_0 = 1.22$ and collectively exert a force on the gas of magnitude $\Gamma_\infty = 2.41$. At the same position in a flow driven by grains having $a_0 = 8 \times 10^{-6}$ cm, $a/a_0 = 1.20$, and $\Gamma_\infty = 2.28$. Note that for the largest grains ($a_0 > 10^{-5}$ cm), this trend is reversed because of the greater intrinsic magnitude of the product $a^2 \dot{Q}_{sp}$ (cf. eq. [2]) for grains of this size.

The last three columns of Table 1 contain information pertaining to the relative motion and degree of coupling of dust and gas. In columns (5) and (6), we compare, respectively, the asymptotic grain velocities obtained from either the assumption of complete momentum coupling, or integration of the grain equation of motion including inertia (see also Fig. 2). In all cases the velocities with which grains move relative to the background gas are highly supersonic ($c_s \approx 5$ km s$^{-1}$). Nevertheless, for larger grains the close agreement between values of $u_{gr}$, $u_{gr}$ (momentum-coupled) $u_{gr}$, $u_{gr}$ (inertia-coupled) and $u_{g}$ is derived by alternative means indicates that such particles are dynamically coupled to the background flow. Further evidence that most of the radiative momentum acquired by large grains is collisionally imparted to the gas can be seen in the last column of Table 1. There we have tabulated the asymptotic value of the quantity ($\dot{f}_{d}(r)/\dot{f}_{d}(\infty)$) as computed during integration of the grain equation of motion. Since $\dot{N}_{d}/\dot{f}_{d}$ is the force per unit volume acting on the gas due to grain-gas collisions, ($\dot{f}_{d}(r)/\dot{f}_{d}(\infty)$) is the fraction of the photon momentum absorbed by grains which is collisionally transferred to the gas (see, e.g., Gilman 1972). When the drag and radiative forces acting on a grain are closely balanced and complete momentum coupling obtains, ($\dot{f}_{d}(r)/\dot{f}_{d}(\infty)$) $\approx 1$; inspection of column (7) in Table 1 reveals that such a description rather accurately portrays the dynamical situation which prevails for $a_0 \geq 8 \times 10^{-6}$ cm. However, further examination of the table discloses that ($\dot{f}_{d}(r)/\dot{f}_{d}(\infty)$) tends toward zero for decreasing $a_0$ in the range $3 \times 10^{-6} \leq a_0 \leq 8 \times 10^{-6}$ cm. Such behavior is indicative of a dynamical state in which the grain motion is completely uncoupled from that of the background gas. In this case, the dominant terms in the equation of motion of a single grain are those describing its inertia (i.e., the left-hand side of eq. [6]) and radiative acceleration. By comparison, $\dot{f}_{d}$ is vanishingly small, and little (if any) of the photon momentum gained by the grain is given via collisions to the gas. We conclude that the solutions obtained for $a_0 \leq 8 \times 10^{-6}$ cm are inconsistent, in the sense that because the actual degree of grain-gas collisional coupling is significantly less than assumed, the force factor $\Gamma$ has been overestimated.

That the forces acting on a collisionally coupled or a completely uncoupled grain balance in different ways is also evident in Figures 3 and 4. In Figure 3a, we show the gas and grain velocities as functions of distance from the condensation radius for a momentum-coupled outflow with $a_0 = 1.5 \times 10^{-3}$ cm. Also shown is the grain velocity distribution by solving the equation of motion (6). The indistinguishability of the grain velocity profiles suggests that the assumption of complete momentum coupling is vindicated for this solution. Further
the grain equation of motion are smaller than those calculated assuming perfect coupling. Because of grain growth and its effect on \( Q_{\text{gr}} \), the radiative force acting on the dust initially falls off somewhat more slowly than \( r^{-2} \). The dependence of the drag force on gas mass density implies that in order to overcome the faster than \( r^{-2} \) decrease in \( \rho \) and still satisfy \( f_{\text{rad}} = f_{\text{drag}} \), \( u_{\text{gr}} \) (and hence the function \( F \); i.e., eq. [7]) must increase. The drift velocities obtained under the assumption of complete coupling therefore tend to be larger than those which result from solving equation (6). Moreover, we note that in the latter equation, the radiative and drag force terms each have the same asymptotic spatial dependence. At large distances (\( r \gg r_{0} \)), \( a, u, \) and \( u_{\text{gr}} \) all approach constant values, and both forces decrease outward as \( r^{-2} \). Hence, the force ratio \( (f_{\text{drag}}/f_{\text{rad}}) \)

substantiation of this conclusion is given in Figure 3b, in which we depict the grain force balance derived from integration of equation (6). At all radii, the radiative and drag forces are virtually identical, and the magnitude of each individually is much greater than that of the grain inertia \( f_{\text{in}} \approx m_{\text{gr}} u_{\text{gr}} du_{\text{gr}}/dr \). Such is not the case for small grains. As is apparent from Figures 4a and 4b, for \( a_{0} = 3 \times 10^{-6} \) cm, significant differences exist between the respective distributions of \( u_{\text{gr}} \). For this solution, \( f_{\text{drag}} \approx f_{\text{in}} \) over most of the flow, and the assumption of complete momentum coupling is vitiated.

It is apparent both from the table and Figures 2 and 4 that, where different, the values of \( u_{\text{gr}} \) derived from integration of

![Figure 3](image)

**Fig. 3.**—(a) Gas \( (u) \) and grain \( (u_{\text{gr}} \) (momentum-coupled)] velocities as functions of \( (r/r_{0}) \) for the wind solution with \( a_{0} = 1.5 \times 10^{-5} \) cm. Also depicted is the grain velocity \( u_{\text{gr}} \) derived from integration of eq. (6), as described in the text. (b) The magnitudes of the radiative (\( f_{\text{rad}} \)), collisional drag \( f_{\text{drag}} \), and inertial \( f_{\text{in}} \) terms in the equation of motion (6) as functions of \( (r/r_{0}) \) for the grain velocity distribution \( u_{\text{gr}} \) of (a).

![Figure 4](image)

**Fig. 4.**—(a) Gas and grain velocity distributions for the momentum-coupled wind solution with \( a_{0} = 3 \times 10^{-6} \) cm, as in Fig. 3a. (b) Radial profiles of \( f_{\text{rad}}/f_{\text{drag}} \) and \( f_{\text{in}} \) for \( a_{0} = 3 \times 10^{-6} \) cm, as in Fig. 3b.
attains a fixed value, suggesting that if the grains have not decoupled from the gas prior to the onset of such steady streaming, they will remain forever momentum-coupled. Alternatively, if the grains are to become decoupled from the background gas, they must do so near the base of the flow where the subsonic rate of expansion leads to a gas density distribution $\rho$ which decreases outward much more rapidly than $r^{-2}$. This is, in fact, what occurs in the last solution listed in Table 1; for $a_0 = 3 \times 10^{-6}$ cm, $(f_{\text{drag}}/f_{\text{rad}})$ decreases from near unity for $r = r_0$ to a value $\approx 0.06$ for $r \approx 2r_0$.

Therefore, the question concerning the validity of the complete momentum coupling hypothesis posed at the outset of this study is equivalent to asking whether or not radiatively accelerated dust grains can drive a wind at all. From the results described above, we conclude that for small grains ($a_0 \approx 5 \times 10^{-6}$ cm), the collisional momentum transfer rate is insufficient to induce expansion of the atmosphere as a whole.

4. DISCUSSION

We have investigated the consistency of dust-driven wind solutions by comparing the results of calculations in which the gas and dust components are assumed to be well coupled collisionally with an independent integration of the dust grain equation of motion including inertia. For the case of a high-luminosity, red giant-like model atmosphere, we find that (1) the resulting mass loss rate is a sharply increasing function of dust grain size, and (2) the assumption of complete momentum coupling appears to break down for grain sizes smaller than $\sim 8 \times 10^{-6}$ cm, suggesting that smaller grains may not be sufficiently coupled to the gas to drive the stellar wind. Our results suggest that if the grains decouple from the gaseous wind in which they are immersed, they do so in the immediate vicinity of the condensation radius $r_0$. An estimate of the degree to which grains and gas are coupled by collisions can be obtained by comparing the magnitudes of the inertial and drag terms in the equation of motion (6). For grain velocities which are supersonic with respect to the gas near the base of the flow, dimensional analysis yields the following expression for the ratio of these forces

$$f_{\text{in}} \sim \frac{m_g}{\pi a^2 pr} \sim \frac{\xi}{\pi a^2 N_{\text{gr}} r},$$

where $\xi \equiv (m_g N_{\text{gr}}/\rho)$ is the ratio of dust-to-gas mass densities. For $r \approx r_0$, grain and gas motions are collisionally coupled (uncoupled) according to whether the quantity $(f_{\text{in}}/f_{\text{drag}})$ given by equation (14) is greater than (less than) 1.

Note that our results differ from those of Berruyer & Frisch (1983), who concluded that the decoupling of grains from gas would occur only asymptotically (i.e., for $r > 10^3 r_0$). Their calculations describe the motion of large ($a_0 = 10^{-5}$ cm) grains of constant size, and start from a reference radius $r_0(=800 R_\odot)$ where the gas number density is assumed to be significantly greater than the values generally attributed to the outer layers of a red giant atmosphere. Apart from these differences, we suspect that the asymptotic decoupling found by Berruyer & Frisch (1983) stems from the gradual decrease in $f_{\text{drag}}(\propto r)$ associated with the continued thermal acceleration of the flow at large distances. This latter behavior is a consequence of the assumed isothermality of the flow and would not obtain with a more realistic description of the wind energy balance.

These results, while interesting, are based on a number of approximations. To improve the range of validity, several improvements should be investigated, including: (1) a better model for grain formation, growth, and sputtering; (2) more accurate values for the Planck mean radiation pressure efficiency, factor, $Q_{\text{rad}}(T_{\text{eff}}, \alpha)$ for silicate grains; (3) inclusion of a spectrum of grain sizes; (4) an improved, physical description of the forces resulting in extension of the hydrostatic portion of the model atmosphere; (5) the effects of different grain characteristics (e.g., density and shape); and (6) solving for consistent flows in cases where the grains appear to only partially couple to the gas. Ideally, these improvements will allow for the application of radiative transfer methods to then synthesize spectral features resulting from the gas and dust, in order to predict Doppler drift signatures.

Among the interesting problems raised if these initial results are borne out are (1) if small grains are selectively expelled from red giant atmospheres, where are the seed nuclei for the larger grains required for dust-driven winds? (2) Will a prolonged period of small particle escape promote chemical differentiation of an atmosphere? (3) If sputtering is effective in reducing grain size, resulting in small particle escape, how can dust-driven winds be sustained? and (4) What is the behavior of small grains that approach molecular sizes, given the prediction of Elitzur, Brown, & Johnson (1989) that radiation pressure on molecules can initiate stellar winds? These questions motivate the suggested improvements outlined above. In any case, the large drift velocities which are characteristic of the wind models discussed in § 3 may be detectable through observations of gas (e.g., molecular lines) and dust (e.g., band structure features) at sufficiently high resolution. In addition, a substantial drift velocity may, over time, lead to spatially resolvable differences in the structure of the gas and dust components of the circumstellar envelope.

We acknowledge useful discussions with Sun Kwok, Philip Judge, and Paul Charbonneau. We are also grateful to an anonymous referee for a careful review of the manuscript. Partial support for this effort came from NASA grant NAGS-1214 to the University of Colorado.

APPENDIX

COULOMB DRAG FORCE ON CHARGED GRAINS

The model described in § 2 does not include treatment of the Coulomb drag experienced by charged grains moving through a partially ionized gas. In the present Appendix, we argue that the magnitude of this force is probably small in comparison with direct collisional drag, under physical conditions of the type thought to prevail in the circumstellar envelopes of those objects most likely to have significant amounts of dust.

Tabak et al. (1975) have noted that given the weak ultraviolet flux from evolved stars with $T_{\text{eff}} \leq 3000$ K, grains acquire charge by accreting charged constituents of the ambient atmospheric gas rather than by the emission of photo electrons. Because the electron
thermal speed is greater than that of the more massive ions, in a steady-state the grains become negatively charged. The magnitude of this charge can be calculated using, for example, the approach described by Spitzer (1978; see also Draine & Salpeter 1979). The Coulomb drag force is then derived by treating a single charged grain as a test particle moving through a Maxwellian plasma, and using the Fokker-Planck equation to compute its deceleration (see, e.g., Boyd & Sanderson 1969). The retardation of the grain's motion is the cumulative result of the interactions between it and the other charged particles contained within a sphere with radius equal to the Debye length, centered on the instantaneous position of the grain. As discussed by Tabak et al. (1975), because of the low degree of ionization of the gas in the envelopes of stars cool enough to have appreciable circumstellar dust, the drag produced by Coulomb interactions with charged particles is small in comparison to the force arising from direct collisions of the more numerous neutral particles with the grain.

To verify this conclusion, we have used the prescription given above to calculate the Coulomb drag force as a function of grain size and relative velocity. Consider a grain with charge $Z_e e$ ($e$ is the electronic charge) moving at velocity $v_{\text{rel}}$ through a uniform gas containing a number density $N_i$ of ions, each of which has (positive) charge $Z_i e$. If $v_{\text{th},i} = (2kT/m_i)^{1/2}$ is the ion thermal speed, the drag force is

$$f_{\text{Coul}} = \frac{2 \pi N_i Z_i^2 Z_e^2 e^4}{kT} \ln \Lambda \left( \frac{v_{\text{rel}}}{v_{\text{th},i}} \right),$$

(A1)

where it has been assumed that $m_e \gg m_i$. In equation (A1)

$$\Lambda = \frac{3}{2Z_i Z_e e^3} \left( \frac{k^3 T^3}{\pi N_e} \right)^{1/2},$$

(A2)

where $N_e$ is the electron number density, and the function $G$ is given by

$$G(x) = \Phi(x) - \frac{2}{\sqrt{\pi}} e^{-x^2},$$

(A3)

where $\Phi(x)$ is the error function. In order to determine $Z_{\text{gr}}$ and evaluate $f_{\text{Coul}}$, we have assumed: (i) conditions of density and temperature like those at the condensation radius in our models ($N = 1.67 \times 10^{10} \text{ cm}^{-3}$, $T = 3000 \text{ K}$); (ii) a gas ionization fraction $f_{\text{ion}}$ in accord with the values inferred from observations of late M giants ($f_{\text{ion}} = 10^{-4}$; Drake, Linsky, & Elitzur 1987); and (iii) an ionic mass and charge consistent with the single-ionization of heavier metallic species ($m_i = 40m_e$, $Z_i = 1$) implied by the assumed $f_{\text{ion}}$. Following Spitzer (1978), we obtain $Z_{\text{gr}} \approx -3.994(aT/e^2)$, where $a$ is the grain radius. Utilizing this value in equations (A1) and (A2) we find that for $3 \times 10^{-6} \leq a \leq 1.5 \times 10^{-5} \text{ cm}$, Coulomb drag is of secondary dynamical importance relative to collisional drag. An example is given in Figure 5, in which the respective drag forces are shown as functions of relative velocity for a grain with $a = 3 \times 10^{-6} \text{ cm}$. At the lowest drift speeds, both forces increase linearly with $v_{\text{rel}}$; for the particular case depicted in Figure 5, $f_{\text{Coul}}/f_{\text{drag}} \approx 5.67 \times 10^{-2}$ when $u_{\text{rel}} < u_{\text{th},i}$. For large drift speeds, $f_{\text{drag}}$ increases as $u_{\text{rel}}^2$ while $f_{\text{Coul}}$ decreases as $u_{\text{rel}}^2$. Hence, we conclude that Coulomb interactions are unlikely to enhance the coupling of small grains to the largely neutral gas contained within the circumstellar envelopes of the coolest red giants.
REFERENCES

Spitzer, L. 1978, Physical Processes in the Interstellar Medium (New York: Wiley), 198