ON POTENTIAL FIELD MODELS OF THE SOLAR CORONA

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ABSTRACT

The current-free approximation has been widely used to infer the magnetic structure of the solar corona from magnetograph observations. In such calculations, the potential field is usually matched directly to the observed line-of-sight component of the photospheric field. However, this procedure is invalid because the magnetograph measurements apply to deep atmospheric layers where the magnetic field is nonpotential and (for observations made in the Fe I λ5250 line) nearly radial. A better approach is to match only the radial component of the potential field to the photospheric data (corrected for line-of-sight projection on the assumption that the field lines are radially oriented), while allowing a discontinuity in the tangential field components. The implied current sheet is a mathematical idealization of the finite "boundary layer" within which the rapidly diverging field lines make their transition to a potential configuration. This procedure yields much stronger polar fields than the conventional line-of-sight method and resolves a number of discrepancies between earlier potential field calculations and observations of polar coronal holes, the brightness distribution of the K-corona, and the shape of the interplanetary current sheet.

Subject headings: interplanetary medium — Sun: corona — Sun: magnetic fields

1. INTRODUCTION

The potential field approximation has often been used to deduce the magnetic structure of the solar corona from measurements of the photospheric field. In a widely adopted model originally proposed by Schatten, Wilcox, & Ness (1969), electric currents are neglected between the photosphere and a spherical "source surface" where the field lines are constrained to be radial, simulating the magnetohydrodynamic effect of the solar wind. At the inner boundary, it has been conventional to match the potential field expansion directly to the line-of-sight component of the photospheric field, a procedure advocated by Alscher & Newkirk (1969) as making the best use of the available magnetic data. An implicit assumption here is that the current-free approximation is applicable at the depth where the photospheric field is measured.

The purpose of this study is to show that the line-of-sight matching procedure does not in fact make good use of the available data, because there is strong evidence that the magnetic field is nearly radial, and therefore nonpotential, at the photosphere. We advocate instead that the observed photospheric field first be corrected for line-of-sight projection and then matched to the radial component of the potential field. We show that this procedure yields much stronger polar fields than the standard method and produces better agreement with high-latitude coronal holes and with white-light structures in the outer corona. We also discuss the relationship of both methods to the observed inclination angles of polar plumes. We begin by reviewing the evidence for a nonpotential, nearly radial photospheric field.

2. EVIDENCE FOR A RADIAIIY ORIENTED PHOTOSPHERIC FIELD

The average inclination of photospheric field lines can be deduced by a number of observational methods. Svalgaard, Duvall, & Scherrer (1978) analyzed magnetograph data taken in the Fe I λ5250 line at the Wilcox Solar Observatory. They tracked the line-of-sight field as it rotated across the disk and determined that it varied as \( \cos \rho \), where \( \rho \) is the angle made by the local radius vector to the line of sight: this is the result expected if the field lines are radial. In addition, they were able to infer the polar field distribution from the annual modulation (due to the Sun's 7°25 axial tilt) of the flux recorded in the polemost apertures of their instrument. They concluded that, poleward of 55° latitude, the field was nearly radial and of the form 11.5 G \( \cos^2 \theta \) (where \( \theta \) is colatitude) during 1976–1977.

The azimuthal component of the photospheric field can also be deduced by comparing the magnetic flux of a region when it is located at the same distance east and west of central meridian. Using this technique, Howard (1974, 1991) found that the east-west inclination angle of field lines of a given polarity was generally less than 10°. Similar values were inferred by Topka, Tarbell, & Title (1992) from the center-to-limb increase in the continuum contrast of facular areas, on the assumption that the variation is solely a function of the angle between the photospheric flux tubes and the line of sight.

The above results were all obtained with relatively weak absorption lines such as Fe I λ5250, which is formed below the solar temperature minimum (see Lites 1973). Magnetic fields recorded using spectral lines originating higher in the atmosphere show a more complex center-to-limb behavior. In daily magnetograms taken in lines such as Fe I λ8688 (now used routinely at the National Solar Observatory/Kitt Peak), individual network elements that are unipolar near disk center appear to become bipolar toward the limb, with the "false" polarity on the limbward side. This fringing effect, which is also observed in sunspots, suggests that the field lines have begun to fan out horizontally at these heights (Chapman & Sheeley 1968; Pope & Mosher 1975). According to Giovanelli (1980; see also Jones 1985), the magnetic field expands to form "canopies" over the supergranule interiors in a distance of only several hundred kilometers above the photosphere.

At the photospheric level, the magnetic field is confined to the network boundaries by the ram pressure of the supergranular flow (see, e.g., Parker 1963; Sheeley, Wang, & DeVore...
The vertical orientation of the field lines can be attributed to the effect of magnetic buoyancy; the upper portion of a flux loop rises faster than its base, causing the legs of the loop to straighten out (see Zwaan 1987). Above the photosphere, the rapid decrease in the density and in the ratio of gas to magnetic pressure allows the magnetic field to relax to a more nearly potential configuration.

3. THE INNER BOUNDARY CONDITION

In this paper, we employ spherical coordinates \((r, \theta, \phi)\), where \(r\) is defined with respect to the Sun's center (so that \(r = R_{\odot}\) at the solar surface), colatitude \(\theta\) is measured from the north pole, and Carrington longitude \(\phi\) increases westward in the direction of the solar rotation. We adopt the "source surface" model of Schatten et al. (1969), according to which the magnetic field \(\mathbf{B}(r, \theta, \phi)\) satisfies the current-free condition \(\nabla \times \mathbf{B} = 0\) within the spherical shell \(R_{\odot} < r < R_{s}\). At the "source surface" \(r = R_{s}\), the tangential components of \(\mathbf{B}\) are constrained to vanish; the implied electric currents are assumed to be present in the region \(r \geq R_{s}\). The inner boundary condition at \(r = R_{\odot}\) makes use of the observed photospheric field, as will be discussed in detail below. We shall henceforth assume that the observations refer to a depth where the field is nearly radial, and that the line-of-sight component, \(B_{\ell o}(\theta, \phi)\), is measured near central meridian, so that \(B_{\ell o} = 0\). Both of these requirements are fulfilled by the Wilcox Solar Observatory and Mount Wilson Observatory monthly synoptic data, which employ the Fe \(\ell 15250\) line and are assembled from daily full-disk magnetogramsweighted around central meridian.

The condition usually imposed at \(r = R_{\odot}\) is continuity between the line-of-sight components of the potential field \(\mathbf{B}\) and the photospheric field \(\mathbf{B}_{\text{phot}}\): thus

\[
B_{\ell r}(R_{\odot}, \theta, \phi) \sin \theta + B_{\ell \theta}(R_{\odot}, \theta, \phi) \cos \theta = B_{\ell o}(\theta, \phi) = B_{\ell o}^{\text{phot}}(\theta, \phi) \sin \theta + B_{\ell \phi}^{\text{phot}}(\theta, \phi) \cos \theta
\]  

(1)

(see, e.g., Altschuler & Newkirk 1969). Since flux conservation requires the radial field components to be continuous at the photosphere \(B_{r} = B_{r}^{\text{phot}}\), the matching condition (1) implies that the \(\theta\)-components are also continuous \(B_{\theta} = B_{\theta}^{\text{phot}}\), in which case no surface currents are present. Thus the photospheric field \(\mathbf{B}_{\text{phot}}\) is constrained by equation (1) to behave like the potential field \(\mathbf{B}\). In particular, since the potential field components \(B_{r}, B_{\theta},\) and \(B_{\phi}\) are generally comparable to each other in magnitude, \(B_{r}^{\text{phot}}\) and \(B_{\theta}^{\text{phot}}\) cannot be neglected in comparison with \(B_{\phi}^{\text{phot}}\). This result is contradicted by the empirical evidence that the photospheric field is nearly radial, and may be regarded as an attempt to make the tail (the current-free extrapolation) wag the dog (the observed photospheric field).

The boundary condition that is consistent with the radial orientation of the photospheric field is the continuity of the radial field components alone:

\[
B_{r}(R_{\odot}, \theta, \phi) = B_{r}^{\text{phot}}(\theta, \phi) = B_{r}^{\text{phot}}(\theta, \phi)/\sin \theta
\]  

(2)

In general, this condition will give rise to a discontinuity in the tangential components of the field, with \(B_{\theta}^{\text{phot}} = B_{\phi}^{\text{phot}} = 0\) on the "photospheric" side of the boundary but \(B_{\theta}\) and \(B_{\phi}\) comparable in magnitude to \(B_{r}\) on the "coronal" side. The implied surface currents reflect the nonpotential nature of the field near the photosphere. In reality, the currents will be spread over a boundary layer extending some height above the photosphere, within which the magnetic field makes a rapid but continuous transition from radial and nonpotential to nonradial and potential.

It is well known that magnetic elements at the photosphere are bunched along the boundaries of supergranules. As noted in § 2, magnetograms taken in spectral lines originating relatively high in the atmosphere show a polarity-doubling effect toward the limb, suggesting that the magnetic field fans out rapidly with height. At distances less than or on the order of 1000 km above the photosphere, the field lines from opposite sides of the supergranules meet and are deflected into a more vertical direction; thereafter, the flux distribution changes relatively slowly with height (see Giovanelli 1980). Thus we can expect the radial matching condition (2) to be valid provided the photospheric field is averaged over an area at least comparable to that of a supergranule. On the other hand, it is clear that the potential field solution must break down very close to the photosphere.

The use of the boundary condition (2) in place of (1) simplifies the computation of the potential field. Subject also to the vanishing of \(B_{\phi}\) and \(B_{\theta}\) at the source surface \(r = R_{s}\), the solution may be expressed in terms of spherical harmonics as

\[
B_{r}(r, \theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} r^{l+1} Y^{l}_{m}(\theta, \phi)
\]  

(3a)

\[
B_{\theta}(r, \theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} c_{lm} \frac{r^{l+1}}{\sin \theta} \frac{\partial Y^{l}_{m}(\theta, \phi)}{\partial \theta}
\]  

(3b)

\[
B_{\phi}(r, \theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} d_{lm} r^{l+1} \sin \theta Y^{l}_{m}(\theta, \phi)
\]  

(3c)

where

\[
a_{lm} = \int_{0}^{2\pi} \int_{0}^{\pi} d\phi \int_{0}^{r} dr \frac{\partial B_{\ell o}^{\text{phot}}(\theta, \phi)}{\partial \theta} Y^{l}_{m}(\theta, \phi)
\]  

(4)

\[
c_{lm} = \frac{R_{s}^{l+2}}{r} \left[ I_{l+1} + (l/r) R_{s}^{2l+1} I_{l+1} + (l/r) R_{s}^{2l+1} I_{l+1} \right]
\]  

(5)

\[
d_{lm} = \left[ \frac{R_{s}^{l+2}}{r} \left[ 1 - (l/r) R_{s}^{2l+1} I_{l+1} + (l/r) R_{s}^{2l+1} I_{l+1} \right] \right]
\]  

(6)

(The asterisk in eq. [4] denotes a complex conjugate.)

If the line-of-sight matching condition (1) is used instead of equation (2), the solution may be expressed in a form similar to equation (3), with \(c_{lm}\) and \(d_{lm}\) again given by equations (5) and (6). However, the coefficients \(a_{lm}\) must now be determined by substituting the spherical harmonic expansions for \(B_{r}\) and \(B_{\theta}\) into equation (1) and inverting the resulting matrix equation (see Altschuler et al. 1977; other techniques have been described by Adams & Pneuman 1976 and Riesebieter & Neubauer 1979).

In the remainder of this paper, we employ both potential field methods to deduce the properties of the Sun's polar fields, the distribution of open magnetic regions, and the shape of the coronal neutral line. The predictions are compared with observations of polar plumes, coronal holes, and the intensity distribution of the K-corona.

In computing the potential fields by either method, we truncate the spherical harmonic expansions above multipole \(l = 17\). We also set \(R_{s} = 2.35 R_{\odot}\), a value that Hoeksema, Wilcox, & Scherrer (1982) derived by modeling the interplanetary sector structure during 1976–1977. For the magnetic...
data we employ monthly synoptic maps from the Wilcox Solar Observatory (WSO), which have a resolution of 72 pixels in longitude by 30 pixels in sine latitude. We note that the size of each pixel considerably exceeds the ~2.5 diameter of a supergranule; thus the measured flux will remain conserved to a height where the current-free approximation becomes valid, as required by equation (2). Following Svalgaard et al. (1978), we multiply the measured field strengths by a factor of 1.8 to compensate for the saturation of the Fe i \( \lambda 5250 \) line profile. No attempt will be made to correct for the annual modulation caused by the Sun’s 7.25 axial tilt.

4. THE SUN’S POLAR FIELDS AND THE INCLINATION OF POLAR PLUMES

4.1. Test Cases

Before turning to the magnetic data, it will be instructive to consider some idealized axisymmetric examples. First, let us suppose that the line-of-sight photospheric field is given (in arbitrary dimensionless units) by

\[ B_{\text{los}}(\theta) = \sin \theta \cos \theta . \quad (7) \]

If the field is assumed to be purely radial at the photosphere, then by equation (2)

\[ B_r(R_\odot, \theta) = \cos \theta . \quad (8) \]

On the other hand, if the line-of-sight component of the potential field is matched directly to \( B_{\text{los}}(\theta) \), it is easy to show that

\[ B_r(R_\odot, \theta) = \left( \frac{2 + \epsilon^2}{3} \right) \cos \theta , \quad (9) \]

where \( \epsilon = R/R_\odot \). In this case, \( B_r \) has the same dipole form as found using the radial matching condition, but its strength is reduced by a factor of \( (2 + \epsilon^2)/3 \) \( (= 0.69 \) for \( R = 2.35 R_\odot \)).

It is also interesting to compare the inclinations of the field lines near the solar surface. We define the inclination angle as

\[ \alpha(r, \theta) \equiv \pm \arctan \left[ \frac{B_r(r, \theta)}{B_{\text{los}}(r, \theta)} \right] , \quad (10) \]

which measures the deviation of a field line from the radial direction at an arbitrary point \((r, \theta)\). The angle \( \alpha \) is taken to be positive if the field line is inclined toward the equator in either hemisphere (thus the minus sign in eq. [10] applies for \( \theta > \pi/2 \)). For the input distribution (7), both methods give

\[ \frac{B_r(R_\odot, \theta)}{B_{\text{los}}(R_\odot, \theta)} = \left( 1 - \frac{\epsilon^2}{2} \right) \tan \theta , \quad (11) \]

where \( B_{\text{los}} \) is evaluated just above the tangential discontinuity when the radial matching condition is used. For small \( \theta \) and \( R = 2.35 R_\odot \), we thus find that \( \alpha(R_\odot, \theta) \approx 0.44\theta \).

We now suppose that the line-of-sight field has the more poleward-concentrated form

\[ B_{\text{los}}(\theta) = \pm \sin \theta \cos^8 \theta \quad (12) \]

(where the plus sign is taken for \( \theta < \pi/2 \)). The assumption of a radially oriented photospheric field gives

\[ B_r(R_\odot, \theta) = \pm \cos^8 \theta , \quad (13) \]

similar to the polar field profile deduced observationally by Svalgaard et al. (1978) near the 1976 sunspot minimum. The corresponding result calculated using the line-of-sight method (with \( R = 2.35 R_\odot \)) is shown by the long-dashed curve in Figure 1a: the amplitude of \( B_r(R_\odot, \theta) \) is reduced by a factor of \( \sim 3 \) compared with the purely radial case (solid curve). On the other hand, the shape of the profile derived by the line-of-sight procedure is even more sharply peaked around the pole than a \( \cos^8 \theta \) distribution. Moreover, \( B_r \) reverses its polarity near latitude 45° in this case; at lower latitudes, where \( B_r \) and \( B_{\text{los}} \) have opposite signs, \( B_r \) provides the dominant contribution to the line-of-sight field.

Figure 1b shows the normalized distribution of field-line inclination angles just above the photosphere: radial method (solid curve); line-of-sight method (long dashed curve). The plotted values \( k(R_\odot, \theta) \) are normalized to the angular distance from the pole and are defined to be positive (negative) if the field line is inclined equatorward (poleward) with respect to the radial direction.
TABLE 1

<table>
<thead>
<tr>
<th>n</th>
<th>(B^{\text{los}}(0)/B^{\text{radial}})</th>
<th>(k^{\text{(radial)}})</th>
<th>(k^{\text{(los)}})</th>
</tr>
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<td>0.444</td>
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<tr>
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<td>1.172</td>
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<td>1.435</td>
<td>1.563</td>
</tr>
<tr>
<td>9</td>
<td>0.353</td>
<td>1.645</td>
<td>1.831</td>
</tr>
<tr>
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<td>0.325</td>
<td>1.842</td>
<td>2.073</td>
</tr>
<tr>
<td>13</td>
<td>0.303</td>
<td>2.022</td>
<td>2.296</td>
</tr>
</tbody>
</table>

* For a line-of-sight field distribution of the form \(\sin \theta \cos \theta\), the columns give the poleward concentration index \(n\), the ratio of polar field strengths inferred from the line-of-sight and radial matching procedures, and the normalized field-line inclination angle \(k = \alpha/\theta\) evaluated near \(r = R_{\odot}, \theta = 0\) using each method. Source surface is at \(r = 2.35 R_{\odot}\).

In the latitude range between 40° and the pole, the radial field approach gives an approximately constant \(k(R_{\odot}, \theta)\) with a value of around 1.6. The line-of-sight method yields somewhat larger field-line inclinations, ranging from \(k \approx 1.7\) near the pole to \(k < 0\) and the field lines bend poleward rather than equatorward. Whichever method is used, the high-latitude field lines are much more strongly inclined toward the equator than was the case for the dipole field distribution, for which \(k \approx 0.44\). The large inclination angles derived in the present example reflect the far stronger poleward concentration of the photospheric flux distribution.

Finally, we consider a generalized line-of-sight distribution of the form

\[B^{\text{los}}(\theta) = B_0 \sin \theta \cos^n \theta.\]  

(15)

For odd \(n\) ranging from 1 to 13, Table 1 lists the ratio between the polar field strengths \(B(R_{\odot}, 0)\) implied by the two methods, as well as the respective values of \(k(R_{\odot}, \theta)\) near \(\theta = 0\). (The tabulated values were calculated analytically from the appropriate finite Legendre-polynomial expansions for \(B_0\) and \(B_0\), again taking \(R_s = 2.35 R_{\odot}\)). As \(n\) increases, the disparity between the inferred polar field strengths grows, with the radial field assumption implying \(B(R_{\odot}, 0) = B_0\) but the line-of-sight approach yielding \(B_0(R_{\odot}, 0) \ll B_0\). The latter method gives slightly larger inclination angles because the derived high-latitude field, even though much weaker, is somewhat more sharply peaked around the pole.

4.2. Polar Fields at Sunspot Minimum and Maximum

We now apply the two procedures to the WSO magnetic data. As an example representative of the period near sunspot minimum, we consider Carrington rotation (CR) 1776 (starting date 1986 May 31). Figure 2a shows the inferred radial component of the photospheric field, averaged over longitude and plotted as a function of latitude. The profiles derived by the two methods are similar in that they are both sharply peaked around the poles. However, above latitude \(\sim 30°\), the line-of-sight procedure gives much smaller values of \(B_0\) (by a factor of \(\sim 3\)) than the radial field approach; in the latitude range \(30°-70°\), the inferred values are even smaller than the observed line-of-sight fields. Below \(\sim 30°\), on the other hand, both methods yield \(B_0 \approx B^{\text{los}}\). The essential point is that the radial field assumption (2) results in much more high-latitude flux than does the line-of-sight matching condition (1).

The disparity between the inferred values of \(B_0\) at the photosphere is reflected in the source surface fields (Fig. 2b); the (longitudinally averaged) field strengths calculated on the basis of equation (2) (solid curve) are 3–4 times higher at all latitudes than those obtained using equation (1) (dashed curve). Also shown in Figure 2b is the latitudinal distribution of field-line inclination angles near the solar surface. The angles have been normalized according to definition (14) and are based on longitudinal averages of \(B_0(R_{\odot}, 0, \phi)\) and \(B_0(R_{\odot}, 0, \phi)\). At high latitudes, the computed values of \(k(R_{\odot}, \theta)\) are relatively constant, with the radial field approach (triangles) giving \(k \approx 1.7\) around latitude \(60°\) and the line-of-sight approach (pluses) yielding slightly larger values (\(k \approx 2.0\)), corresponding to a...
somewhat more sharply peaked polar field. The behavior at lower latitudes is more erratic; the inclination angles derived by either method fluctuate between positive and negative values.

It is interesting to compare the calculated inclination angles with the orientation of polar plumes, which are observed in white light during solar eclipses and are thought to be aligned along the large-scale magnetic field. Saito (1958, 1965) has compiled measurements of polar plumes during a number of eclipses. He found that the inclination angle $\alpha$ scales linearly with angular distance from the pole (i.e., $k \approx$ constant) down to a latitude of $\sim 60^\circ$, as also noted by Waldmeier (1956). For the three eclipses in his sample which occurred nearest to sunspot minimum (1900, 1954, and 1963), he obtained values of $k$ near the solar limb ranging from $\sim 1.6$ to $\sim 1.9$. (It should be noted that the polar plumes are observed only at heights well above the photosphere proper.) Allowing for the rather large uncertainties and scatter in the measurements, these values are in excellent agreement with the calculated ones, but cannot be used to rule out one method or the other.

Although the calculations are not displayed here, we obtained very similar results for other Carrington rotations near the 1986 minimum. We also found that, whichever potential field method was used, the computed inclination angles decrease rapidly with radial distance (at fixed latitude). This result is consistent with the measurements of Waldmeier (1956) for the 1954 eclipse: he found that $k \approx 1.4$ at $r = 1.1 R_\odot$, but $k \approx 1.05$ at $r = 1.3 R_\odot$. As was already recognized during the 1920s, the large equatorward inclinations of polar plumes imply that the large-scale field near the solar surface is nondipolar. (This result seems to have been forgotten in recent times; see, e.g., Pneuman, Hansen, & Hansen 1978.) Campbell, Moore, & Baker (1923) interpreted the eclipse observations near sunspot minimum in terms of a fictitious bar magnet aligned with the Sun's axis; the length of the magnet had to be taken as $\approx 3/2$ of a solar diameter (corresponding to $k \sim 2$) in order to match the measured polar plume angles. A similar model was adopted by many subsequent investigators (e.g., van de Hulst 1950; Saito 1958, 1965; Nesmyanovich 1963; Waldmeier 1965). From our present perspective, the observed polar-plume inclinations are simply a consequence of the large-scale photospheric field being sharply peaked around the poles near sunspot minimum, more like a $\cos^6 \theta$ than a $\cos \theta$ distribution. The physical mechanism responsible for the formation of such "topknot" polar fields is the meridional surface flow, which continually transports magnetic flux from decaying active regions to the poles and concentrates it there (see DeVore, Sheeley, & Boris 1984; Wang, Nash, & Sheeley 1989; Sheeley, Wang, & DeVore 1989).

Near sunspot maximum, the large-scale field is highly non-axisymmetric and most of the flux is located at relatively low latitudes. Figure 3 compares the distributions of $B_j(R_\odot, \theta, \phi)$ derived by the two methods for CR 1821 (starting date 1989 October 9). Although the maps agree at low latitudes, significant differences in the strength and polarity of the inferred fields are apparent at high latitudes, with the radial method again yielding greater amounts of flux near the poles. (The field-line inclination angles did not show any regular behavior and have not been plotted.)

5. CORONAL HOLES

In this section we use both procedures to locate the footpoint areas of open field lines, which will be assumed to represent coronal holes (see, e.g., Levine et al. 1977; Pneuman et al. 1978; Levine 1982; Wang & Sheeley 1990b). We define as "open" those field lines that reach the source surface at $r = 2.35 R_\odot$.

In order to illustrate conditions near sunspot minimum, we again consider CR 1776. The open field regions inferred using the radial and line-of-sight methods are shown in Figures 4a and 4b, respectively. In constructing each map, we have traced a total of $72 \times 36$ field lines, spaced at uniform 5° intervals in longitude and latitude at the source surface, down to the solar surface. The resulting footpoint areas are indicated by gray or white light during solar eclipses and are thought to be aligned with the orientation of polar plumes, which are observed in white light during solar eclipses and are thought to be aligned along the large-scale magnetic field. Saito (1958, 1965) has compiled measurements of polar plumes during a number of eclipses. He found that the inclination angle $\alpha$ scales linearly with angular distance from the pole (i.e., $k \approx$ constant) down to a latitude of $\sim 60^\circ$, as also noted by Waldmeier (1956). For the three eclipses in his sample which occurred nearest to sunspot minimum (1900, 1954, and 1963), he obtained values of $k$ near the solar limb ranging from $\sim 1.6$ to $\sim 1.9$. (It should be noted that the polar plumes are observed only at heights well above the photosphere proper.) Allowing for the rather large uncertainties and scatter in the measurements, these values are in excellent agreement with the calculated ones, but cannot be used to rule out one method or the other.

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Near sunspot maximum, the large-scale field is highly non-axisymmetric and most of the flux is located at relatively low latitudes. Figure 3 compares the distributions of $B_j(R_\odot, \theta, \phi)$ derived by the two methods for CR 1821 (starting date 1989 October 9). Although the maps agree at low latitudes, significant differences in the strength and polarity of the inferred fields are apparent at high latitudes, with the radial method again yielding greater amounts of flux near the poles. (The field-line inclination angles did not show any regular behavior and have not been plotted.)

5. CORONAL HOLES

In this section we use both procedures to locate the footpoint areas of open field lines, which will be assumed to rep-
Fig. 4.—Distribution of open field regions over the solar surface during CR 1776 (sunspot minimum), as inferred using (a) the radial and (b) the line-of-sight approach. Source surface is at \( r = 2.35 \, R_\odot \). Gray stippling indicates the footpoint areas of positive-polarity open field lines; black stippling indicates the footpoint areas of negative-polarity open field lines. For comparison, (c) displays the corresponding NSO/Kitt Peak synoptic map taken in the He \( \lambda 10830 \) absorption line; here the lightest areas represent coronal holes.

In Figure 6, we have plotted the fraction \( \eta \) of the solar surface occupied by open magnetic field during the interval 1976 August to 1991 July (CR 1645 — 1844), as computed using the radial (solid curve) and line-of-sight (dotted curve) procedures. Except at sunspot maximum, the radial approach yields consistently larger values of \( \eta \) because of the greater size of its polar holes. The average values of \( \eta \) over the entire 15 yr interval are 0.14 (radial) and 0.11 (line of sight). Both methods indicate that the area covered by open field lines is greatest during the declining phase of the cycle, when the polar holes are accompanied by large equatorward extensions (see Wang & Sheeley 1990b).

By applying the line-of-sight technique to NSO/Kitt Peak data taken in the Fe \( \lambda 5250 \) line, Pneuman et al. (1978) deduced open field regions that were consistently smaller than the coronal holes appearing on Skylab X-ray and He \( \pi \) \( \lambda 304 \) images during 1973–1974. As may be seen from their Figure 3, holes may still be present at this time, although the calculations do not show corresponding open field regions; the disagreement may be due to errors in the polar field measurements.)
In this section, we derive the shape of the source-surface neutral line using both extrapolation methods. Adopting an idealized density model in which the coronal plasma is assumed to be concentrated around the neutral sheet, we then calculate the resulting white-light intensity distributions and compare them with SOLWIND K-corona observations at 3.5 R⊙.

We shall make the usual assumption that the neutral sheet, where \( B(r, \theta, \phi) = 0 \), extends radially outward from the source surface (located at \( r = R_⊙ = 2.35 R_⊙ \)). The scattering electrons are taken to be distributed uniformly in \( \theta \) and \( \phi \) along the neutral sheet, with the density \( n_e \) falling off with radius as \( \exp \left[-8.6(1 - R_⊙/r)\right] \), corresponding to hydrostatic equilibrium at a temperature of \( 1.6 \times 10^6 \) K. The results are insensitive to the radial variation of \( n_e \); our choice is based on an empirical fit to a number of coronal streamers by Dollfus & Martres (1977). For simplicity, we suppose that the electrons are confined to a 1 pixel (5°) wide band centered around the neutral sheet; more refined models that take into account the density variation within coronal holes will be described elsewhere (Nash & Wang 1992).

We consider only the component of radiation with electric vector tangential to the solar surface. The total intensity of such photons, Thomson-scattered into the line of sight, is proportional to the integral

\[
\int_{-\infty}^{\infty} n_e[(1 - u)C + uD]ds
\]

(see, e.g., Billings 1966). Here \( ds \) denotes a path element along the line of sight, \( u \) is a limb darkening coefficient which we set equal to 0.6 (compare Billings 1966; Munro & Jackson 1977), and \( C \) and \( D \) are functions of the angle \( \gamma \) between a tangent from the scattering point \( P \) to the solar surface and a line connecting \( P \) to the Sun’s center:

\[
C(\gamma) = \frac{4}{3} - \cos \gamma - \frac{1}{3} \cos^3 \gamma;
\]

\[
D(\gamma) = \frac{8}{3} \left[ 5 + \sin^2 \gamma - \frac{\cos^2 \gamma}{\sin \gamma} (5 - \sin^2 \gamma) \ln \left( \frac{1 + \sin \gamma}{\cos \gamma} \right) \right].
\]

Because our objective is to model the location and shape of the coronal white-light structures rather than their absolute brightness, the intensities will be normalized arbitrarily.

We construct a synoptic map of simulated K-coronal intensity by calculating, at regular intervals during the given Carrington rotation, the latitudinal distribution of intensity as it would appear above the Sun’s limb at a fixed radius \( r = 3.5 R_⊙ \). Because each line-of-sight integration employs magnetic data from the preceding and following 180° of longitude, the resulting map contains contributions from the adjacent two Carrington rotations.

We have applied this procedure to CR 1746 (starting date 1984 March 3) and CR 1762 (starting date 1985 May 14); the results are displayed in Figures 7 and 8, respectively. In each figure, the top row shows the source-surface neutral line, as derived using the radial (left panel) and line-of-sight (right panel) methods; the corresponding simulated intensity maps are given in the middle row. Finally, the bottom row displays (in duplicate) the observed distribution of tangentially polarized white-light intensity at 3.5 R⊙, based on west limb data from the Naval Research Laboratory’s SOLWIND corona- graph experiment. An arbitrary background intensity has been subtracted from the latter maps.
Fig. 7.—K-coronal structures for CR 1746 (1984 March). Top panels show the shape of the source-surface neutral line (black), as derived by extrapolating WSO magnetograph measurements using the radial (left) and line-of-sight (right) methods. Alternatively, these maps indicate the angular distribution of electron density, which is assumed to be concentrated at the neutral line. Middle panels show the corresponding simulated patterns of scattered intensity, with black indicating the brightest structures and white representing the regions of lowest intensity. For comparison, bottom panels (which are identical to each other) display the SOLWIND coronal intensity patterns at \( r = 3.5 R_\odot \) during CR 1746. Here an arbitrary background intensity has been subtracted from the data and the brightest structures are again denoted by black.

Fig. 8.—K-coronal structures for CR 1762 (1985 May). See Fig. 7 legend.
From an inspection of Figures 7 and 8, it is apparent that the radial method yields a much flatter neutral line than does the line-of-sight approach. This difference, which holds for both Carrington rotations, reflects the great disparity between the derived polar fields. In particular, the radial field assumption implies a much stronger axisymmetric dipole moment, which dominates at the source surface and acts to confine the neutral sheet to low latitudes. Correspondingly, the intensity patterns computed using the radial procedure are more compressed toward the equator than those derived from the line-of-sight approach. The flattening is greater for CR 1762 than for CR 1746 because there is less low-latitude activity at the later time, resulting in a more nearly aligned dipole moment. For both Carrington rotations, the radial method gives much better agreement with the observed intensity patterns.

It is interesting to note that the brightest structures in the simulated intensity maps occur where the neutral line runs horizontally \( (\partial B_r / \partial \phi = 0) \). At these locations, the neutral sheet is viewed edge-on and has its maximum thickness along the line of sight. As the scattering regions rotate behind or in front of the plane of the sky, they move to higher apparent latitudes (except at the equator), producing the arclike features seen in both the simulated and observed intensity maps. The arcs are most prominent at high latitudes and always bend away from the equator. The brightness maxima are located along the equatormost portions of the arcs, and coincide in latitude with the horizontal segments of the neutral line. The importance of such geometrical effects in determining the appearance of coronal streamers has been recognized by earlier investigators (see, e.g., Hansen et al. 1969; Bohlin & Garrison 1974).

The tendency for potential field extrapolations based on the line-of-sight matching condition to place the neutral line poleward of the observed coronal intensity maxima was first noticed by Pneuman et al. (1978). They noted that the discrepancy could be reduced by artificially increasing the line-of-sight photospheric field strengths above 70° latitude and concluded that the measured polar fields might be too low. Alternatively, they suggested that perspective effects in the K-coronal observations might be responsible for the discrepancy, but this possibility appears to be ruled out by the present model calculations, which include the line-of-sight integration.

Wilcox, Scherrer, & Hoeksema (1980) encountered a similar difficulty in modeling the interplanetary sector structure during 1976 using the conventional extrapolation technique. As pointed out by Burlaga, Hundhausen, & Zhao (1981), their computed neutral line extended to higher latitudes than either the observed heliospheric current sheet or the maximum brightness curves in K-coronameter data. Subsequently, Hoeksema et al. (1982) remedied this problem by adding to their magnetic data the 11.5 G \( \cos^8 \theta \) polar field deduced by Svalgaard et al. (1978). They justified this procedure on the grounds that computed magnetograph observations do not adequately represent the polar fields. However, the correction that they adopted was in fact derived from the same WSO measurements that they employed in their original potential field calculations. The simple resolution of this "paradox" is to recognize, with Svalgaard et al. (1978), that the photospheric field is radially oriented—and that it is therefore inconsistent to apply the line-of-sight matching condition.

7. CONCLUSIONS

Our main conclusions may be summarized as follows:

1. The solar magnetic field is nonpotential and on average nearly radial at the depth where it is measured in the Fe i \( \lambda 5250 \) line.

2. The conventional procedure of matching the line-of-sight component of a potential field directly to the observed photospheric field is invalid, because it assumes that the current free approximation is applicable at the photospheric level. Constraining the line-of-sight component of the photospheric field to be the same as that of a potential field generally requires the radial and nonradial components of the photospheric field to be comparable in magnitude, contradicting conclusion (1).

3. The procedure consistent with conclusion (1) is first to correct the observed photospheric field for line-of-sight projection by dividing by \( \sin \theta \), and then to match it to the radial component of the potential field. In general, this method yields a discontinuity in the tangential field components, which vanish at the photosphere but are finite just above it. The implied current sheet is a mathematical idealization of the "boundary layer" in which the field makes the transition from radial and nonpotential to nonradial and potential.

4. The radial field procedure yields much stronger polar fields (by a factor of \(~ 3\)) than the line-of-sight approach.

5. Both methods give \( B_r / B \), ratios (evaluated just above the solar surface) consistent with eclipse observations of polar plumes, which are strongly inclined toward the equator near sunspot minimum. The large field-line inclination angles reflect the fact that the photospheric field is far more concentrated toward the poles at this time than a dipole field.

6. At high latitudes, the open field regions predicted using the radial field approach are considerably larger than those inferred by the line-of-sight method and show better agreement with the observed polar holes, which typically extend down to a latitude of \(~ 60°\) near sunspot minimum. Employing Mount Wilson data and assuming a radial photospheric field, Wang & Sheeley (1990a) calculated that \(~ 20\% \) of the solar surface was covered by open magnetic field during 1973–1974 (see their Fig. 1c), in agreement with Skylab coronal hole observations (Bohlin 1977). Thus the tendency found by Levine et al. (1977) and Pneuman et al. (1978) for their computed open field regions to cover less area than the Skylab coronal holes can be at least partially attributed to their use of the line-of-sight matching procedure.

7. The neutral line topology calculated using the radial field method is consistent with observations of the K-coronal intensity distribution and of the interplanetary current sheet. By contrast, because it greatly underestimates the polar field strength, the line-of-sight technique yields a neutral line extending to much higher latitudes than indicated by the observations. Although this problem has been remedied by artificially introducing a polar field correction (Hoeksema et al. 1982; Hoeksema 1984; Hoeksema & Scherrer 1986), the radial field approach obviates the need for such an inconsistent procedure.

The results presented here should make it apparent that it is in fact the radial matching condition that makes the best use of the magnetograph data, contrary to the original claim of Altschuler & Newkirk (1969). In future, the reliability of the coronal extrapolations based on this method may be improved further by including the effect of current sheets in the outer corona and by employing direct measurements of the radial photospheric field component by vector magnetographs.

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