LINEAR RESISTIVE MAGNETOHYDRODYNAMIC COMPUTATIONS OF RESONANT
ABSORPTION OF ACOUSTIC OSCILLATIONS IN SUNSPOTS

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ABSTRACT

A numerical study of the resonant absorption of $p$-modes by sunspots is carried out in linear resistive MHD. The sunspot is idealized as a cylindrical axisymmetric flux tube stratified only in the radial direction and surrounded by a uniform unmagnetized plasma. First, the results of Lou in viscous MHD are reproduced in resistive MHD, and, hence, it is shown that the absorption efficiency is independent of the actual dissipation mechanism. Next, the parameter domain investigated by Lou is substantially enlarged to higher $m$-values, larger sunspots, and sunspot models with a twisted magnetic field. A parametric study reveals that the efficiency of the absorption mechanism depends significantly on both the equilibrium model and the characteristics of the $p$-modes. The overall picture resulting from this numerical survey of the relevant parameter domain is that the resonant absorption of $p$-modes is more efficient in larger sunspots with twisted magnetic fields. This is particularly true for $p$-modes with higher azimuthal wave numbers.

Subject headings: MHD — Sun: oscillations — Sun: sunspots

1. INTRODUCTION

Recent observations by Braun, Duvall, & Labonte (1987, 1988) of high-degree $p$-mode oscillations in regions around sunspots have revealed that sunspots act as strong absorbers of $p$-mode wave energy. Adopting a cylindrical coordinate system centered on the spot, Braun et al. (1987, 1988) determined the amplitudes of the waves traveling inward and outward the spot, and found that as much as 50% of the acoustic wave power can be lost. In a subsequent investigation, Braun, Labonte, & Duvall (1990) explored the horizontal spatial distribution of high-degree $p$-mode absorption in solar active regions. They found that the absorption reaches a maximum in the visible sunspot, but that it is not limited to the location of the visible spot and it is also associated with magnetic fields in the surrounding plage. The discovery that sunspots are strong absorbers of acoustic wave energy opens up a new avenue for sunspot seismology, in that the effect of active regions upon solar oscillations can be directly observed. The aim is then to use observations of $p$-mode oscillations outside sunspots to derive information on the conditions inside the spots. This of course requires a basic theoretical model that describes the observed properties of $p$-mode oscillations, in particular the strong absorption. Hollweg (1988) was the first to discuss the possibility of resonant absorption of $p$-mode oscillations in sunspots. He used a planar geometry and approximated the spot by a uniform plasma combined with a thin nonuniform plasma. In this nonuniform plasma the equilibrium quantities vary sharply from constant values characteristic of the sunspot to their constant photospheric values. In Hollweg's analysis the resonant absorption of $p$-modes occurs in this thin nonuniform plasma layer. Here "thin" means that if $a$ is the width of the nonuniform layer and $R$ is the radius of the sunspot, then $a/R \ll 1$. Hollweg assumed that the Eulerian perturbation of total pressure is constant across the dissipation layer. This assumption combined with the approximation of a thin nonuniform layer enabled Hollweg to obtain analytic expressions for the absorption coefficients. On the whole, Hollweg arrived at the conclusion that resonant absorption cannot explain the substantial observed $p$-mode absorption.

Lou (1990) studied the absorption of $p$-mode oscillations in sunspots using the equations of viscous compressible MHD. As idealized background model Lou used a cylindrical magnetic flux tube in magnetostatic equilibrium with its surroundings. The equilibrium physical variables are functions of the radial coordinate $r$ only, and the equilibrium magnetic field is purely longitudinal such that the magnetic field lines are straight. Lou's numerical results indicate that resonant absorption can be very efficient in absorbing power of nonaxisymmetric $p$-mode oscillations in sunspots. Typical values for the absorption coefficient obtained by Lou are on the order of 40%-50% in reasonable agreement with the observed values. Lou suggested that the discrepancy between his numerical solutions and Hollweg's (1988) pessimistic analytical estimates could be related to Hollweg's simplifying assumptions. In particular, Lou questioned Hollweg's assumption that the Eulerian perturbation of the total pressure is constant across the dissipation layer. Sakurai, Goossens, & Hollweg (1991a) determined the fundamental conservation laws and the jump conditions for the classic Alfvén and slow resonances in linear MHD. In a subsequent paper (Sakurai, Goossens, & Hollweg 1991b) they used these jump conditions to study the resonant absorption of $p$-mode oscillations by a magnetic flux tube. They considered an equilibrium magnetic field with straight field lines and showed that in this case the Eulerian perturbation of total pressure is the conserved quantity across the dissipation layer as assumed by Hollweg (1988). Sakurai et al. (1991b) used a crude cylindrical equilibrium model in which the sunspot is approximated by two constant plasmas separ-
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2. MATHEMATICAL FORMULATION

The resonant absorption of p-mode oscillations in sunspots is studied in the framework of linear resistive MHD. The resistive MHD equations are linearized around an ideal static equilibrium. As a model for this static equilibrium we adopt a cylindrically symmetric magnetic flux tube in which the equilibrium quantities are functions of the radial distance to the axis of symmetry only. In a system of cylindrical coordinates the equilibrium force balance equation and the ideal gas law are given by

\[ \frac{d}{dr} \left[ p_0(r) + \frac{B_0^2(r)}{2\mu} \right] = -\frac{B_0^2(r)}{\mu r}, \] (1)

\[ p_0(r) = \frac{\mathcal{A}}{\mu} \rho_0(r) T_0(r). \] (2)

In these equations \( \rho_0(r), p_0(r), T_0(r), \) and \( B_0(r) \) are, respectively, the equilibrium density, plasma pressure, temperature, and magnetic field. The equilibrium magnetic field has both axial and azimuthal components. Here \( \mathcal{A} \) and \( \mu \) are, respectively, the Boltzmann constant and the mean molecular weight, and \( \mu \) is the magnetic permeability which is approximated by its value in vacuum. Equations (1)-(2) provide only two constraints for five equilibrium variables \( \rho_0, p_0, T_0, \) and \( B_0 \). As a consequence we are free to specify three of the equilibrium variables. At the boundary \( r = R \) of the sunspot the total pressure (plasma pressure and magnetic pressure) and the normal component of the magnetic field have to be continuous. The second boundary condition is automatically satisfied in the present equilibrium since the radial component of the equilibrium magnetic field is identically zero. In the idealized equilibrium state considered in the present paper the magnetic field in the sunspot drops gradually to zero with the increase of radial distance \( r \), and the nonuniform sunspot is surrounded by a nonmagnetic and uniform plasma, so that for \( r > R \) the equilibrium density, pressure, and temperature are constant. The first boundary condition then reduces to the continuity of plasma pressure \( p_0(r) \).

The resistive MHD equations that govern the linear displacements about this equilibrium state can be written in the (dimensionless) form

\[ \frac{\partial \rho_1}{\partial t} = -\mathbf{v} \cdot (\nabla \rho_1), \] (3)

\[ \rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p_1 + (\mathbf{v} \times \mathbf{B}_0) \times \mathbf{B}_1 + (\mathbf{v} \times \mathbf{B}_1) \times \mathbf{B}_0, \] (4)

\[ \rho_0 \frac{\partial T_1}{\partial t} = \rho_0 \mathbf{v} \cdot \nabla T_0 - (\gamma - 1)\rho_0 T_0 \nabla \cdot \mathbf{v}, \] (5)

\[ \frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) - \nabla \times (\eta \nabla \times \mathbf{B}_1). \] (6)

A subscript 1 denotes an Eulerian perturbation, except for the velocity. As the equilibrium velocity is assumed to be identically zero (i.e., static equilibrium), there is no need to have subscripts 0 and 1 as far as the velocity is concerned. The Eulerian (and also the Lagrangian) perturbation is denoted as \( \mathbf{v} \). The resistivity, \( \eta \), is assumed to be constant in the present paper, and the ratio of the specific heats, \( \gamma \), is taken to be 5/3. Equations (3)-(6) are, respectively, the continuity equation, the momentum equation for a nonviscous plasma, the equation for...
the variation of the internal energy, and the induction equation which includes the Ohmic term due to the finite electric conductivity of the plasma. Notice that there is no restriction to incompressible plasmas. Equations (3)-(6) form a system of eight partial differential equations for eight unknowns, viz. $\rho_1$, $v_r$, $v_\theta$, $v_z$, $T_1$, $B_\theta$, $B_\phi$, and $B_z$. The equations (3)-(6) are written in dimensionless form. The distance is normalized to the radius of the sunspot ($R$), while the magnetic field, the plasma pressure, the temperature, and the density are normalized to, respectively, $B_0(0)$, $B_0^2(0)/\mu_0$, $V_0^2(\partial/\partial r)$, and $\rho_0(0)$. The time is expressed in Alfvén transit times, $t_a = R/V_\phi$, with $V_\phi = B_0(0)/\sqrt{\mu_0 \rho_0(0)}$. The velocity, $v$, is normalized to $V_\phi$ and the resistivity to $\mu R V_\phi$. In the following, we always use dimensionless quantities except in § 4 where the results are presented and discussed.

Since the equilibrium variables depend on $r$ only, the perturbed quantities can be Fourier analyzed with respect to the ignorable spatial coordinates $\varphi$ and $z$ as

$$f_1(r, \varphi, z; t) = f_1(r; t) \exp\left(\text{i} m \varphi + \text{i} k z\right),$$

where $f_1$ denotes any of the perturbed quantities. The azimuthal wave number $m$ has to be an integer, and $k$ is the longitudinal wave number. The linear displacements in the sunspot are driven by the $p$-mode oscillations that impinge on the spot. The $p$-mode oscillations have real frequencies determined by the global structure of the solar atmosphere and interior. A dissipative system that is driven periodically at a fixed frequency by an external source will reach an asymptotic or stationary state. In this asymptotic state all the perturbed quantities oscillate harmonically with the frequency of the externally imposed $p$-mode oscillation. The temporal behavior of every perturbed quantity $f_1$ is then

$$f_1(r; t) = f_1(r) \exp\left(-\text{i} \omega t\right),$$

where $\omega$ is the real frequency of the driving acoustic wave. In the nonmagnetic region surrounding the sunspot the behavior of the plasma is adequately described by the equations of ideal MHD. In this region the radial dependency of the linear displacements is governed by a differential system of the second order. As the background is nonmagnetic and uniform, the equations simplify. In terms of the perturbed plasma pressure, $p_1$, and the radial component of the Lagrangian displacement, $\xi_r$, for example, we get

$$\frac{d}{dr} \left( r \frac{dp_1}{dr} \right) + \left( K^2 - \frac{m^2}{r^2} \right) r p_1 = 0,$$

where $\xi_r$ and the constant $K^2$ are defined by

$$\xi_r = -\text{i} \omega \xi_r, \quad K^2 = (\omega^2 - k^2 c^2)/c^2,$$

with $c^2$ the square of the sound velocity. Equations (9)-(12) apply to the unmagnetized and uniform plasma surrounding the sunspot ($r \geq 1$). As we are considering $p$-mode oscillations that have a propagating wave character in the nonmagnetic plasma, we have $K^2 > 0$ ($\omega^2 > k^2 c^2$), and we can view $K$ as the radial (or horizontal) wave number outside the sunspot. The solutions to equations (9)-(10) then can be written in the form

$$p_{1e} = \alpha_1 H_m^{(1)}(Kr) + \alpha_2 H_m^{(2)}(Kr),$$

$$\xi_{re} = \frac{\alpha_1}{\rho_0 \omega^2} \left[ \alpha_1 H_m^{(1)}(Kr) + \alpha_2 H_m^{(2)}(Kr) \right],$$

where $p_{1e}$ and $\xi_{re}$ denote, respectively, the dimensionless plasma pressure and radial component of the Lagrangian displacement in the external region. In equations (13)-(14) $H_m^{(1)}(z)$ and $H_m^{(2)}(z)$ are the Hankel functions of, respectively, the first and the second order, and a prime on these symbols denotes the derivative of the Hankel functions with respect to their argument. Here $\rho_0e$ is the dimensionless equilibrium density outside the sunspot, and $K$ and $\omega$ are the dimensionless radial wavenumber and the dimensionless frequency. Solutions (13)-(14) are valid for $r \geq 1$.

When we consider positive values of $\omega$, the time dependency (8) then implies that $H_m^{(1)}$ corresponds to the outgoing wave, while $H_m^{(2)}$ corresponds to the incoming wave. The constant $\alpha_2$ in front of the incoming wave scales the complete solution, and the quantity we are interested in is the ratio $\alpha_2/\alpha_1$. The oscillations in the sunspot are driven by the $p$-mode oscillations outside the sunspot. In the mathematical formulation this is taken into account by imposing boundary conditions at the sunspot boundary which connect the $p$-mode oscillations outside the sunspot to the oscillations inside the sunspot. At the sunspot boundary we must have continuity of the normal component of the Lagrangian displacement, of the Lagrangian perturbation of total pressure, and of the Lagrangian perturbation of all three components of the magnetic field. Hence, in $r = 1$ we must have

$$\xi_r = \xi_{re}, \quad p_{1e} + B_0 \cdot B_1 - B_{0e} \xi_r = p_{1e}, \quad B_{1e} - (m B_{0e} + k B_0) \xi_r = 0,$$

$$B_{1e} + \frac{dB_{0e}}{dr} \xi_r = 0,$$

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$$\xi_r = \xi_{re}, \quad p_{1e} + B_0 \cdot B_1 - B_{0e} \xi_r = p_{1e}, \quad B_{1e} - (m B_{0e} + k B_0) \xi_r = 0,$$

$$B_{1e} + \frac{dB_{0e}}{dr} \xi_r = 0,$$

These five boundary conditions, together with two regularity conditions at the axis, viz., for the radial components of both $v$ and $B$, guarantee a unique solution to this problem for a given $\alpha_2$. In the idealized equilibrium state adopted here for the sunspot the magnetic field vanishes at $r = 1$: $B_{0e}(1) = B_{0e}(1) = 0$. This results in a substantial simplification of the conditions (15b)-(15e). First in conditions (15b)-(15e) the terms containing $B_{0z}$ and $B_{0e}$ vanish. Second, conditions (15d)-(15e) are identically satisfied. This can be readily seen from the expressions for $B_{1e}$ and $B_{1z}$:

$$B_{1e} = \left( k B_{0e} + \frac{m}{r} B_{0e} \right) \xi_c - B_{0e} \text{ div } \xi + \left( \frac{B_{0e}}{r} - \frac{dB_{0e}}{dr} \right) \xi_r,$$

$$B_{1z} = \left( k B_{0e} + \frac{m}{r} B_{0e} \right) \xi_c - B_{0e} \text{ div } \xi - \frac{dB_{0e}}{dr} \xi_r.$$

At $r = 1$, we have

$$B_{1e} = -\frac{dB_{0e}}{dr} \xi_c,$$

$$B_{1z} = -\frac{dB_{0e}}{dr} \xi_c,$$

implying that conditions (15d)-(15e) are identically satisfied.

3. NUMERICAL FORMULATION

Let us now focus on the determination of the solution inside the sunspot. We use the linearized equations of resistive MHD
to describe the driven oscillations in this region and their resonant absorption in a resistive layer around the ideal Alfvén resonance point. The Fourier-decomposition (eq. [7]) and the assumed time dependency (eq. [8]) enable us to rewrite these equations as a set of ordinary linear differential equations. With the perturbed magnetic field expressed in terms of a vector potential \( A \), that is, \( B = \mathbf{V} \times A \), this yields

\[
-i\omega \frac{\rho}{r} v_1 = -\frac{\rho_0}{r} v_1 - \frac{\rho_0}{r} \frac{v_2}{v_2} - \frac{m\rho_0}{rB_{0z}} v_3, \tag{16a}
\]

\[
-i\omega \frac{\rho_0}{r} v_2 = -\left( \begin{array}{c} \frac{T_0}{r} \rho + \frac{\rho_0}{r} \phi \end{array} \right) - \frac{B_{0z}}{r} a_2 + \frac{mB_{0z}}{r} a_1
\]

\[
\frac{k}{r} \left( \frac{rB_{0z}}{a_1} + \frac{1}{r} \frac{rB_{0z}}{a_3} - \frac{mB_{0z}}{a_3} \right)
\]

\[
+ \frac{k^2B_{0z}}{r} a_2 - B_{0z} \left( \frac{a_2}{r} - \frac{ma_2}{r} \right)
\]

\[
- \frac{B_{0z}}{r} (rka_1 - ra_3) - \frac{m^2}{r} B_{0z} a_3
\]

\[
+ \frac{mkB_{0z}}{r} a_2, \tag{16b}
\]

\[
-i\omega \left( \frac{rT_0}{B_{0z}} v_2 + \frac{\rho_0 B_{0z} v_3}{B_{0z}} \right) = \frac{mT_0}{r} \rho + \frac{m\rho_0}{r} \phi - \frac{m}{r} (rB_{0z}) a_3 + \frac{k}{r} (rB_{0z}) a_2
\]

\[
+ \frac{mB_{0z}}{r} a_2 - \frac{mB_{0z}}{r} a_1 - \frac{mkB_{0z}}{r} a_3, \tag{16c}
\]

\[
-i\omega \left( \frac{\rho_0 B_{0z} v_2}{B_{0z}} + \frac{\rho_0 B_{0z}^2}{B_{0z}} \right) v_3 = \frac{mT_0}{r} B_{0z} \rho + \frac{m\rho_0}{r} B_{0z} \phi
\]

\[
+ \frac{mB_{0z}}{r} \phi + \frac{kT_0}{r} B_{0z} a_2 + \frac{kp_0 B_{0z}}{r} a_2
\]

\[
- \frac{m}{r} \left[ \frac{B_{0z}}{r} (rB_{0z}) a_3 + \frac{k}{r} \left( \frac{B_{0z}}{r} (rB_{0z}) a_2 + \frac{B_{0z}}{r} (rB_{0z}) a_3 \right) \right], \tag{16d}
\]

\[
-i\omega \frac{\rho_0}{r} \phi = -\frac{\rho_0}{r} T_0 v_1 - \frac{\rho_0}{r} T_0 (\gamma - 1) v_1
\]

\[
+ \frac{m\rho_0}{r} T_0 (\gamma - 1) v_2 - \frac{m\rho_0}{r} T_0 B_{0z} (\gamma - 1) v_3
\]

\[
- \frac{kp_0}{r} T_0 (\gamma - 1) v_3, \tag{16e}
\]

\[
-i\omega a_1 = v_2 + \eta \frac{m}{r} a_2 - \eta \frac{m^2}{r} a_1 - \eta rk^2 a_1 + \eta rka_3, \tag{16f}
\]

\[
-i\omega a_2 = -\frac{B_{0z}}{r} v_1 + \eta \frac{mk}{r} a_3 - \eta \frac{k^2}{r} a_2 + \eta \left( \frac{a_2}{r} - \frac{ma_2}{r} \right), \tag{16g}
\]

\[
-i\omega a_3 = B_{0z} v_1 - \eta (rka_1 - ra_3) - \eta \frac{m^2}{r} a_3 + \eta \frac{mk}{r} a_2. \tag{16h}
\]

Here, and in what follows, the prime denotes the derivative with respect to the dimensionless distance \( r \). The dimensionless state variables \( \rho, v_1, v_2, v_3, \phi, a_1, a_2 \) and \( a_3 \) are defined as

\[
\rho = r\rho_1, \quad \phi = rT_1, \quad v_1 = rv, \quad a_1 = iA_1, \quad v_2 = i(B_0 v_\phi - B_{0\phi} v_3), \quad a_2 = rA_2, \quad v_3 = irv_3, \quad a_3 = A_3. \tag{17}
\]

The system (eq. [16]) of eight ODEs for the eight unknowns defined in equation (17) has order 6 in \( r \) and has to be supplied with proper boundary conditions. The most convenient form of the boundary conditions (eq. [15]) and the above-mentioned regularity conditions at the axis is, in terms of the state variables (eq. [17]),

\[
[v_1]_{r=1} = -\frac{iK}{\rho_0 \omega} [z_1 H_{m}^{s}(K) + z_2 H_{m}^{s}(K)], \tag{18a}
\]

\[
[p_1]_{r=1} = \frac{i\rho_0(1)}{K} H_{m}^{s}(K) v_1(1), \tag{18b}
\]

\[
[m a_3 - ka_2]_{r=1} = 0, \tag{18c}
\]

\[
[k a_1 - a'_3]_{r=1} = -\frac{i}{\omega} B_{0\phi}(1)v_1(1), \tag{18d}
\]

\[
[a'_2 - ma_1]_{r=1} = -\frac{i}{\omega} B_{0\phi}(1)v_1(1), \tag{18e}
\]

\[
[v_1]_{r=0} = 0, \tag{18f}
\]

\[
[a_2]_{r=0} = 0, \tag{18g}
\]

where we have used equations (13) and (14) and the fact that \( B_{0\phi}(1) = B_{0\phi}(1) = 0 \) in the considered equilibrium model. Conditions (18a)-(18e) correspond to the conditions (15a)-(15e). As explained in the previous section, conditions (15d)-(15e) are identically satisfied when the equilibrium magnetic field vanishes at \( r = 1 \), as is the case here. Nonetheless, it is convenient to have these conditions in terms of the state variables (eq. [17]) for computing surface terms that appear when applying the Galerkin method. Conditions (18f)-(18g) are the regularity conditions that have to be satisfied at the axis. In total we have seven conditions which yield a mathematically well-defined problem and guarantee a unique solution (and a unique value of \( z_2 \)) for every given \( z_2 \). Conditions (18a)-(18b) show that \( z_2 \) is just a factor that scales the complete solution. Multiplying the amplitude of the incoming wave by, say, 10 results in multiplying the complete solution by 10. Conditions (18b)-(18g) are used to determine the solution in the sunspot. Once the solu-
tion in the sunspot is known, condition (18a) is used to compute the ratio $\alpha/\kappa$ and to fix the external solution.

The system (eq. [16]) of ordinary differential equations can be written in the form

$$L u = 0,$$

where $L$ denotes the linear matrix operator, and the state $u$ is defined as

$$u^T = (\dot{\rho}, v_1, v_2, v_3, \dot{T}, a_1, a_2, a_3).$$

The system of ODEs (eq. [19]) is solved in its weak form. The vector $u(r)$ is a weak solution if for any function $w(r)$ of the admissible Sobolev space satisfying the appropriate boundary conditions, the scalar product $\langle Lu, w \rangle$ vanishes (Strang & Fix 1973). We solved the system (eq. [16]) by means of a combination of the Galerkin procedure and the finite-element method. In this numerical technique the components of the state vector $u$ are approximated by a finite linear combination of local expansion functions. In our formulation, a combination of cubic Hermite and quadratic finite elements is used for the spatial discretization in order to increase the accuracy of the method (see Poedts et al. 1990a). This yields

$$u(r) \approx \sum_{j=1}^{N} a_j^k h_j^1(r) + \sum_{k=1}^{8} a_k^2 h_k^2(r), \quad k = 1, \ldots, 8,$$

with $a_j^1$ and $a_k^2$ the constant coefficients that have to be determined and $h_j^1(r)$ and $h_k^2(r)$ are the finite elements. Cubic Hermite spline functions are used for $v_1, a_2,$ and $a_3,$ and quadratic finite elements for $\dot{\rho}, v_2, v_3, \dot{T},$ and $a_1.$ The introduction of two orthogonal shape functions per interval and per component of $u$ raises the number of unknowns to $16N$, with $N$ the number of grid points.

In the Galerkin method, which is applied here, the basis functions $h_j^1(r)$ and $h_k^2(r)$ are used in the weak form. This leads to a set of linear algebraic equations of the form

$$(A + i\omega B)u = f,$$

with $u$ the vector of $16N$ expansion coefficients. The matrix $B$ is positive definite, but $A$ is non-Hermitian. Both $A$ and $B$ possess a tridiagonal block structure owing to the use of finite elements as expansion functions. The driving term $f$ results from the implementation of the boundary conditions (eq. [18]). The conditions (18c), (18f), and (18g) are essential boundary conditions and are imposed on the basis functions themselves, that is, the Sobolev space of the basis functions is limited to those functions that satisfy these conditions. The boundary conditions (18b), (18d), and (18e), on the other hand, are called natural boundary conditions as they are satisfied automatically when they are substituted in the surface terms that appear in the weak form by partial integrations. In the weak form of equations (16a)-(16h) the underlined terms in equations (16b), (16g), and (16h) are partially integrated. The substitution of the boundary conditions (18b), (18d), and (18e) in the resulting surface terms yields

$$-[v^T \left( \frac{T_0}{r} \frac{\dot{\rho}}{\rho} + \frac{\rho_0}{r} \dot{T} \right) v_k v_1]_{r=1} = -[v^T \psi_1]_{r=1}$$

$$= -v^T \left( \frac{\rho_0}{K} \frac{H_m^2(K)}{H_m^1(K)} v_1 \right) l - \frac{4i\omega \gamma K}{\pi K H_m^{1.5}(K)} v_1,$$

where the superscript asterisk denotes the complex conjugate. Note that when $B_{0}(0) = 0,$ boundary condition (18d) becomes an essential boundary condition and needs to be imposed in the same way as the other essential boundary conditions (see above). The same applies to condition (18e) when $B_{0}(1) = 0.$ The first right-hand-side term in equation (23a) and the right-hand-side terms in equations (23b) and (23c) yield contributions to the coefficient matrix $(A + i\omega B)$ in equation (22). Owing to the local nature of the finite elements, the matrix modifications needed to impose the natural boundary conditions are limited to the last subblock of the coefficient matrix. The second right-hand-side term in equation (23a) yields, for the same reason, only one nonzero component of the driving term $f.$ Note that this source term is proportional to $\alpha_2,$ the amplitude of the incident wave which scales the whole solution. Once the solution has been determined, the amplitude of the outgoing wave, $\alpha_2,$ and, hence, the absorption coefficient, defined as

$$\alpha_{2} = \frac{|\alpha_2|^2 - |\alpha_1|^2}{|\alpha_2|^2},$$

can be determined by means of the boundary condition (18a).

4. RESULTS

Our first computations of absorption coefficients of p-mode oscillations in sunspots were carried out for the equilibrium models that Lou (1990) used in his numerical study of resonant absorption in viscous MHD. The first aim of these computations is to give a numerical proof that resonant absorption is indeed independent of the actual dissipation mechanism, which to the best of our knowledge has not yet been done. This then enables us to compute absorption rates using the equations of resistive MHD rather than the more complicated equations of viscous MHD, and the second aim is then to extend Lou's results over a substantially larger domain of the parameter space.

The equilibrium models used by Lou are characterized by analytical distributions of density and pressure,

$$\rho_0(x) = \rho_0(0)\left(1 + \exp \left[-\lambda(x - 1)^2\right]\right)/(1 + e^{-x}),$$

$$\rho_0(x) = \rho_0(0)\left(1 + \exp \left[-\lambda(x - 1)^2\right]\right) \times \left(1 + \frac{1}{2} \exp \left[-\lambda(x - 1)^2\right]\right),$$

where $x = r/R,$ $\rho_0(0)$ is the density on the magnetic axis, and $\rho_0,$ the constant photospheric plasma pressure outside the sunspot. Note that equations (25) and (26) are not dimensionless. In this section, dimensional quantities are used in order to facilitate the comparison of the presented results with the results obtained by Lou (1990) and with the observational data of Braun et al. (1988). The dimensional quantities used in the previous sections are noted with a subscript asterisk in the present section. The parameter $\lambda$ controls the sharpness of the transition between the magnetic ($x \leq 1$) and the nonmagnetic ($x \geq 1$) regions. In case of a straight magnetic field ($B_0 = 0$) it
is straightforward to integrate the equation of magnetostatic equilibrium (1) to give

$$p_0(x) + \frac{B_0^2}{2\mu} = p_0e. \quad (27)$$

It is obvious that equations (25)-(27) give a relatively good first approximation of the variation of density, pressure, and magnetic field in a sunspot.

The limitations of this simple model become clear when the following ratios are considered

$$\frac{p_0(1)}{p_0(0)} = \frac{2}{1 + e^{-\lambda}} - \frac{1}{3} \left(1 + e^{-\lambda}\right)^{1 + \frac{1}{2}} \frac{\lambda}{1 + \frac{1}{2}}, \quad (28)$$

$$\frac{B_0^2(0)/\mu}{p_0e} = 1 - \frac{1}{3} \left(1 + e^{-\lambda}\right) \frac{\lambda}{2}, \quad (29)$$

$$\frac{B_0^2(0)/2\mu}{p_0e} = 1 - \frac{1}{3} \left(1 + e^{-\lambda}\right) \frac{\lambda}{2}. \quad (30)$$

Unless $\lambda$ is small, which implies little variation in $p_0(x)$, $p_0(x)$, and $B_0(x)$, the ratios (28)-(30) hardly vary with $\lambda$ and approximate, respectively, 2, 3, and 4. This means that this equilibrium model cannot represent density contrasts larger than a factor 2 and pressure contrasts larger than a factor 3. In addition, equation (30) shows that once a value of the external pressure and of $\lambda$ are chosen, $B_0(0)$ is also fixed. That is the reason why Lou had to take $B_0(0) = 2000$ G for an external pressure $p_0e = 2.4 \times 10^6$ Pa. The choice $\lambda = 0$ leads to constant values of the equilibrium quantities and corresponds to a uniform sunspot and no absorption. When $\lambda$ is increased the sunspot becomes progressively more nonuniform.

The frequency of the acoustic oscillation is $\omega = 0.02 \text{ rad s}^{-1}$ and $K = 1 \times 10^8$ m$^{-1}$. The radius of the sunspot $R = 8.4 \times 10^6$ m, and total photospheric pressure $p_0e = 2000$ G and total photospheric pressure $P = 2.4 \times 10^6$ Pa as in Lou (1990) to make a comparison with Lou's results straightforward. On each figure results are dis-
Fig. 2.—Absorption coefficient \( \alpha \) as a function of the radius of the sunspot for a sunspot with a straight magnetic field. The sharpness parameter is \( \lambda = 5 \). The remaining quantities \( B_0(0) = 0.2T \), \( \omega = 0.02 \text{ rad s}^{-1} \), \( K = 1 \times 10^{-6} \text{ m}^{-1} \), and \( m = 1, 2, 3, 5 \) have the same values as in Fig. 1.

played for oscillations with azimuthal wave number \( m = 1, 2, 3, \) and 5. These four values of \( m \) are considered to see how the absorption coefficients vary in function of the azimuthal wave number of the oscillation. Since the magnetic field is straight, the absorption is independent of the sign of the azimuthal wave number, and it suffices to consider positive values of \( m \) only. Figures 1–3 correspond to Lou’s Figures 3–5. On his Figures 3–5 Lou shows only results for oscillations with azimuthal wave number \( m = 1 \), and our Figures 1–3 extend Lou’s Figures 3–5 to the azimuthal wave numbers different from 1. Figures 1–3 give the absorption coefficients for \( p \)-mode oscillations with frequency \( \omega = 2 \times 10^{-2} \text{ rad s}^{-1} \), respectively, in function of the sharpness parameter \( \lambda \) for \( R = 4.2 \times 10^6 \text{ m} \) and \( K = 1 \times 10^{-6} \text{ m}^{-1} \) (Fig. 1), in function of the radius of the spot for \( K = 1 \times 10^{-6} \text{ m}^{-1} \) and \( \lambda = 5 \) (Fig. 2), and in function of the horizontal wavenumber \( K \) for \( \lambda = 5 \) and \( R = 4.2 \times 10^6 \text{ m} \) (Fig. 3a) and \( R = 6.3 \times 10^6 \text{ m} \) (Fig. 3b). Lou’s results for oscillations with \( m = 1 \) are also displayed on our Figures 1–3.

First of all, Figures 1–3 show that all of Lou’s results which were obtained in viscous MHD are recovered here in resistive MHD. All results shown in the present paper, except those in Figure 4, are obtained with \( R_m = 5 \times 10^5 \), where \( R_m \) is the magnetic Reynolds number (\( = \eta^{-1} \)). Computations for test cases over a wide range of \( R_m \) values from \( 10^4 \) up to \( 10^9 \) reveal little or no variation of the absorption coefficient with respect to \( R_m \), as is illustrated in Figure 4. This is in agreement with earlier numerical results by Poedts et al. (1989), and the analytical prediction by Kapraff & Tataronis (1977) that the rate of resonant absorption is independent of the size of the dissipative mechanism provided it is small. Figures 1–4 present the first numerical proof of the fact that, for resonant absorption, the energy absorption rate is independent of the actual dissipation mechanism and of its size.

Let us now focus on how the resonant absorption coefficient varies with the radius of the spot, the horizontal wave number, the azimuthal wave number, and the frequency of the incident acoustic wave. Note first that we get absorption coefficients up to 50% as observed, but that there is no absorption for \( m = 0 \). Figure 1, in agreement with Lou’s Figure 4, shows that for \( \lambda = 5 \) the absorption is a monotonically decreasing function of the azimuthal wave number \( m \). However, for acoustic waves with an azimuthal wave number \( m > 1 \), the maximal absorption is systematically shifted toward higher values of \( \lambda \) as \( m \) increases. In addition, the absorption is no longer a monotonically decreasing function of \( m \) in an equilibrium with sufficiently large \( \lambda \). Take as an example an equilibrium with \( \lambda = 20 \). In that case the absorption is largest for \( m = 2 \), and even the absorption for \( m = 3 \) is larger than that for \( m = 1 \). A high value of \( \lambda \) corresponds to a more rapid variation of the equilibrium quantities in the outer part of the sunspot, as can be seen on Lou’s Figure 1. More rapid variations of equilibrium quantities in the outer part of the sunspot favor absorption of acoustic waves with \( m > 1 \). Most of the computations presented in this paper have been carried out for equilibria with \( \lambda = 5 \), and the results presented on the following figures are all for \( \lambda = 5 \). In view of Figure 1 this choice of the
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equilibrium parameter $\lambda$ favors absorption for $m = 1$ in comparison to absorption for higher values of $m$.

In Figure 2 the absorption coefficient is depicted as a function of the radius $R$ for different values of $m$. The frequency $\omega$ and $K$ are constant here ($\omega = 0.02$ rad s$^{-1}$, $K = 1 \times 10^{-6}$ m$^{-1}$). For a radius $R < 7 \times 10^6$ m the absorption is highest for $m = 1$ and monotonically decreases with increasing $m$. For given $m$ the absorption coefficient is first an increasing function of $R$, it attains a maximum and then decreases when $R$ is further increased. As we go to higher values of $m$, the increasing behavior of the absorption coefficient as a function of the radius $R$ is extended over larger intervals of $R$, and the maximum is steadily shifted toward higher values of $R$. For a radius $R > 7.5 \times 10^6$ m the absorption is larger for $m = 2$ than that for $m = 1$. Large spots are on the whole more efficient absorbers of $p$-mode oscillation energy and, in particular, absorb $m > 1$ oscillations better than smaller sunspots.

In Figures 3a and 3b the absorption coefficient is depicted as a function of the horizontal wave number $K$ for different values of $m$. The frequency $\omega$ and the radius $R$ are constant ($\omega = 0.02$ rad s$^{-1}$, $R = 4.2 \times 10^6$ m [Fig. 3a], and $R = 6.3 \times 10^6$ m [Fig. 3b]). The variation of the absorption coefficient as a function of $K$ is very similar to that as a function of $R$. For a given value of $m$ the absorption coefficient first increases, attains a maximum, and steeply decreases with increasing value of $K$. In Figure 3a, which applies to a relatively small sunspot with $R = 4.2 \times 10^6$ m, the absorption is always largest for $m = 1$ and monotonically decreases with increasing $m$. However, in Figure 3b where $R = 6.3 \times 10^6$ m, this is no longer the case, and we have an interval of $K$-values from about $0.40 \times 10^{-6}$ m$^{-1}$ to $0.9 \times 10^{-6}$ m$^{-1}$ where the absorption is largest for $m = 2$, rather than for $m = 1$.

Figures 5a and 5b show how the absorption coefficient varies as a function of the frequency of the incident wave. The radius of the spot $R$ and the horizontal wave number $K$ are held constant ($K = 1 \times 10^{-6}$ m$^{-1}$, $R = 4.2 \times 10^6$ m [Fig. 5a] and $R = 6.3 \times 10^6$ m [Fig. 5b]). Again the behavior of the absorption coefficient is similar to that found in Figures 2 and 3. For a given $m$ the absorption coefficient first increases, attains a maximum, and then decreases with increasing frequency. For frequencies $\omega < 2.7 \times 10^{-2}$ rad s$^{-1}$ (Fig. 5a) and $\omega < 2.1 \times 10^{-2}$ rad s$^{-1}$ (Fig. 5b) the absorption is highest for $m = 1$ and is a monotonically decreasing function of $m$. As the acoustic wave has a higher azimuthal wave number $m$, the increasing behavior of the absorption as a function of frequency is extended over larger intervals of $\omega$, and the maximum is steadily shifted to higher values of $\omega$. For $\omega > 2.7 \times 10^{-2}$ rad s$^{-1}$ (Fig. 5a) the absorption is largest for $m = 2$ rather than for $m = 1$. For a sunspot with a larger radius (Fig. 5b, $R = 6.3 \times 10^6$ m) this behavior becomes more pronounced. For $\omega > 2.1 \times 10^{-2}$ rad s$^{-1}$ the absorption is largest for $m = 2$, and as we go to $\omega > 2.6 \times 10^{-2}$ rad s$^{-1}$ we even have larger absorption for $m = 3$ than for $m = 1$. Note that we get absorption coefficients up to 50%-60% for $m = 2$ and $m = 3$ at $\omega \approx 3 \times 10^{-2}$ rad s$^{-1}$. On the whole, “high” frequency oscillations seem to favor absorption for larger azimuthal wave numbers. This behavior is similar to that found by Sakurai et al. (1991b) for a somewhat different cylindrical equilibrium state. In their Figure 2 these authors plotted for given $kR$ the absorption coefficient as a function of $K/k$; that is, as a function of frequency. They also found large absorption coefficients and there also the absorption coefficient for $m = 1$ is often smaller than the absorption coefficients for $m > 1$.© American Astronomical Society • Provided by the NASA Astrophysics Data System
The results presented in our Figures 1–3 and 5 reveal a dependence of the absorption coefficient on $m$ which is by far more complicated than can be deduced from Lou's Figure 6. This figure shows a strong and monotonically decreasing behavior. Our results indicate that the variation of the absorption coefficient in terms of the azimuthal wave number largely depends on the equilibrium quantities like $\lambda$ and radius $R$ and on the characteristics of the incident acoustic wave like the frequency $\omega$ and the horizontal wave number $K$. On the whole, our results indicate that for oscillations with relatively low frequencies interacting with small sunspots that have smooth variations of the equilibrium quantities (small $\lambda$) the absorption coefficient is a decreasing function of the azimuthal wave number $m$. However, if any of these conditions is not satisfied, this clear-cut behavior is no longer present. In general, larger sunspots with rapid variations of the equilibrium quantities and "high" frequency oscillations seem to favor absorption at higher azimuthal wave numbers. Chitre & Davila (1990, 1991) claim that in a cylindrical flux tube with a straight magnetic field ($B_\phi = 0$) the absorption of acoustic waves depends on the azimuthal wave number as $1/m^2$. The results of the present paper and also the results obtained by Sakurai et al. (1991b) contradict this simple $1/m^2$ dependence of the absorption coefficient.

Let us now comment on the solutions of the driven oscillation in the sunspot. In Figure 6 we have plotted the components $v_1$, $v_2$, and $v_3$ of the velocity (Fig. 6a) and the Eulerian perturbation of total pressure (Fig. 6b). The quantity $v_1$ is the velocity component normal to the equilibrium magnetic surfaces. The quantities $v_2$ and $v_3$ are components in the magnetic surfaces, and $v_3$ is perpendicular to the magnetic field lines. Since $B_\phi \equiv 0$ here, $v_3$ is parallel to the magnetic field lines. Figure 6 is for an equilibrium with $\lambda = 5$, a sunspot with radius $R = 4.2 \times 10^6$ m, and a wave with $\omega = 0.02$ rad s$^{-1}$, azimuthal wave number $m = 1$, and horizontal wave number $K = 1 \times 10^{-6}$ m$^{-1}$. The ideal singularity is at $x = x_0 = 0.36$. Figure 6 first of all shows that the component $v_2$ (i.e., in the magnetic surfaces and perpendicular to the magnetic field lines) is dominant. The component is characterized by strong remnants of the ideal $1/(x - x_0)$ singularity and the ideal $\delta(x - x_0)$ contribution. The other components are an order of magnitude smaller. The normal component $v_1$ shows traces of the ideal $\ln|x - x_0|$ singularity and the ideal jump [or $H(x - x_0)$ contribution]. The parallel component of velocity $v_2$ and the Eulerian perturbation of total pressure are very smoothly varying functions which do not show any characteristic behavior around the ideal singularity $x = x_0$. There is not any indication of remnants of the ideal $\ln|x - x_0|$ singularity or the ideal jump contribution. This is a numerical verification of the analytic prediction by Sakurai et al. (1991a) that there is no singularity and no jump in the parallel component of velocity and in the Eulerian perturbation of pressure in an equilibrium with straight magnetic field lines ($B_\phi = 0$). It also justifies the approach by Hollweg (1988) and Sakurai et al. (1991b).

An important shortcoming of the equilibrium models used by Lou is that they do not produce absorption of axisymmetric waves ($m = 0$) in contrast with the observed absorption of these waves. As already explained, axisymmetric waves are not absorbed in a cylindrical equilibrium with straight magnetic field lines. Absorption of axisymmetric waves requires curved magnetic field lines and hence the equilibrium magnetic field should have both longitudinal and azimuthal components.

We have modified Lou's equilibrium models by including an azimuthal magnetic field component. We have kept the density and the pressure unchanged and have chosen a simple polynomial expression for the azimuthal magnetic field, viz.,

$$B_{\phi\text{max}}(x) = -4B_{\phi\text{max}}x(x - 1),$$

where $B_{\phi\text{max}}$ is the maximal value of $B_\phi$, which is attained in $x = \frac{1}{2}$. This choice makes $B_\phi$ vanish on the axis of the sunspot ($x = 0$) as it should do to have a finite electric current there and at the boundary of the spot ($x = 1$) since we consider an idealized sunspot with a magnetic field that gradually goes to zero as we move to the boundary of the spot. This azimuthal field $B_\phi$ requires an axial electric current which reverses sign at $x = \frac{3}{4}$. In addition this axial current does not vanish at the edge of the loop and produces a surface current there. In Lou's
original model there is already a surface current since the axial magnetic field which has a nonzero derivative at \( x = 1 \) requires an azimuthal electric current which does not vanish at the edge of the loop.

The equation of magnetostatic equilibrium (1) can be integrated to give

\[
\frac{B_0^2(x)}{2\mu} = p_0(x) + B_{0zm}(0) \left( \frac{4}{3} - 16x^2 + \frac{80}{3} x^3 - 12x^4 \right).
\]

From this equation it follows for \( x = 0 \) that

\[
\frac{B_0^2(0)}{2\mu} = \frac{1 - (1/3)(1 + e^{-\lambda})(1 + (1/2)e^{-\lambda})}{1 - 8Q^2/3}
\]

\[
\lim_{\lambda \to \infty} \frac{2}{3 - 8Q^2} \lim_{\lambda \to 0} \frac{2}{3} = 3
\]

where

\[
Q = \frac{B_{0zm}}{B_{0z}(0)}
\]

measures the relative importance of the azimuthal magnetic field compared to the axial field. Again \( p_0(x) \), \( B_{0zm}(0) \), and \( B_{0z}(0) \) cannot be chosen arbitrarily. Once \( p_0 \), \( Q \), and \( \lambda \) are chosen, \( B_{0z}(0) \) is fixed.

The dimensionless pressure and magnetic field now take the form

\[
p_{0*} = \frac{p_0(x)}{B_{0z}(0)/\mu} = \frac{1 - 8Q^2/3}{2[1 - (1/3)(1 + e^{-\lambda})(1 + (1/2)e^{-\lambda})]}
\]

\[
\lim_{\lambda \to \infty} \frac{2}{3 - 8Q^2} \lim_{\lambda \to 0} \frac{2}{3} = 3
\]

\[
B_{0z*}(x) = -4Q(x - 1)
\]

\[
B_{0zm}(x) = 2[p_{0*} - p_0(x)]
\]

\[
+ Q^2 \left( \frac{8}{3} - 32x^2 + \frac{160}{3} x^3 - 24x^4 \right)^{1/2}
\]

The observations (Lites & Skumanich 1990) indicate that \( B_{0zm}(0) \) is about 200 to 300 G, at least at the photosphere. We therefore set \( Q = B_{0zm}/B_{0z}(0) = 0.1 \). The effect of the azimuthal magnetic field on the equilibrium structure can easily be retraced. Since \( B_{0z} \) enters through terms in \( Q^2 \) this effect is relatively small.

Once values for \( p_{0*} \), \( Q \), and \( \lambda \) are chosen, the dimensionless equilibrium is completely determined. The dimensionless formulation of resonant absorption then contains the same four dimensionless parameters as in the straight field case, namely \( \omega_A, K_a = KR, k_a = kR \), and \( m \). As before we use equation (12) to solve for \( k_a = kR \) for given \( \omega_A \) and \( K_a \). We then compute the square of the dimensionless Alfvén frequency as

\[
\omega_A^2(x) = \left[ k_a B_{0zm}(x) + \frac{m}{x} B_{0zm}(x) \right]^2 / \rho_{0*}(x).
\]

Again for a resonance to occur inequalities (eq. [36]) must be satisfied. But now the azimuthal magnetic field changes the profile of \( \omega_A^2(x) \) for nonaxisymmetric waves \( m \neq 0 \), and resonant absorption will occur at different locations compared to the straight field case as illustrated in Figure 7 where we have plotted \( \omega_A^2(x) \) versus \( x \) for \( m = 0, \pm 1, \pm 2, \pm 3, \) and \( \pm 5 \). For \( m = 0 \) the profile of \( \omega_A^2(x) \) is only indirectly changed as the longitudinal magnetic field is slightly changed by including the azimuthal field in the equilibrium, but more importantly—in contrast to the straight field case—there is now resonant absorption of axisymmetric waves.

Part of our results are presented in Figures 8–10. As in Figures 1–5 these results are again for a sunspot with \( B_{0z}(0) = 2000 \) G and \( \lambda = 5 \). In addition to the axial magnetic field there is now an azimuthal field with \( B_{0zm} = 200 \) G \( (Q = 0.1) \). On each figure results are shown for waves with azimuthal wave number \( m = 0, \pm 1, \pm 2, \pm 3, \) and \( \pm 5 \). In contrast to the straight field case there is now absorption for \( m = 0 \), and also the absorption for waves with opposite azimuthal wave numbers \( \pm m \) is different.

On Figure 8 the variation of the absorption coefficient in terms of the radius of the spot \( R \) is displayed. The horizontal wave number \( K \) and the frequency \( \omega \) are kept constant at \( 1 \times 10^{-6} m^{-1} \) and \( 0.02 \text{ rad s}^{-1} \) respectively. Figure 8 has to be compared with its straight field twin Figure 2. Apparently, the
effect of the azimuthal magnetic field on the absorption is different for different \( m \) values. For \( m = 1 \) the effect of a nonzero \( B_0 \), is an overall reduction of the absorption coefficient for \( R \) varying from \( 4.2 \times 10^6 \) m up to \( 8.4 \times 10^6 \) m. The reduction is substantial and amounts to more than 50\% for \( R = 8.4 \times 10^6 \) m. In contrast to Figure 2, the variation of the absorption coefficient for \( m = 1 \) in terms of the radius of the sunspot is now monotonically decreasing at least in the interval \( 4.2 \times 10^6 \) m \(< R \leq 8.4 \times 10^6 \) m. It might be that the initially increasing behavior of the absorption coefficient in terms of \( R \) which is present in Figure 2 is also present here but for radii substantially smaller than \( 4.2 \times 10^6 \) m. For \( m = 2, 3 \) and 5 the azimuthal magnetic field leads to a substantial increase in absorption, and we get absorption coefficients up to 60\%. For these wave numbers the behavior of the absorption coefficients in terms of the radius is very similar to that depicted in Figure 2. As \( R \) is increased the absorption coefficient first increases to a maximum and then steadily decreases. Again, as for a straight magnetic field, larger sunspots are better absorbers of \( p \)-mode oscillations with higher azimuthal wave numbers than smaller spots and are on the whole better absorbers. This is in agreement with recent observations by Braun & Duvall (1990) who found that acoustic waves are more efficiently absorbed in giant spots than in medium sized spots. The absorption efficiency for waves with negative azimuthal wave numbers \( m < 0 \) differs now from that for positive \( m \)-values. There is substantial absorption for \( m = -1 \), and \( m = -2 \), little absorption for \( m = -3 \), and no absorption for \( m = -5 \). The reason for the absence of absorption for \( m = -5 \) is that the frequency of the considered \( p \)-mode oscillation is outside the Alfvén continuum. In other words, condition (36) is not satisfied for \( m = -5 \) and the considered frequency as is illustrated in Figure 7. Notice that the absorption for \( m = -1 \) is, in general, larger than that for \( m = 1 \) and that there is also an \( R \)-interval in which the absorption for \( m = -2 \) beats that for \( m = 1 \). As anticipated, axisymmetric modes (\( m = 0 \)) are now also absorbed although the absorption is always modest, 10\% or smaller. On the whole, the absorption is larger in the case of a curved magnetic field than in the straight field case.

On Figures 9a and 9b the variation of the absorption coeffi-
Let us now comment on the solutions of the driven oscillation in the sunspot. In Figure 11 we have plotted the normal component $v_1$, the perpendicular component $v_\perp$ and the parallel component $v_\parallel$ of the velocity (Fig. 11a) and the Eulerian perturbation of total pressure (Fig. 11b). Figure 11 corresponds to Figure 6, but now there is an azimuthal magnetic field with $Q = 0.1$. The ideal singularity is slightly shifted and is now at $x = x_A = 0.41$. Again the perpendicular component $v_\parallel = v_2$ dominates and has the same behavior as in Figure 6. The other components are an order of magnitude smaller and now have traces of the ideal $\ln |x - x_A|$ singularity and the ideal jump. The Eulerian perturbation of pressure now also shows traces of the ideal $\ln |x - x_A|$ singularity and the ideal jump. Nevertheless these traces are only weakly present in $v_\parallel$ and $P$ since the azimuthal magnetic field is weak compared to the axial magnetic field. This again confirms the analytic prediction by Sakurai et al. (1991a) that the $\ln |x - x_A|$ singularity and the jump are proportional to the ratio $B_x B_z / B^2$ which is small in the present equilibrium.

Figure 11.—Same as Fig. 6, but now for a sunspot with a twisted magnetic field, $Q = 0.1$. 

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The results obtained in the present paper might be compared with those found by Poedts et al. (1990b) although the background and the external driving are different. The most striking difference might be that Poedts et al. (1990b) found cases where all of the wave energy is absorbed, while here a maximum of 60%-70% of incident wave energy is absorbed. The main reason for this difference is that Poedts et al. (1990b) performed a systematic scan of the Alfvén continuum and in doing so found external kink modes giving rise to complete absorption. Here we preferred to take observed values of frequency and reasonable values of $K$ and $R$, and we did not focus on possible quasi-modes which might produce complete absorption. A search for quasi-modes and complete absorption is currently being carried out and has proven to be successful (Poedts, Stenuit, & Goossens 1991).

5. CONCLUSIONS

We have investigated the resonant absorption of $p$-modes by sunspots by means of numerical simulations in linear resistive MHD. We first confirmed Lou's viscous MHD results and provided the first numerical proof of a remarkable feature of resonant absorption, viz., the fact that the amount of power absorbed is independent of the specific dissipation mechanism. In addition, we showed that the amount of absorption is independent of the plasma resistivity $\eta$ in the limit $\eta \to 0$. Next, Lou's results were extended, and the resonant absorption of $p$-mode oscillations by sunspots is investigated by means of a parametric study of the dependence of the absorption coefficient on both the equilibrium quantities and the characteristics of the incident $p$-modes. This extensive parametric study was performed for Lou's equilibrium model and for an extension of this equilibrium with an azimuthal magnetic field component. In this extended equilibrium model, the magnetic field lines are curved, and this breaks down the $\pm m$ symmetry in the absorption and makes the absorption of $m = 0$ modes possible. The overall impression from this parametric survey is that the resonant absorption of $p$-modes is more efficient in larger sunspots with twisted magnetic fields, and that this is particularly so for higher azimuthal wave numbers. Also let us recall that these computations are for $\lambda = 5$ and that acoustic waves with high $m$-values are more efficiently absorbed in an equilibrium with a higher $\lambda$. The decreasing behavior of the absorption coefficient with increasing values of $m$ (Lou 1990) is found not to be a general feature of resonant absorption in sunspots, and the $1/m^2$ dependence for straight fields claimed by Chitre & Davila (1990, 1991) is shown not to exist. As a matter of fact, our results reveal that the variation of the absorption coefficient in terms of the azimuthal wave number largely depends, in a very complicated way, on both the equilibrium quantities and the characteristics of the incident acoustic mode. Let us finally recall that these results are obtained for rather artificial equilibrium models which have fixed ratios of magnetic pressure to external plasma pressure. These results should not be overinterpreted, but they do show that resonant absorption is an efficient mechanism for absorbing $p$-mode oscillations which depends in a very complicated way on the equilibrium and the characteristics of the $p$-modes.

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