MAGNETIC FLUX TUBES IN A 3D CHAOTIC FLOW FIELD

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ABSTRACT We argue that if the magnetic fields in the solar interior are of a fibril (flux tube) form, the field strength can be amplified by fluid motions up to a limiting value $B_{as}$, which is determined by a balance between magnetic tension and aerodynamic drag. We find $B_{as} \simeq 3.7 \rho^{2/3} l^{2/3} v_i^{4/3} \Phi^{-1/3}$, where $\rho$ is the mass density, $l$ is the size scale of the most vigorous motions, $v_i$ is the velocity amplitude associated with this size scale, and $\Phi$ is the magnetic flux per fibril. We discuss implications of this limit for our understanding of the solar cycle.

Keywords: solar cycle; dynamo; magnetic flux tubes; activity

INTRODUCTION

An important component of any solar or stellar dynamo model is the amplification of magnetic fields through the stretching of magnetic field lines. In kinematic models of dynamos, the flow fields that drive field amplification are specified a priori; it is assumed that the magnetic fields can be carried by the flow without resistance. However, as the field strength increases, there is an increasingly important back reaction by the Lorentz force which acts to resist the fluid motions. This force is ignored in kinematic models. Under the assumption that magnetic fields are concentrated into thin flux tubes surrounded by field-free plasma, we investigate the role of magnetic tension in limiting the growth of magnetic energy.

In order to carry out this investigation, we have developed a numerical algorithm for the solution of the equation of motion of a single, closed magnetic flux tube moving in 3 dimensions. Our algorithm is based on solving the momentum equation in a novel flux tube coordinate system in which the mesh is spaced uniformly along the tube. The form of the momentum equation solved is

$$2\rho \frac{dv_{\perp}}{dt} = \frac{B^2}{\pi} \frac{\partial^2 r}{\partial s^2} + C_D \rho \left( \frac{v_{f\perp} - v_{\perp}}{\sqrt{\pi} \Phi / B} \right),$$

where $v_{\perp}$ is the velocity of plasma in the flux tube in the directions normal to the flux tube orientation itself, $v_{f\perp}$ is the velocity of the fluid surrounding the flux tube in the normal directions, $B$ is the magnetic field strength, $C_D$ is the aerodynamic drag coefficient (assumed to be unity), and $\Phi$ is the magnetic flux.
in the tube. Note that in this study we have neglected the buoyancy force (but see Fisher, McClymont and Chou 1991 for the effects of field amplification on flux tube equilibria near the base of the convection zone of stellar interiors). The vector \( r(s) \) describes the space curve delineated by the tube, where \( s \) is distance along the tube measured from some reference point. The magnetic field strength \( B \) is related to the total length \( L \) of the closed flux tube through the equation \( B/L = B_0/L_0 \), where \( B_0 \) and \( L_0 \) are the initial values. Further details of the numerical code will be described in Fisher, Patten and DeLuca (1992).

The flow field \( \mathbf{v}_f \) in which the flux tube is embedded is an "ABC" or Beltrami flow (see e.g. Gilbert 1991). The components of \( \mathbf{v}_f \) are given by

\[
\begin{align*}
\mathbf{v}_{fx} &= v_l \left[ B \sin(y/l) + C \cos(z/l) \right], \quad (2a) \\
\mathbf{v}_{fy} &= v_l \left[ C \cos(z/l) + A \sin(x/l) \right], \quad (2b) \\
\mathbf{v}_{fz} &= v_l \left[ A \cos(x/l) + B \sin(y/l) \right], \quad (2c)
\end{align*}
\]

where we have assumed \( A = B = C = 1 \).

**SYNOPSIS OF SIMULATIONS**

We have performed several simulations of an initially circular flux ring embedded in the flow described above. Important characteristics of the ABC flow are: (1) the fluid streamlines are generally not closed; (2) the flow has a characteristic scale size \( l \) and velocity amplitude \( v_l \). The motion of the flux ring is governed by a competition between the magnetic tension force, whose amplitude depends both on the magnetic field strength and the radius of curvature of the field lines, and the aerodynamic drag force, whose amplitude is proportional to the square of the relative normal velocity of the tube with respect to the fluid.

Initially, the ring is stretched and distorted by fluid motions, but in so doing, the magnetic field strength of the ring increases, causing the tension force to increase more quickly than does the drag force. Eventually, an asymptotic solution is reached wherein the tension and drag forces balance. This behavior is similar to what Parker (1982a,b) found in his investigation of the motion of magnetic fibrils in steady 2-D flows and convective rolls.

The field strength achieved by the asymptotic solution can be understood by the following simple argument: The tightest bends in the asymptotic flux tube shape are presumably places where the flux tube is withstanding the full brunt of the flow field against it; the amplitude of the drag force here must be \( F_d \approx C_D \rho v_l^2/(\pi \Phi /B)^{1/2} \), where \( \Phi \) is the magnetic flux in the tube and \( C_D \) is the drag coefficient (assumed to be unity). On the other hand, the amplitude of the magnetic tension force in the tight bends must be \( F_t \approx B^2/(4\pi l) \), where we have estimated the radius of curvature of the tube to be the characteristic length scale in the flow field. Equating the two forces and solving for asymptotic field strength \( B_{as} \) yields

\[
B_{as} \approx \frac{(4\pi C_D/\sqrt{\pi})^{2/3} \rho^{2/3} l^{2/3} v_l^{4/3}}{\Phi^{1/3}} \quad (3)
\]

where \((4\pi C_D/\sqrt{\pi})^{2/3} \approx 3.7\). Inserting values of the flux tube parameters from the simulation (\( \Phi \approx 10^{18} \ Mx \), \( \rho \approx 0.2 \ g \ cm^{-3} \), \( l = 10^{10} \ cm \), and \( v_l = 10^4 \ cm \ s^{-1} \)) yields
$B_{as} \approx 1.3 \times 10^6$ G, which compares favorably to the value of $B_{as}$ actually reached in the simulation of $1.46 \times 10^6$ G. Other simulations with the same flux tube parameters, but a different starting position for the ring results in somewhat different equilibrium tube shapes, and values of $B_{as}$ which differ to some degree, but the asymptotic field strength seems to be given roughly by equation (3). Values of the magnetic field strength $B_{as}$ are in general much greater than the equipartition value, $B_{eq} \approx (4\pi \rho v^2)^{1/2}$. Using the above values of the parameters, we find that $B_{as}$ is roughly 50 times larger than $B_{eq}$.

**IMPLICATIONS FOR THE SOLAR CYCLE**

(1) Equation (3) shows that fluid motions on the largest spatial scales are most important; small scale motions are unable to move the tube because the tension forces resist most strongly at small scales and because there is less fluid energy at small scales.

(2) It is a simple matter to get field strengths far in excess of their equipartition value.

(3) Recent reconnection studies (e.g. DeLuca and Craig, 1992) suggest that in a collision between two flux tubes, reconnection resulting in a new flux tube configuration can happen on a time scale short compared to other dynamic time scales. This suggests that changes in magnetic topology required by a dynamo model can be accomplished easily using a flux tube paradigm.

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**REFERENCES**